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Massless Limit for Spin 2

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Abstract

In these notes we use the construction of a field theory for the spin-2 fields, using an extended auxiliary-field formalism. In order to impose sufficient constraints on the $h^{\mu\nu}$-fields one vector auxiliary field $\eta^{\mu}(x)$, and a scalar field $\epsilon(x)$ was needed. Here, we solve the field equation for the one-particle states, i.e. we solve the inhomogeneous Klein-Gordon equation for $h^{\mu\nu}(x)$ and study the massles limit for its solutions. Using these solutions, we derive again the one-particle propagator, both for the massive and the massless case.

We found that by choosing the constants suitably, and performing a couple of gauge transformations, we can eliminate the unwanted helicity components in the massless limit. Thereby we arrive at a satisfactorily massless spin-2 theory.

I. INTRODUCTION

In these notes we use the construction of a field theory for the spin-2 fields, using an extended auxiliary-field formalism [1, 2]. In order to impose sufficient constraints on the $h^{\mu\nu}$-fields one vector auxiliary field $\eta^\mu(x)$, and a scalar field $\epsilon(x)$ was needed. Here, we solve the field equation for the one-particle states, i.e. we solve the inhomogeneous Klein-Gordon equation for $h^{\mu\nu}(x)$ and study the massles limit for its solutions. Using these solutions, we derive again the one-particle propagator, both for the massive and the massless case.
The result is that we can indeed eliminate the helicities $\lambda = 0, \pm 1$ from the tensor-field $h^{\mu\nu}$, in the massless limit.

In our work on the quantization of the spin-2 fields [2], we used the symmetric $h_{\mu\nu}$-tensor field, and the two auxiliary fields $\eta^\mu(x)$ and $\epsilon(x)$, with the Lagrangian

$$\mathcal{L}_{\eta\epsilon} = \mathcal{L}_2 + M_2 \partial_\mu h^{\mu\nu} \eta_\nu + M_2^2 H^\mu_\mu \epsilon + \frac{1}{2} b M_2^2 \eta^\mu \eta_\mu \ , \tag{1.1}$$

with

$$\begin{aligned}\mathcal{L}_2 = & \frac{1}{4} \partial^\alpha h^{\mu\nu} \partial_\alpha h_{\mu\nu} - \frac{1}{2} \partial^\mu h^{\mu\nu} \partial^\alpha h_{\alpha\nu} - \frac{1}{4} \partial_\nu h^\beta_\beta \partial^\nu h^\alpha_\alpha + \frac{1}{2} \partial_\alpha h^{\alpha\beta} \partial_\beta h^\nu_\nu \\ & - \frac{1}{4} M_2^2 h^{\mu\nu} h_{\mu\nu} + \frac{1}{4} M_2^2 h^\mu_\mu h^\nu_\nu \ .\end{aligned} \tag{1.2}$$

In the following, we denote the mass by $M_2 \equiv M$.
For the normalization of our solutions, the commutation relations of the field operators are important. Using the Dirac quantization method, and using a vector and a scalar auxiliary field, the obtained field commutators read

$$[\epsilon(x), \epsilon(y)] = -\frac{3}{8} \frac{b(1-b)^2}{(3+b)^3} \ i\Delta(x-y; M_\epsilon^2) \ , \tag{1.3a}$$

$$[\eta^\mu(x), \epsilon(y)(y)] = -\frac{3}{4} \frac{(1-b)}{(3+b)^2} \frac{1}{M} \ \partial^\mu i\Delta(x-y; M_\epsilon^2) \ , \tag{1.3b}$$

$$\begin{aligned}[\eta^\mu(x), \eta^\nu(y)] = & \frac{1}{2} \left[g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{bM^2} \right] i\Delta(x-y; M_\eta^2) \\ & + \frac{3}{2b(3+b)} \frac{\partial^\mu \partial^\nu}{M^2} \ i\Delta(x-y; M_\epsilon^2) \ ,\end{aligned} \tag{1.3c}$$

$$\left[h^{\mu\nu}(x), h^{\alpha\beta}(y)\right] = \left[\frac{1}{2} \left(g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} \right) - \frac{1}{3} g^{\mu\nu} g^{\alpha\beta} + \ldots \right] i\Delta(x-y; M^2) \ , \tag{1.3d}$$

$$- \left[\frac{1}{6} \frac{b}{3+b} g^{\mu\nu} g^{\alpha\beta} + \ldots \right] i\Delta(x-y; M_\epsilon^2) \ , \tag{1.3e}$$

and commutators between $h^{\mu\nu}$ and $\epsilon(y), \eta^\mu(y)$. The dots in the square brackets above denote terms with $\partial^\mu, ..., \partial^\beta$. These are unimportant since we couple the spin-2 field to a conserved energy-momentum tensor $T^{\mu\nu}$. The masses M_η and M_ϵ are given in [2] in terms of M_2 and the b-parameter, see furtheron in this paper.
It is one of the aims of this investigation to find a theory which allows (i) a smooth massless limit, and (ii) a perturbation expansion in the small mass M. For the latter to be meaningful, it is necessary that the theory satisfies the following requirements: (i) no-ghosts, (ii)

unitarity, and (iii) a correct massless limit. This would open the possibility of giving a small mass to the graviton without destroying e.g. the correct prediction for the perihelium of Mercury.

The contents of this paper is as follows. In section II we give the coupled Klein-Gordon equations for the spin-2, the spin-1, and spin-0 one-particle wave-functions. We give the explicit form of the helicity wave-functions in momentum-space in terms of the helicity polarization tensor for spin-2 and vector for spin-1. In section III the spin-2 polarization vectors are studied, in particularly the the massless limits of these helicity polarizations is described. In section IV the massless limit of the wave-functions studied. Analyzed are the conditions on the parameters for the vanishing or decoupling of the "wrong" helicity components. Section V is devoted to the question whether representation for the spin-2 propagator etc. can be found that allows a smooth massless limit, such that $M \neq 0$ the theory contains besides the spin-2 propagator also a physical acceptable spin-0 propagator. Finally, we finish this paper by some conclusions in section VI. Appendix A contains some miscelaneous note, and in appendix B we analyse the use of gauge symmetry for the removal of dipole-ghost terms.

II. ONE-PARTICLE SOLUTION

The one-particle wave-functions, corresponding to the $h^{\mu\nu}$-, η^μ-, and ϵ-fields satisfy the following Klein-Gordon equations [1]:

$$\left(\Box + M^2\right)\ h^{\mu\nu}(x) = -\mathcal{M}(1+b)\left(\partial^\mu\eta^\nu - \partial^\nu\eta^\mu\right) + 2\mathcal{M}^2\frac{1+b}{1-b}\ g^{\mu\nu}\ \epsilon(x)\ , \tag{2.1a}$$

$$\left(\Box - bM^2\right)\ \eta^\mu(x) = -2\mathcal{M}\frac{1+b}{1-b}\ \partial^\mu\epsilon(x)\ , \tag{2.1b}$$

$$\left(\Box - \frac{2b}{3+b}M^2\right)\ \epsilon(x) = 0\ . \tag{2.1c}$$

with the constraint

$$\partial\cdot\eta(x)\ =\ \frac{4}{1-b}\ \mathcal{M}\ \epsilon(x)\ . \tag{2.2}$$

For the following, we introduce the short-hand notations

$$M_\eta^2 = -bM^2\ \ ,\ \ M_\epsilon^2 = -\frac{2b}{3+b}M^2\ . \tag{2.3}$$

Working in Fourier space, and using the spectral representations for the vector and scalar fields,

$$h^{\mu\nu}(x)\ =\ \sum_\lambda\sum_i\int\frac{d^4p}{(2\pi)^4}\ \widetilde{h}_i^{\mu\nu}(p,\lambda)\ \delta\left(p^2 - M_i^2\right)\ e^{-ip\cdot x}\ , \tag{2.4a}$$

$$\eta^\mu(x)\ =\ \sum_\lambda\sum_i\int\frac{d^4p}{(2\pi)^4}\ \widetilde{\eta}_i^\mu(p,\lambda)\ \delta\left(p^2 - M_i^2\right)\ e^{-ip\cdot x}\ , \tag{2.4b}$$

$$\epsilon(x)\ =\ \int\frac{d^4p}{(2\pi)^4}\ \widetilde{\epsilon}(p)\ \delta\left(p^2 - M_\epsilon^2\right)\ e^{-ip\cdot x}\ , \tag{2.4c}$$

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[1] An alternative is to use a parameter $\beta \equiv 1/b$. So, later the point $b = \pm\infty \leftrightarrow \beta = 0$.

Then, we obtain the following solutions for the vector and scalar one particle solutions, i.e. plane wave solutions,

$$\widetilde{\epsilon}(p) = a_\epsilon\ \delta\left(p^2 - M_\epsilon^2\right)\ , \tag{2.5a}$$

$$\begin{aligned}\widetilde{\eta}^\mu(p,\lambda) &= a_\eta\ \varepsilon^\mu(p,\lambda)\delta\left(p^2 - M_\eta^2\right)\\ &\quad -2ia_\epsilon\frac{1+b}{1-b}\frac{\mathcal{M}p^\mu}{M_\epsilon^2 - M_\eta^2}\delta\left(p^2 - M_\epsilon^2\right)\ .\end{aligned} \tag{2.5b}$$

Here, $\varepsilon^\mu(p,\lambda)$ are the spin-1 polarization vectors. In the following we consider momenta $p^\mu = (E_p, 0, 0, p)$ with $p^2 = M^2$ and the standard corresponding spin-1 polarization vectors as

$$\epsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}}\left(0, \mp 1, -i, 0\right)\quad ,\quad \epsilon^\mu(0) = \frac{1}{M}\left(p, 0, 0, E_p\right)\ , \tag{2.6}$$

and it is obvious that $p\cdot\epsilon(p,\lambda) = 0$.

Now we check that these special solutions satisfy the constraints (2.2). We find that

$$\begin{aligned}-ip\cdot\eta &= -2a_\epsilon\frac{1+b}{1-b}\frac{M_\epsilon^2}{M_\epsilon^2 + bM^2}\mathcal{M}\delta(p^2 - M_\epsilon^2)\\ &= -2a_\epsilon\frac{1+b}{1-b}\left[1 - b\frac{3+b}{2b}\right]\mathcal{M}\delta(p^2 - M_\epsilon^2)\\ &= +4a_\epsilon\frac{1}{1-b}\mathcal{M}\delta(p^2 - M_\epsilon^2)\ = \frac{4}{1-b}\mathcal{M}\ \tilde{\epsilon}(p)\quad \text{(Q.E.D.)}\end{aligned}$$

Then, the special solutions for the $h^{\mu\nu}(x)$ satisfy

$$\begin{aligned}&\left(-p^2 + M^2\right)\widetilde{h}^{\mu\nu}(p,\lambda) = +ia_\eta(1+b)\mathcal{M}\left(p^\mu\varepsilon^\nu + p^\nu\varepsilon^\mu\right)\delta\left(p^2 - M_\eta^2\right)\\ &+4a_\epsilon\frac{(1+b)^2}{1-b}\mathcal{M}^2\frac{p^\mu p^\nu}{M_\epsilon^2 - M_\eta^2}\delta\left(p^2 - M_\epsilon^2\right)\\ &+2a_\epsilon\frac{1+b}{1-b}\ g^{\mu\nu}\ \mathcal{M}^2\delta\left(p^2 - M_\epsilon^2\right)\\ =\ &+ia_\eta(1+b)\mathcal{M}\left(p^\mu\varepsilon^\nu(p,\lambda) + p^\nu\varepsilon^\mu(p,\lambda)\right)\delta\left(p^2 - M_\eta^2\right)\\ &+2a_\epsilon\frac{1+b}{1-b}\left\{g^{\mu\nu} + 2\frac{3+b}{b}\frac{p^\mu p^\nu}{M^2}\right\}\ \mathcal{M}^2\delta\left(p^2 - M_\epsilon^2\right)\ .\end{aligned} \tag{2.7}$$

So, for the special solution we get

$$\begin{aligned}&\widetilde{h}^{\mu\nu}(p,\lambda) = +ia_\eta\frac{\mathcal{M}}{M^2}\left(p^\mu\varepsilon^\nu(p,\lambda) + p^\nu\varepsilon^\mu(p,\lambda)\right)\delta\left(p^2 - M_\eta^2\right)\\ &+\frac{2}{3}a_\epsilon\frac{3+b}{1-b}\left\{g^{\mu\nu} + 2\frac{3+b}{b}\ \frac{p^\mu p^\nu}{M^2}\right\}\ \frac{\mathcal{M}^2}{M^2}\delta\left(p^2 - M_\epsilon^2\right)\ .\end{aligned} \tag{2.8}$$

III. SPIN-2 POLARIZATION VECTORS

We work with the 4-momenta of the form $p^\mu = (p^0, 0, 0, p)$, for which we choose the spin-1 polarization vectors in the standard form:

$$\epsilon^\mu(\pm 1) \;=\; \frac{1}{\sqrt{2}}\,(0, \mp 1, -i, 0) \quad , \quad \epsilon^\mu(0) = \frac{1}{M}\,(p, 0, 0, E_p) \ , \tag{3.1}$$

Then, the spin-2 polarization vectors are, up to a gauge transformation,

$$\varepsilon^{\mu\nu}(p, \lambda = +2) \;=\; \varepsilon^\mu(p, +1)\ \varepsilon^\nu(p, +1) \ , \tag{3.2a}$$

$$\varepsilon^{\mu\nu}(p, \lambda = +1) \;=\; \frac{1}{\sqrt{2}}\left(\varepsilon^\mu(p, +1)\ \varepsilon^\nu(p, 0) + \varepsilon^\mu(p, 0)\ \varepsilon^\nu(p, +1)\right) \ , \tag{3.2b}$$

$$\varepsilon^{\mu\nu}(p, \lambda = 0) \;=\; \frac{1}{\sqrt{6}}\left(2\varepsilon^\mu(p, 0)\ \varepsilon^\nu(p, 0) + \varepsilon^\mu(p, +1)\ \varepsilon^\nu(p, -1) + \varepsilon^\mu(p, -1)\ \varepsilon^\nu(p, +1)\right) , \tag{3.2c}$$

and similarly for $\lambda = -1, -2$.

<u>**The massless limit polarization vectors**</u>: We first note that

$$\varepsilon^\mu(p, \lambda = 0) \;=\; \frac{p^\mu}{M} + \left(-\frac{M}{2p}, 0, 0, \frac{M}{2p}\right) + O(M^3) \tag{3.3}$$

Then, we have that

$$\varepsilon^{\mu\nu}(p, \lambda = +1) \;\sim\; \frac{1}{\sqrt{2}M}\left(\varepsilon^\mu(p, +1)\ p^\nu + p^\mu(p, 0)\ \varepsilon^\nu(p, +1)\right) \ , \tag{3.4}$$

$$\varepsilon^{\mu\nu}(p, \lambda = 0) \;\sim\; \sqrt{\frac{2}{3}}\frac{p^\mu p^\nu}{M^2} + \frac{1}{\sqrt{6}}\left(g^{\mu\nu} - \frac{p^\mu \tilde{p}^\nu + \tilde{p}^\mu p^\nu}{2p\cdot\tilde{p}}\right) \ , \tag{3.5}$$

where we introduced $\tilde{p}^\mu = (p^0, -\mathbf{p})$, and used the identity

$$\varepsilon^\mu(p, +1)\ \varepsilon^\nu(p, -1) + \varepsilon^\mu(p, -1)\ \varepsilon^\nu(p, +1) = g^{\mu\nu} - \frac{p^\mu \tilde{p}^\nu + \tilde{p}^\mu p^\nu}{2p\cdot\tilde{p}} \ . \tag{3.6}$$

IV. THE MASSLESS LIMITS

We first add to the special solution for $h^{\mu\nu}(x)$ the solution of the homogeneous equation:

$$\begin{aligned}\widetilde{h}^{\mu\nu}(p, \lambda) \;\Rightarrow\; & \varepsilon^{\mu\nu}(p, \lambda)\ \delta\left(p^2 - M^2\right) + \\ & + i a_\eta \frac{\mathcal{M}}{M^2}\left(p^\mu \varepsilon^\nu(p, \lambda) + p^\nu \varepsilon^\mu(p, \lambda)\right)\delta\left(p^2 - M_\eta^2\right) \\ & + \frac{2}{3}a_\epsilon \frac{3+b}{1-b}\left\{g^{\mu\nu} + 2\frac{3+b}{b}\ \frac{p^\mu p^\nu}{M^2}\right\}\ \frac{\mathcal{M}^2}{M^2}\delta\left(p^2 - M_\epsilon^2\right) \ .\end{aligned} \tag{4.1}$$

Now, we want to analyze under what conditions on the parameters and gauges the components of $\widetilde{h}^{\mu\nu}(p, \lambda)$ vanish for the helicities $\lambda = 0, \pm 1$ in the limit $M \to 0$.

(i) In order to fix the scale mass $\mathcal{M}$ the commutator $[\eta^\mu(x), \eta^\nu(y)]$ in equation (1.3e) gives

$$a_\eta^2 = -1/2 \ \rightarrow (M/\mathcal{M})^2 = 1 \ , \tag{4.2}$$

from which we choose $\mathcal{M} = M$.

(ii) $\underline{\lambda = +1}$: In this case we have that

$$\begin{aligned} \widetilde{h}^{\mu\nu}(p,+1) &= \left[\varepsilon^{\mu\nu}(p,+1) + i a_\eta \frac{\mathcal{M}}{M^2}\left(p^\mu \varepsilon^\nu(p,+1) + p^\nu \varepsilon^\mu(p,+1)\right)\right]\delta(p^2) \\ &\rightarrow \frac{1}{\sqrt{2}M}\left(p^\mu \varepsilon^\nu(p,+1) + p^\nu \varepsilon^\mu(p,+1)\right) + i a_\eta \frac{\mathcal{M}}{M^2}\left(p^\mu \varepsilon^\nu(p,+1) + p^\nu \varepsilon^\mu(p,+1)\right) \Rightarrow 0 \end{aligned} \tag{4.3}$$

and we find , for $\mathcal{M} = M$, the condition

$$a_\eta = \frac{i}{\sqrt{2}}\frac{M}{\mathcal{M}} \ . \tag{4.4}$$

(iii) $\underline{\lambda = +0}$: Similarly, in this case we have that

$$\begin{aligned} \widetilde{h}^{\mu\nu}(p,0) \ \rightarrow & \ \sqrt{\frac{2}{3}}\frac{p^\mu p^\nu}{M^2} + \frac{1}{\sqrt{6}}\left(g^{\mu\nu} - \frac{p^\mu \tilde{p}^\nu + \tilde{p}^\mu p^\nu}{2p\cdot\tilde{p}}\right) \\ & + i a_\eta \frac{\mathcal{M}}{M}\frac{p^\mu p^\nu}{M^2} + \frac{2}{3} a_\epsilon \frac{3+b}{1-b}\left\{g^{\mu\nu} + 2\frac{3+b}{b}\frac{p^\mu p^\nu}{M^2}\right\}\frac{\mathcal{M}^2}{M^2} \Rightarrow 0 \ . \end{aligned} \tag{4.5}$$

Now, since we couple the $h^{\mu\nu}(x)$-field always to a conserved energy-momentum tensor $T^{\mu\nu}$, with $\partial_\mu T^{\mu\nu} = 0$ and $T^{\mu\nu} = T^{\nu\mu}$, the terms proportional to the four-momentum p^μ are not important. In appendix B paragraph we will demonstrate moreover that these terms can be 'gauged away'. However, in order to arrive at a decent massless spin-2 theory we must cancel the $g^{\mu\nu}$-terms. This leads to the condition, taking again $\mathcal{M} = M$,

$$\frac{1}{\sqrt{6}} + \frac{2}{3}\frac{3+b}{1-b}\frac{\mathcal{M}^2}{M^2} a_\epsilon = 0 \ . \tag{4.6}$$

A solution for this is $b = \pm\infty$ and $a_\epsilon = \sqrt{3/8}\cdot M^2/\mathcal{M}^2$.

(iv) Now, in order to exclude other solutions we look at the commutator for the scalar $\epsilon(x)$-field. This reads

$$[\epsilon(x), \epsilon(y)] \ = \ -\frac{3}{8}\frac{b(1-b)^2}{(3+b)^3}\ i\Delta(x-y; M_\epsilon^2) \sim -a_\epsilon^2\ i\Delta(x-y; M_\epsilon^2) \ . \tag{4.7}$$

Since $M_\epsilon^2 < 0$ the scalar field is ghost-like, so that there is a $(-)$-sign in the commutator for the annihilation and creation operators. This justifies the last relation in (4.7). Comparing (4.7) with the solution from (4.6) one obtains the relation

$$a_\epsilon^2 = \frac{3}{8}\frac{(1-b)^2}{(3+b)^2}\frac{M^4}{\mathcal{M}^4} = \frac{3}{8}\frac{b(1-b)^2}{(3+b)^3} \Rightarrow \frac{b}{b+3} = 1 \ , \tag{4.8}$$

and therefore $b = \pm\infty$ (Q.E.D.).

We close this section by noting that indeed for $b = \pm\infty$ the commutator in (1.3e) for $h^{\mu\nu}$ in essence becomes properly that for a massless spin-2 theory if we take $M, M_\epsilon \rightarrow 0$.

V. SMOOTH LIMIT AND THE BRANS-DICKE THEORY

In the previous section we studied the double limit: $M \to 0; b \to \infty$. It is the purpose of this section to find a representation of the propagator for the $h^{\mu\nu}$-field which allows a smooth massless limit and such that for $M \neq 0$ the theory contains besides a spin-2 propagator, also a physically acceptable spin-0 propagator. In other words a so-called Brans-Dicke type of theory.

From the commutators in (1.3e) we obtain the propagator by the replacement

$$\Delta(x-y;M^2) \to \Delta_F(x-y;M^2) \ . \tag{5.1}$$

Then, we have apart from irrelevant terms for the $h^{\mu\nu}$-field the Feynman propagator

$$\begin{aligned} D_F^{\mu\nu,\alpha\beta}(x-y) \ &= \ \left[\frac{1}{2}\left(g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha}\right)-\frac{1}{3}g^{\mu\nu}g^{\alpha\beta}\right] \ \Delta_F(x-y;M^2) \\ &-\frac{1}{6}\frac{b}{3+b}g^{\mu\nu}g^{\alpha\beta} \ \Delta_F(x-y;-\frac{2b}{3+b}M^2) \ . \end{aligned} \tag{5.2}$$

We note that in this propagator for the range $-3 < b < 0$ the first term describes a physical massive spin-2 particle, and the second term describes a physical massive spin-0 particle. Rewriting this propagator as

$$\begin{aligned} D_F^{\mu\nu,\alpha\beta}(x-y) \ &= \ \left[\frac{1}{2}\left(g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha}\right)-\frac{1}{3}g^{\mu\nu}g^{\alpha\beta}\right] \ \Delta_F(x-y;M^2) \\ &+\left[-\frac{1}{6}+\frac{1}{2}\frac{1}{3+b}\right] \ g^{\mu\nu}g^{\alpha\beta} \ \Delta_F(x-y;-\frac{2b}{3+b}M^2) \ , \end{aligned} \tag{5.3}$$

and keeping b in the above range, we find in the massless limit that the propagator describes a massless spin-2 and a massless spin-0, giving a Brans-Dicke type of theory.

It is one of the aims of this investigation to find a theory which allows (i) a smooth massless limit, and (ii) a perturbation expansion in the small mass M. For the latter to be meaningful, it is necessary that the theory satisfies the following requirements:

1. No-ghost: $M_\epsilon^2 > 0 \to b/(3+b) < 0$,
2. Unitarity: $b/(3+b) < 0$,
3. Correct massless limit: $b/(3+b) \to +1-\Delta, \quad \Delta > 0$.

Clearly, requirement 3) is in conflict with 1) and 2) if $\Delta = 0$, i.e. for a pure spin-2 theory in the massless limit. So, at best we could end up with is a satisfactory Brans-Dicke theory!

Next we introduce the following parametrization and definition

$$b = \alpha + \beta\frac{\Lambda^2}{M^2} \quad , \quad \kappa(\beta, M^2/\Lambda^2) \equiv \frac{b}{3+b} = \frac{\beta+\alpha\mu^2}{\beta+(3+\alpha)\mu^2} \ , \tag{5.4}$$

where we introduced $\mu^2 = M^2/\Lambda^2$, and which means that $M_\eta \to -\beta\Lambda^2$ in the limit $M \to 0$, which is well defined. Then, we can write

$$\begin{aligned} D_F^{\mu\nu,\alpha\beta}(x-y) &= \left[\frac{1}{2}\left(g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha}\right)-\frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right]\ \Delta_F(x-y;M^2)+ \\ &\quad \frac{1}{6}g^{\mu\nu}g^{\alpha\beta}\left[\Delta_F(x-y;M^2)-\kappa\ \Delta_F(x-y;-2\kappa\ M^2)\right] \\ &\equiv \bar{D}_F^{\mu\nu,\alpha\beta}(x-y,M^2)+\Delta D_F^{\mu\nu,\alpha\beta}(x-y;\kappa,M^2) \end{aligned} \tag{5.5}$$

In passing, we notice that $\kappa \to 1$ in the limit $M \to 0$, independently of the value of the parameter β. Therefore, in the limit $M \to 0$ the extra piece $\Delta D_F^{\mu\nu;\alpha\beta} \to 0$, and we get the proper massless spin-2 propagator, independent of β. In momentum space we have

$$\begin{aligned} \Delta\widetilde{F}_F^{\mu\nu;\alpha\beta}(p) &= \frac{i}{6}g^{\mu\nu}g^{\alpha\beta}\left[\frac{1}{p^2-M^2+i\delta}-\frac{\kappa}{p^2+2\kappa M^2+i\delta}\right] \\ &= \frac{1}{6}(1-\kappa)g^{\mu\nu}g^{\alpha\beta}\frac{p^2+3\kappa M^2/(1-\kappa)}{p^2+2\kappa M^2+i\delta}\cdot\frac{1}{p^2-M^2+i\delta}\ . \end{aligned} \tag{5.6}$$

We note that

$$1-\kappa = 1-\frac{b}{b+3} = \frac{3M^2/\Lambda^2}{\beta+(3+\alpha)M^2/\Lambda^2} \to 0 \ \text{ in the limit } M \to 0\ . \tag{5.7}$$

Therefore, in the limit $M \to 0$ we obtain the proper propagator for the massless spin-2 theory, independent of the parameter β!

For $M \neq 0$, taking $\alpha = 0$, and having β in the 'physical domain': $-3M^2/\Lambda^2 < \beta < 0$, the extra p^2-dependent factor has a pole at $p^2 = -2\kappa M^2 > 0$. So, $\Delta\widetilde{F}_F^{\mu\nu;\alpha\beta}(p)$ presents an acceptable singularity structure, i.e. a non-tachyon like, for representing a physical spin-0 propagator.
However, we can not take the massless limit keeping β fixed in the 'physical domain'! In doing this the spin-0 becomes tachyonic before reaching the massless limit. Therefore, the massive and massless theories are not connected in a satisfactory way, such that we can compute small mass corrections!!

VI. CONCLUSIONS

We found that by choosing the constants suitably, and performing a couple of gauge transformations, we can eliminate the unwanted helicity components in the massless limit. Thereby we arrive at a satisfactorily massless spin-2 theory. This in accordance with the Dirac quantization method for spin-2 fields using auxiliary vector and scalar fields.
However, we did not succeed giving a small mass to the graviton without destroying the correct prediction for the perihelium of Mercury. Therefore, so far in our treatment of the spin-2 field with the Dirac quantization method a discontineous change in the predictions persists, as claimed by Van Dam and Veltman [3, 4].

APPENDIX A: MISCELANEOUS NOTES

The $h^{\mu\nu}$-propagator $D^{\mu\nu;\alpha\beta}$ contains, apart from irrelevant terms proportional to the p^μ-vector, three terms. The first and the second term with respectively $g^{\mu\alpha}g^{\nu\beta}$ and $g^{\mu\beta}g^{\nu\alpha}$ have coefficients $C_1 = C_2 = 1/2$, apart from a factor $1/(p^2 - M^2 + i\delta)$. The third, i.e. the $g^{\mu\nu}g^{\alpha\beta}$-term, in the $h^{\mu\nu}$-propagator has in momentum space the coefficient C_3 with

$$\begin{aligned}
C_3 \quad &= \quad -\left[\frac{1}{3}\frac{1}{p^2 - M^2 + i\delta} + \frac{1}{6}\frac{b}{b+3}\frac{1}{p^2 - M_\epsilon^2 + i\delta}\right] \\
&= \quad -\frac{1}{6}\left(2 + \frac{b}{b+3}\right)\frac{1}{p^2 - M^2 + i\delta} - \frac{1}{6}\frac{b}{b+3}\frac{M_\epsilon^2 - M^2}{(p^2 - M^2 + i\delta)(p^2 - M_\epsilon^2 + i\delta)} \\
&= \quad -\frac{1}{6}\left(2 + \frac{b}{b+3}\right)\frac{1}{p^2 - M^2 + i\delta} + \frac{b(b+1)}{2(b+3)^2}\frac{M^2}{(p^2 - M^2 + i\delta)(p^2 - M_\epsilon^2 + i\delta)} \\
&\overset{M\to 0}{\Longrightarrow} \quad -\frac{1}{6}\left(2 + \frac{b}{b+3}\right)\frac{1}{p^2 + i\delta} \ ,
\end{aligned} \tag{A1}$$

and the coefficient in parentheses $(\ldots)$ tends to the correct value $-1/2$ for $b \to \infty$.

APPENDIX B: GAUGE-SYMMETRY AND THE DIPOLE-GHOST TERM

The gauge transformations are of the form

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + X_{\mu,\nu} + X_{\nu,\mu} \ . \tag{B1}$$

1. We construct now a gauge transformation such that in momentum space

$$\tilde{h}_{\mu\nu}(p) \to \tilde{h}_{\mu\nu}(p) + a\left(\tilde{p}_\mu p_\nu + p_\mu \tilde{p}_\nu\right)/\tilde{p}\cdot p \ . \tag{B2}$$

The function $X(x)$ which corresponds to (B2) is given by

$$X_\mu(x) = ia\int \frac{d^4p}{(2\pi)^4}\ \frac{\tilde{p}_\mu}{\tilde{p}\cdot p}\ \delta\left(p^2 - M^2\right)\ e^{-ip\cdot x} \ , \tag{B3}$$

which is easily seen to give the transformation (B2). For $M^2 = 0$, the function $X_\mu(x)$ satisfies the equation $\Box X_\mu(x) = 0$.

2. The gauge transformation that gives in momentum space

$$\tilde{h}_{\mu\nu}(p) \to \tilde{h}_{\mu\nu}(p) + b\frac{p_\mu p_\nu}{M^2} \ , \tag{B4}$$

is materialized by choosing

$$X_\mu(x) = i\frac{b}{2}\int \frac{d^4p}{(2\pi)^4}\ \frac{p_\mu}{M^2}\ \delta\left(p^2 - M^2\right)\ e^{-ip\cdot x} \ , \tag{B5}$$

which shows that indeed the terms proportional to p^μ in the fields $\widetilde{h}^{\mu\nu}(p,\lambda)$ can be "gauged" away for $M = 0$.

[1] J. W. Wagenaar, *Pion-Nucleon Scattering in Kadyshevsky Formalism and Higher Spin Field Quantization*, Ph.D. thesis University of Nijmegen, July 2009, e-Print: arXiv:0904.1398[nucl-th].
[2] J. W. Wagenaar and T.A. Rijken, *On the Quantization of the Higher Spin Fields*, e-Print: arXiv:0905.3893[hep-th].
[3] H. Van Dam and M. Veltman, Nucl. Phys. **B32**, 397 (1970); G.R.G. **3**, 215 (1972).
[4] D.G. Boulware and S. Deser, Phys. Rev. **D6**, 3368 (1972).