THEF-NIJM 09.01
arXiv:0904.1398

Pion-Nucleon Scattering in Kadyshevsky Formalism

and

Higher Spin Field Quantization

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Higher Spin Field Quantization

een wetenschappelijke proeve op het gebied der
Natuurwetenschappen, Wiskunde en Informatica

Proefschrift

ter verkrijging van de graad van doctor
aan de Radboud Universiteit Nijmegen
op gezag van de Rector Magnificus
prof. mr. S.C.J.J. Kortmann,
volgens besluit van het College van Decanen
in het openbaar te verdedigen op vrijdag 10 juli 2009
om 10.30 uur precies

door

Jan Willem Wagenaar

geboren op 13 maart 1981
te Minnertsga

Promotor:

Prof. dr. R.H.P. Kleiss

Copromotor:

Dr. T.A. Rijken

Manuscriptcommissie:

Prof. dr. S.J. de Jong	
Prof. dr. P.J.G. Mulders	Vrije Universiteit Amsterdam
Prof. dr. E.L.M.P. Laenen	Nikhef/Universiteit van Amsterdam, Universiteit Utrecht
Prof. dr. R.G.E. Timmermans	KVI/Rijksuniversiteit Groningen
Dr. W.J.P. Beenakker	

ISBN 978-90-9024197-5

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πN-Scattering in Kadyshevsky Formalism

Chapter 1

Introduction

When we consider all subatomic particles, excluding the gauge bosons, i.e. the force carrying particles, and the not-yet observed Higgs particle, we divide them into three groups: the leptons, the mesons and the baryons. The particles in the last two groups together are called hadrons. The three names of the groups originate from the Greek words: *leptos*, *mesos* and *barys*, meaning small, intermediate and heavy. These names refer to the mass of the first (and lightest) particles of these groups that were discovered (such as the electron (lepton) and the proton (baryon)). Later, when other members were discovered, these names were not appropriate anymore. For instance the τ-lepton (1777 MeV/c^2) is much heavier then the proton (938 MeV/c^2). Nevertheless, these names are still used for reasons we will soon encounter. This part of the thesis is about the strong interaction between mesons and baryons and in particular pions and nucleons (a nucleon is a proton or a neutron).

The study of strong meson-baryon interactions has a long history that goes back to the year 1935 in which Yukawa predicted the existence of mesons as carriers of the strong nuclear force [1]. After the discovery of the first mesons (the charged pions) by Powell and collaborators in 1947 [2], Yukawa was awarded the Nobel prize in 1949 [1].

A lot of new particles were discovered and the categorization of them led Gell-Mann and Ne'eman to propose their Eightfold way [3]. Collections of particles form representations of a mathematical group: $SU_f(3)$ (f stands for *flavour*) and the elements of the fundamental representation are called *quarks*. It got Gell-mann the Nobel prize in 1969. These quarks are considered as the elementary building blocks of matter. In the view of quarks a meson consists of a quark and an anti-quark and a baryon of three quarks.

[1] Powell got the Nobel prize in 1950.

Because of the Δ^{++} problem - it seemed that this particle had a totally symmetric ground state wave function, which is forbidden by the Pauli-exclusion principle - quarks were assigned an additional degree of freedom: *colour*. This led to the development of a theory describing the interaction between the quarks and therefore also describing the nuclear force in which the force carriers are gluons [4]. This theory is called: *Quantum Chromodynamics* (QCD). It got Gross, Wilczek and Politzer the Nobel prize in 2004. Despite its successes, as for instance *asymptotic freedom* (at very short distances quarks are free) and *confinement* (a quark can never be isolated), it has a major difficulty. Due to its perturbative character it can not be applied at low energies and it is therefore not capable of the describing hadron scattering processes in a simple way.

In order to be able to describe hadron scattering processes effective theories based on the idea of Yukawa can be used. For instance in baryon-baryon scattering or in nucleon-nucleon scattering specifically, the baryons are treated as the effective elementary particles and the mesons are the force carrying particles, which are being exchanged. Already since the seventies the Nijmegen group has, successfully, constructed models describing such interactions based on this idea. The Nijmegen models are considered to be one of the best in the world [5]. For a complete list of the Nijmegen models and articles see [6].

In light of QCD baryons and mesons are colourless and therefore mesons are the only reasonable option to be used as exchanged particles in baryon-baryon scattering in order to describe the strong force at medium and long range $r \gtrsim 1fm$. Also there are several models that form a bridge between the hadron phenomenology on the one hand and the QCD basis on the other hand. Main idea in this is to describe the coupling constants used in phenomenological models by means of the QCD based models. Examples of these models are for instance QCD sum rules [7]. Furthermore, we would like to mention the Quark Pair Creation (QPC) $^3P_0$ model [8, 9] where the mesons and baryons are represented by their constituent quarks. This model is supported by the so-called Flux-Tube model [10]: a lattice QCD based model in which the quarks and flux-tubes are the basic degrees of freedom.

Recently, the Nijmegen group broadened its horizon by including besides the baryon-baryon models also meson-baryon models [11]. The work in this (part of the) thesis can be regarded as an extension of [11], since we also consider meson-baryon scattering or pion-nucleon, more specifically. The reason for considering pion-nucleon scattering is, besides the interest in its own, that there is a large amount of experimental data. Also using $SU_f(3)$ symmetry the extension to other meson-baryon systems is easily made. Last but not least we would like to mention the connection to photo-production models.

Compared to [11] our focus is more on the theoretical background. For instance we formally include what is called "pair suppression", whereas this was assumed in [11]. Pair suppression comes down to the suppression of negative energy contributions. For the first time, at least to our knowledge, we incorporate pair suppression in a covariant and frame independent way. This may particularly be interesting for relativistic many body theories. In order to have this covariant and frame independent pair suppression, we use the Kadyshevsky formalism. This formalism covariantly, though frame dependently [2], separates positive and negative energy contributions. It is introduced and discussed in chapter 2.

Problems may arise in the comparison of results in the Kadyshevsky and the Feynman formalism, when couplings containing derivatives and/or higher spin fields are used. This seeming problem is discussed and solved in chapter 3. The Kadyshevsky formalism is applied to the pion-nucleon system in chapter 4, starting with the meson exchange processes. This is continued in chapter 5, which deals with the baryon sector. Here, also pair suppression is properly introduced and incorporated. In chapter 6 we use the helicity basis and the partial wave expansion to solve the integral equation (see section 2.4 and 6.2) and to introduce the experimental observable phase-shifts.

1.1 Conventions and Units

Throughout this thesis we will use $\hbar = c = 1$. For the metric we use $g^{\mu\nu} = diag(1, -1, -1, -1)$. As far as the definition of the gamma- and the Pauli spin- matrices we use the convention of [12]. In all other cases we explicitly mention what convention we use.

1.2 Meson-Baryon Scattering Kinematics

We consider the pion-nucleon or more general the (pseudo) scalar meson-baryon reactions

$$M_i(q) + B_i(p, s) \rightarrow M_f(q') + B_f(p', s') \ . \tag{1.1}$$

where M stands for a meson and B is a baryon. For the four momentum of the baryons and mesons we take, respectively

$$\begin{aligned} p_c^\mu &= (E_c, \mathbf{p}_c) \quad , \quad where \quad E_c = \sqrt{\mathbf{p}_c^2 + M_c^2} \ , \\ q_c^\mu &= (\mathcal{E}_c, \mathbf{q}_c) \quad , \quad where \quad \mathcal{E}_c = \sqrt{\mathbf{q}_c^2 + m_c^2} \ . \end{aligned} \tag{1.2}$$

[2]By frame dependent we mean: dependent on a vector n^μ (see chapter 2).

Here, c stands for either the initial state i or the final state f. In some cases we find it useful to use the definitions (1.2) for the intermediate meson-baryon states n.

In chapter 2 we will introduce the four vector n^μ and quasi particles with initial and final state momenta $n\kappa$ and $n\kappa'$, respectively. Therefore, the usual overall four-momentum conservation is generally replaced by

$$p + q + \kappa\, n = p' + q' + \kappa'\, n \ . \tag{1.3}$$

As (1.3) and (1.1) make clear a "prime" notation is used to indicate final state momenta; no prime means initial state momenta. We will maintain this notation (also for the energies) throughout this thesis, unless indicated otherwise.

Furthermore we find it useful to introduce the Mandelstam variables in the Kadyshevsky formalism

$$\begin{aligned} s_{pq} &= (p+q)^2 \ , \quad s_{p'q'} = (p'+q')^2 \ , \\ t_{p'p} &= (p'-p)^2 \ , \quad t_{q'q} = (q'-q)^2 \ , \\ u_{p'q} &= (p'-q)^2 \ , \quad u_{pq'} = (p-q')^2 \ , \end{aligned} \tag{1.4}$$

where s_{pq} and $s_{p'q'}$ etc, are only identical for $\kappa' = \kappa = 0$. These Mandelstam variables satisfy the relation

$$2\sqrt{s_{p'q'}s_{pq}} + t_{p'p} + t_{q'q} + u_{pq'} + u_{p'q} = 2\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right) \ . \tag{1.5}$$

The total and relative four-momenta of the initial, final, and intermediate channel ($c = i, f, n$) are defined by

$$P_c = p_c + q_c \ , \ k_c = \mu_{c,2}\, p_c - \mu_{c,1}\, q_c \ , \tag{1.6}$$

where the weights satisfy $\mu_{c,1} + \mu_{c,2} = 1$. We choose the weights to be

$$\begin{aligned} \mu_{c,1} &= \frac{E_c}{E_c + \mathcal{E}_c} \ , \\ \mu_{c,2} &= \frac{\mathcal{E}_c}{E_c + \mathcal{E}_c} \ . \end{aligned} \tag{1.7}$$

Since in the Kadyshevsky formalism all particles are on their mass shell, the choice (1.7) means that always $k_c^0 = 0$.

In the center-of-mass (CM) system $\mathbf{p} = -\mathbf{q}$ and $\mathbf{p}' = -\mathbf{q}'$, therefore

$$\begin{aligned} P_i = (W, \mathbf{0}) \ , \quad & P_f = (W', \mathbf{0}) \ , \\ k_i = (0, \mathbf{p}) \ , \quad & k_f = (0, \mathbf{p}') \ , \end{aligned} \tag{1.8}$$

where $W = E + \mathcal{E}$ and $W' = E' + \mathcal{E}'$. Furthermore we take $n^\mu = (1, \mathbf{0})$.

Also we take as the scattering plane the xz-plane, where the 3-momentum of the initial baryon is oriented in the positive z-direction. This is indicated in figure 1.1 and will be of importance in chapter 6.

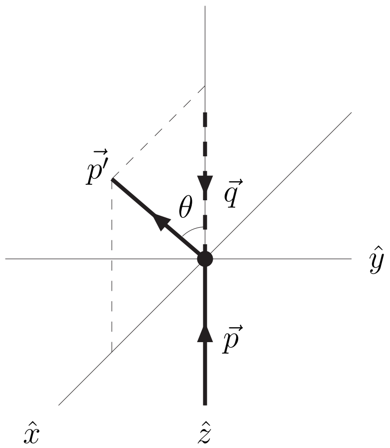


Figure 1.1: *The xz scattering plane in the CM system*

In the CM system the unpolarized differential cross section is defined to be

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{|\mathbf{p}'|}{2|\mathbf{p}|} \sum \left| \frac{M_{fi}}{8\pi\sqrt{s}} \right|^2 , \tag{1.9}$$

where the amplitude M_{fi} is defined in section 2.3 and the sum is over the spin components of the final baryon.

Chapter 2

Kadyshevsky Formalism

In the canonical treatment scattering and decay processes are usually described using the Dyson formula for the S-matrix in the interaction picture, defined in many textbooks as for instance [12, 13]. From this S-matrix Feynman rules are obtained, which are considered as the building blocks of the theoretical description of particle scattering and decay processes. Equivalently, Kadyshevsky developed an alternative formalism starting from the same S-matrix [14, 15, 16, 17], which leads to the so called Kadyshevsky rules.

The difference between both formalisms lies in the treatment of the Time Ordered Product (TOP). In the Feynman formalism the TOP leads to a covariant propagator and intermediate particles go off the mass shell. In the Kadyshevsky formalism the Heaviside step functions of the TOP are replaced by ones with a covariant argument and as a whole they are considered as quasi particle propagators. As a result all (intermediate) particles are on the mass shell and the number of diagrams is increased ($1 \rightarrow n!$ at order n) as in old-fashioned perturbation theory. Four momentum conservation at the vertices is only guaranteed when the quasi particles are included.

2.1 S-Matrix

As mentioned before the S-matrix is defined in many textbooks, like for instance [12, 13], as

$$S = T\left[exp\left(-i\int d^4x\mathcal{H}_I(x)\right)\right] . \tag{2.1}$$

However, to discuss the Kadyshevsky formalism we use an equivalent form for the S-matrix

$$\begin{aligned} S &= 1+\sum_{n=1}^{\infty}(-i)^n\int_{-\infty}^{\infty} d^4x_1\ldots d^4x_n\ \theta(x_1^0-x_2^0)\ldots\theta(x_{n-1}^0-x_n^0) \\ &\qquad \times\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_n)\ . \end{aligned} \tag{2.2}$$

Next, a time like vector n^μ is introduced in the Heaviside step function (or θ-function).

$$\begin{aligned} n^2 &= 1 \quad , \quad n^0>0 \\ \theta(x^0) &\rightarrow \theta[n\cdot x]\ . \end{aligned} \tag{2.3}$$

This will not cause any effect on the S-matrix. Assuming that the S-matrix defined in (2.2) is Lorentz-invariant, and realizing that the S-matrix containing this vectors n^μ is identical to (2.2) in the frame where $n^\mu=(1,\mathbf{0})$, it follows that they are equivalent in all frames because the expression in (2.2) is manifest Lorentz-invariant.

That the introduction of the n^μ-vector (2.3) does not cause any effect can also be seen by looking at the difference $\theta[n(x-y)]-\theta(x^0-y^0)$. Key point is that this difference is unequal to zero in a region outside the light-cone, where the S-matrix does not have a meaning anyway. Consider the surface $n\cdot(x-y)=0$ in the following

$$\begin{aligned} (x-y)^2 &= (x^0-y^0)^2-|\vec{x}-\vec{y}|^2 \\ &= \frac{1}{n_0^2}\left(\vec{n}\cdot(\vec{x}-\vec{y})\right)^2-|\vec{x}-\vec{y}|^2 \end{aligned} \tag{2.4}$$

Now, $0\leq(\vec{n}\cdot(\vec{x}-\vec{y}))^2\leq|\vec{n}|^2.|\vec{x}-\vec{y}|^2$. Considering those limits in (2.4) yields

$$\begin{aligned} (x-y)^2 &\geq -|\vec{x}-\vec{y}|^2<0 \\ (x-y)^2 &\leq \frac{|\vec{n}|^2}{n_0^2}|\vec{x}-\vec{y}|^2-|\vec{x}-\vec{y}|^2=\frac{-1}{1+|\vec{n}|^2}|\vec{x}-\vec{y}|^2<0\ . \end{aligned} \tag{2.5}$$

From this we see that $n\cdot(x-y)=0$ is a surface outside the light cone, hence the difference $\theta[n(x-y)]-\theta(x^0-y^0)$ is also a region outside the light-cone, marked by an arced area in figure 2.1

As a next step the θ-function is written as a Fourier integral which can best be understood considering the reverse order

$$\begin{aligned} \frac{i}{2\pi}\int d\kappa_1\frac{e^{-i\kappa_1 n\cdot(x-y)}}{\kappa_1+i\varepsilon} &= \begin{cases} n\cdot x<n\cdot y & \rightarrow \qquad 0 \\ n\cdot x>n\cdot y & \rightarrow \ -\frac{i}{2\pi}\oint d\kappa_1\frac{e^{-i\kappa_1 n\cdot(x-y)}}{\kappa_1+i\varepsilon}=1 \end{cases} \\ &= \theta[n\cdot(x-y)]\ , \end{aligned} \tag{2.6}$$

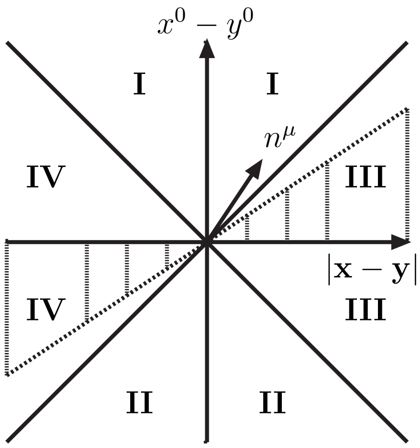


Figure 2.1: *Light-cone. The dashed lines mark the points $n \cdot (x - y) = 0$. In the regions I and II: $(x - y)^2 > 0$, and in the regions III and IV: $(x - y)^2 < 0$.*

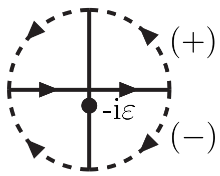


Figure 2.2: *Closing the integral* $\int d\kappa_1 \frac{e^{-i\kappa_1 n\cdot(x-y)}}{\kappa_1 + i\varepsilon}$

where the closing of the integral to make it a cauchy contour integral is schematically exposed in figure 2.2. Momentum space is enlarged by also including $n\kappa$, representing the momentum of a so-called *quasi particle*. The Fourier integral in (2.6) represents therefore the propagation of a quasi particle in a Kadyshevsky diagram.

2.2 Wick Expansion

Although we have not yet said anything about the interaction Hamiltonian we assume, for the moment, that it is just minus the interaction Lagrangian: $\mathcal{H}_I = -\mathcal{L}_I$ [1]. We will have a closer look on this matter in chapter 3.

[1] Of course we should say interaction Lagrangian/Hamiltonian *density*. This is and will be omitted for convenience throughout this thesis, unless indicated otherwise.

The interaction Lagrangian is always our staring point. Since it is a product of fields, also, via the interaction Hamiltonian, the S-matrix will contain a product of fields. In Feynman theory this product, together with the product of θ-functions, is rewritten in terms TOPs, which lead to Feynman propagators Δ_F using Wick's theorem for TOPs.

In Kadyshevsky formalism this can not be done, because the θ-functions are used as quasi-particle propagators. Instead we will use Wick's theorem for ordinary products which states that such a product can be rewritten in terms of Normal Ordered Products (NOP) of (contracted) fields (see for instance [18])

$$\begin{aligned} A_1 A_2 \ldots A_n &= N(A_1 A_2 \ldots A_n) + N(\underbrace{A_1 A_2} \ldots A_n) + N(\underbrace{A_1 A_2 A_3} \ldots A_n) + \ldots \\ &\quad + N(\underbrace{A_1 A_2}\underbrace{A_3 A_4} \ldots A_n) + \ldots \ , \end{aligned} \tag{2.7}$$

where the $\underbrace{\quad}$ symbol means a contraction of fields. These contractions need to be taken out of the NOP in the following way

$$\begin{aligned} N(\underbrace{A_1 A_2 A_3} \ldots) &= (-1)^{n_A} N(\underbrace{A_1 A_3} A_2 \ldots) \ , \\ N(\underbrace{A_1 A_2} \ldots) &= \underbrace{A_1 A_2} N(\ldots) \ , \end{aligned} \tag{2.8}$$

where $n_A = 1(0)$ in case of fermions (bosons). These contractions are vacuum expectation values of fields. This becomes clear when we look at the following example for hermitean scalar fields

$$\begin{aligned} \phi(x)\phi(y) &= \int \frac{d^3p d^3k}{(2\pi)^6 4E_p E_k} \left[a(p)a(k)e^{-ipx}e^{-iky} + a(p)a^\dagger(k)e^{-ipx}e^{iky} \right. \\ &\qquad \left. + a^\dagger(p)a(k)e^{ipx}e^{-iky} + a^\dagger(p)a^\dagger(k)e^{ipx}e^{iky} \right] \ , \\ N\left[\phi(x)\phi(y)\right] &= \int \frac{d^3p d^3k}{(2\pi)^6 4E_p E_k} \left[a(p)a(k)e^{-ipx}e^{-iky} + a^\dagger(k)a(p)e^{-ipx}e^{iky} \right. \\ &\qquad \left. + a^\dagger(p)a(k)e^{ipx}e^{-iky} + a^\dagger(p)a^\dagger(k)e^{ipx}e^{iky} \right] \ , \\ \phi(x)\phi(y) &= N\left[\phi(x)\phi(y)\right] + \int \frac{d^3p d^3k}{(2\pi)^6 4E_p E_k} \left[a(p), a^\dagger(k) \right] e^{-ipx}e^{iky} \\ &= N\left[\phi(x)\phi(y)\right] + \langle 0|\phi(x)\phi(y)|0\rangle \ . \end{aligned} \tag{2.9}$$

Comparing this with (2.7) and (2.8) we see that

$$\underbrace{A_1 A_2} = \langle 0|A_1 A_2|0\rangle \ . \tag{2.10}$$

In (2.9) we already used the commutation relation of the creation and annihilation operators given in (2.44).

These vacuum states are called Wightman functions. The ones used in this thesis are exposed below in (2.11)

$$\begin{aligned}
\langle 0|\phi(x)\phi(y)|0\rangle &= \Delta^{(+)}(x-y)\ ,\\
\langle 0|\psi(x)\bar{\psi}(y)|0\rangle &= S^{(+)}(x-y) = \Lambda^{(1/2)}(\partial)\ \Delta^{(+)}(x-y)\ ,\\
\langle 0|\bar{\psi}(x)\psi(y)|0\rangle &= S^{(-)}(x-y) = -\Lambda^{(1/2)}(-\partial)\ \Delta^{(+)}(x-y)\ ,\\
\langle 0|\phi_\mu(x)\phi_\nu(y)|0\rangle &= \Delta^{(+)}_{\mu\nu}(x-y) = \Lambda^{(1)}_{\mu\nu}(\partial)\ \Delta^{(+)}(x-y)\ ,\\
\langle 0|\psi_\mu(x)\bar{\psi}_\nu(y)|0\rangle &= S^{(+)}_{\mu\nu}(x-y) = \Lambda^{(3/2)}_{\mu\nu}(\partial)\ \Delta^{(+)}(x-y)\ ,\\
\langle 0|\bar{\psi}_\nu(x)\psi_\mu(y)|0\rangle &= S^{(-)}_{\mu\nu}(x-y) = -\Lambda^{(3/2)}_{\mu\nu}(-\partial)\ \Delta^{(+)}(x-y)\ , \qquad (2.11)
\end{aligned}$$

where $\Delta^+(x-y)$ is defined in (D.1) and

$$\begin{aligned}
\Lambda^{(1/2)}_{\mu\nu}(\partial) &= (i\partial\!\!\!/ + M)\ ,\\
\Lambda^{(1)}_{\mu\nu}(\partial) &= \left(-g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{M^2}\right)\ ,\\
\Lambda^{(3/2)}_{\mu\nu}(\partial) &= -\,(i\partial\!\!\!/ + M)\left(g_{\mu\nu} - \frac{1}{3}\,\gamma_\mu\gamma_\nu + \frac{2\partial_\mu\partial_\nu}{3M^2}\right.\\
&\qquad\left. + \frac{1}{3M}\left(i\partial\!\!\!/_\mu\gamma_\nu - \gamma_\mu i\partial\!\!\!/_\nu\right)\right)\ . \qquad (2.12)
\end{aligned}$$

In momentum space these functions (2.11) lead to

$$\begin{aligned}
\Delta^{(+)}(P) &= \theta(P^0)\delta(P^2-M^2)\ ,\\
S^{(+)}(P) &= \Lambda^{(1/2)}(P)\ \theta(P^0)\delta(P^2-M^2)\ ,\\
S^{(-)}(P) &= \Lambda^{(1/2)}(-P)\ \theta(P^0)\delta(P^2-M^2)\ ,\\
\Delta^{(+)}_{\mu\nu}(P) &= \Lambda^{(1)}_{\mu\nu}(P)\ \theta(P^0)\delta(P^2-M^2)\ ,\\
S^{(+)}_{\mu\nu}(P) &= \Lambda^{(3/2)}_{\mu\nu}(P)\ \theta(P^0)\delta(P^2-M^2)\ ,\\
S^{(-)}_{\mu\nu}(P) &= \Lambda^{(3/2)}_{\mu\nu}(-P)\ \theta(P^0)\delta(P^2-M^2)\ . \qquad (2.13)
\end{aligned}$$

These are the functions we use in the Kadyshevsky rules (section 2.3). As can be seen from (2.11) and (2.13) we have removed the minus signs from the $S^{(-)}$-functions. We come back to this point when discussing the Kadyshevsky rules in the next section (section 2.3)

2.3 Kadyshevsky Rules

In the previous sections (sections 2.1 and 2.2) we have discussed the basic ingredients of the S-matrix in Kadyshevsky formalism. Its elements can, just as in Feynman theory, be represented by diagrams: Kadyshevsky diagrams.

Since the basic starting points are the same as in Feynman theory we take a general Feynman diagram and give the Kadyshevsky rules from there on to construct the amplitude M_{fi}. Here, we define the amplitude as

$$S_{fi} = \delta_{fi} - i(2\pi)^2\delta^4\left(P_f - P_i\right)\, M_{fi}\ , \tag{2.14}$$

where $P_{f/i}$ is the sum of the final/initial momenta.

Kadyshevsky Rules:

1) Arbitrarily number the vertices of the diagram.

2) Connect the vertices with a quasi particle line, assigned to it a momentum $n\kappa_s$. Attach to vertex 1 an incoming initial quasi particle with momentum $n\kappa$ and attach to vertex n an outgoing final quasi particle with momentum $n\kappa'$ [2].

3) Orient each internal momentum such that it leaves a vertex with a lower number than the vertex it enters. If two fermion lines with opposite momentum direction come together in one vertex assign a + symbol to one line and a − to the other. Each possibility to do this yields a different Kadyshevsky diagram.

4) Assign to each internal quasi particle line a propagator $\frac{1}{\kappa_s+i\varepsilon}$.

5) Assign to all other internal lines the appropriate Wightman function of (2.13). Assign to a fermion line with a ± symbol: $S^{(\pm)}(P)$ (see **3)**).

6) Impose momentum conservation at the vertices, including the quasi particle lines.

7) Integrate over the internal quasi momenta: $\int_{-\infty}^{\infty} d\kappa_s$.

8) Integrate over those internal momenta not fixed by momentum conservation at the vertices: $\int_{-\infty}^{\infty} \frac{d^4P}{(2\pi)^3}$.

9) Include a − sign for every fermion loop.

[2]Obviously these quasi particles may not appear as initial or final states, since they are not physical particles. However, since we use Kadyshevsky diagrams as input for an integral equation (see section 2.4) we allow for external quasi particles.

10) A factor minus between two graphs that differ only by the interchange of two identical external fermions (just as in Feynman theory, see for instance [12]).

11) Repeat the various steps for all different numberings in **1**.

It is clear from **3)** and **11)** that one Feynman diagram leads to several Kadyshevsky diagrams. Generally, one Feynman diagram leads to $n!$ Kadyshevsky diagrams, where n is the number of vertices (or; the order). Especially for higher order diagrams this leads to a dramatic increase of labour. Fortunately, we will only consider second order diagrams.

A few remarks need to be made about these rules as far as the choice of definition is concerned. In **3)** we have followed [14] to orient the internal momenta. Furthermore we have chosen to use the integral representation of the θ-function as in (2.6) instead of its complex conjugate. Since the θ-function is real, this is also a proper representation, originally used in the papers of Kadyshevsky. To understand why we have chosen to deviate from the original approach, consider the S-matrix (2.2), again.

In each order S_n there is a factor $(-i)^n$ already in the definition. In that specific order there are $(n-1)$ θ-functions, each containing a factor i from the integral representation (2.6). Therefore, every S_n will, regardless the order, contain a factor $(-i)$. Hence, the amplitude M_{fi}, defined in (2.14), will not contain overall factors of i, anymore.

As mentioned before the momentum space $S^{(-)}(P)$-functions used in the Kadyshevsky rules (2.13) differ from their coordinate space analogs defined in (2.11) by an overall minus sign. The reason for that is twofold. In many cases the Wightman functions $S^{(-)}(x-y)$, including the overall minus sign, appear in combination with the NOP: $N(\psi\bar{\psi}) = -N(\bar{\psi}\psi)$. Therefore, the minus signs cancel. In all other cases the Wightman functions $S^{(-)}(x-y)$ appear in fermion loops and are therefore responsible for the fermion loop minus sign in **9)**, since every fermion loop will contain an odd number of $S^{(-)}(x-y)$ functions. We stress that this method of defining the Kadyshevsky rules for fermions differs from the original one in [16].

Although it is tempting to demonstrate the Kadyshevsky rules here, we postpone that to chapter 3.

2.4 Integral Equation

To describe complete two body scattering processes use can be made of the Bethe-Salpeter (BS) equation [19], which is a fully relativistic two particle scattering integral equation. It needs to be mentioned, however, that it is by definition unsolvable, since the input of the integral equation is already an infinite set of amplitudes. This will become clear in section 2.4.1. Therefore, approximations have to be made as far as the input is concerned. In [11] are, besides this fact, other approximations made in the BS equation to come to a three dimensional integral equation, which, then, is used to solve the problem.

The integral equation in Kadyshevsky formalism [15] is also by definition unsolvable in the same way as the BS equation. This we will see in section 2.4.2. However, the Kadyshevsky integral equation is a three dimensional integral equation, which comes about in a natural way, without any approximation . In the following two subsections we are going to discuss the BS equation (section 2.4.1) and the Kadyshevsky integral equation (section 2.4.2). This to see the difference between them clearly.

2.4.1 Bethe-Salpeter Equation

To understand how the Bethe-Salpeter equation comes about we imagine to have the following interaction Hamiltonian

$$\mathcal{L}_I(x) = g\,\bar{\psi}\psi\cdot\phi_1 + g\,\phi_a\phi_b\cdot\phi_1 = -\mathcal{H}_I(x)\ , \tag{2.15}$$

where we use subscripts a and b to indicate outgoing and incoming scalar fields, respectively. The interaction Hamiltonian (2.15) serves as basic ingredient of the S-matrix as used in Feynman theory (2.1). When we consider πN-scattering up to the fourth order, the relevant contributions are $S^{(2)}$ and $S^{(4)}$

$$\begin{aligned} S^{(2)} &= \frac{(-i)^2}{2!}\int d^4x_1 d^4x_2\, T\left[\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\right]\ , \\ &= -g^2\int d^4x_1 d^4x_2\, N[\bar{\psi}(x_1)\psi(x_1)\phi_a(x_2)\phi_b(x_2)]\, T[\phi_1(x_1)\phi_1(x_2)]\ , \end{aligned}$$

$$
\begin{aligned}
S^{(4)} &= \frac{(-i)^4}{4!}\int d^4x_1d^4x_2d^4x_3d^4x_4\, T\left[\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\mathcal{H}_I(x_3)\mathcal{H}_I(x_4)\right] \ , \\
&= g^4\int d^4x_1d^4x_2d^4x_3d^4x_4\, N[\bar{\psi}(x_1)\psi(x_3)\phi_a(x_2)\phi_b(x_4)]\, T[\phi_1(x_1)\phi_1(x_2)] \\
&\qquad \times T[\phi_1(x_3)\phi_1(x_4)]T[\psi(x_1)\bar{\psi}(x_3)]T[\phi_c(x_2)\phi_d(x_4)] + \dots \ .
\end{aligned}
\tag{2.16}
$$

The ellipsis indicate those terms that also appear when the TOP is fully expanded, using Wick's theorem. They are not exposed because they do not contain a product πN Feynman propagators ($\Delta_F(x-y;m_\pi^2)$ and $S_F(x-y;M_N^2)$); they are said to be πN-irreducible.

Performing all integrals, collecting all factors of i and (2π) and sandwiching between initial and final πN states, the contributions up to fourth order are

$$
\begin{aligned}
& S_{4,\ fi}(p'q';pq) = S^{(2)}_{fi} + S^{(4)}_{fi} \\
=\ & -i(2\pi)^4\delta^4(P_f-P_i)M_{fi}(p'q';pq) \\
& -i(2\pi)^4\delta^4(P_f-P_i)\int d^4P\ M_f(p'q';p_cq_c)\left[\frac{i}{(2\pi)^4}\,\Delta_F(q_c)\,S_F(p_c)\right] \\
& \qquad \times M_i(p_cq_c;pq) + \dots \ ,
\end{aligned}
\tag{2.17}
$$

where the internal momenta q_c and p_c are expressible in terms of incoming/outgoing momenta and the loop momentum P. In (2.17) one has to realize that for instance $M_f(p'q';p_cq_c)$ does not contain a final state spinor u. A similar thing accounts for $M_i(p_cq_c;pq)$, which does not contain an initial state spinor.

In order to generate all terms, equation (2.17) becomes an integral equation, where the first term $M(p'q';pq)$ is the driving term. Those terms that are pion-nucleon irreducible indicated in (2.16) by the ellipsis, as mentioned before, are also put in the driving term.

Taking these consideration into account, the BS equation reads (see also figure 2.3)

$$
\begin{aligned}
M_{fi}(p'q';pq) &= M^{irr}_{fi}(p'q';pq) \\
&\quad + \sum_n \int d^4P_n\ M^{irr}_f(p'q';P_n)\,G(P_n)\,M_i(P_n;pq)\ , \\
G(P_n) &= \frac{i}{(2\pi)^4}\,\Delta_F(P_n)\,S_F(P_n)\ ,
\end{aligned}
\tag{2.18}
$$

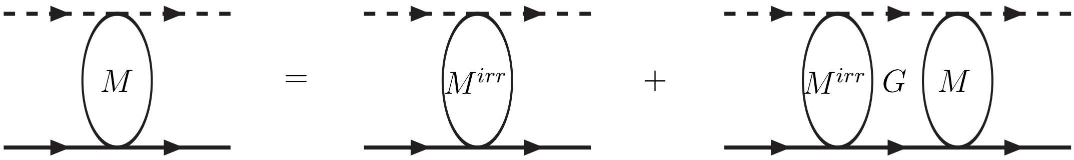


Figure 2.3: *Bethe-Salpeter equation*

[3] where the sum in (2.18) stand for all intermediate meson-baryon channels.

As mentioned before the driving term in (2.18) contains the set of all pion-nucleon irreducible diagrams. Since this set is infinite, the BS equation is unsolvable by definition.

Similar to the remarks about $M_f(p'q';p_cq_c)$ in the text below (2.17), (2.18) is strictly speaking not correct. This is because the first term on the rhs of (2.18) contains an initial and final state spinor, whereas this same expression M^{irr} in the second term on the rhs of (2.18) does not. This accounts for the whole iteration. A simple way out is to consider (2.18) as an operator equation (so, no initial and final state spinors) or to consider only initial or final states.

An even better solution is to split the fermion propagator in a positive and negative energy contribution and include their spinors, present in the projection operator of the propagator, in $M_f(p'q';p_cq_c)$ and $M_i(p_cq_c;pq)$. Then (2.18) becomes schematically

$$M_{++} \;=\; M^{irr}_{++} + M^{irr}_{+-}\,G_-\,M_{-+} + M^{irr}_{++}\,G_+\,M_{++}\;, \tag{2.19}$$

where a "+" stands for a u spinor and a "$-$" for a v spinor. This is what is done in [11], where also the boson propagator is split in positive and negative energy contributions. In [11] only positive energy contributions are considered motivated by the assumption of pair suppression.

2.4.2 Kadyshevsky Integral Equation

In Kadyshevsky formalism we use the S-matrix as exposed in (2.2). Using the same interaction Hamiltonian as in (2.15) the relevant S-matrix contributions

[3]Obviously, Δ_F and S_F in (2.18) do not have the same argument. The notation is merely meant to indicate that P_n is the only free variable over which the integral runs.

up to fourth order are

$$
\begin{aligned}
S^{(2)} &= (-i)^2 \int d^4x_1 d^4x_2\, \theta[n(x_1-x_2)]\mathcal{H}_I(x_1)\mathcal{H}_I(x_2) \\
&= -g^2 \int d^4x_1 d^4x_2\, N[\bar{\psi}(x_1)\psi(x_1)\phi_a(x_2)\phi_b(x_2)] \\
&\qquad \times \Big[\theta[n(x_1-x_2)]\langle 0|\phi_1(x_1)\phi_1(x_2)|0\rangle \\
&\qquad + \theta[n(x_2-x_1)]\langle 0|\phi_1(x_2)\phi_1(x_1)|0\rangle\Big] \ , \\
S^{(4)} &= (-i)^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4\, \theta[n(x_1-x_2)]\theta[n(x_2-x_3)]\theta[n(x_3-x_4)] \\
&\qquad \times \mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\mathcal{H}_I(x_3)\mathcal{H}_I(x_4) \\
&= g^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4\, N[\bar{\psi}(x_1)\psi(x_3)\phi_a(x_2)\phi_b(x_4)] \\
&\qquad \times \Big[\theta[n(x_1-x_2)]\langle 0|\phi_1(x_1)\phi_1(x_2)|0\rangle \\
&\qquad +\theta[n(x_2-x_1)]\langle 0|\phi_1(x_2)\phi_1(x_1)|0\rangle] \\
&\qquad \times \theta[n(x_2-x_3)]\langle 0|\psi(x_1)\bar{\psi}(x_3)|0\rangle\langle 0|\phi_c(x_2)\phi_d(x_4)|0\rangle \\
&\qquad \times \Big[\theta[n(x_3-x_4)]\langle 0|\phi_1(x_3)\phi_1(x_4)|0\rangle \\
&\qquad +\theta[n(x_4-x_3)]\langle 0|\phi_1(x_4)\phi_1(x_3)|0\rangle] + \dots \ .
\end{aligned} \tag{2.20}
$$

Again, the ellipsis indicate terms that are πN-irreducible, but now in the sense of the Kadyshevsky propagators $\Delta^{(+)}(x-y;m_\pi^2)$ and $S^{(+)}(x-y;M_N^2)$ in which the orientation is also important (see figure 2.4).

Performing all integrals, collecting all factors of i and (2π) and sandwiching between initial and final πN states, again, the contributions up to fourth order are

$$
\begin{aligned}
S_4(p'q';pq) &= S^{(2)} + S^{(4)} = -i(2\pi)^4\delta^4(P_f-P_i)M_{00}(p'q';pq) \\
&\quad -i(2\pi)^4\delta^4(P_f-P_i)\int d^4P\, d\kappa\ M_{0\kappa}(p'q';p_nq_n) \\
&\quad \times \left[\frac{1}{(2\pi)^3}\frac{1}{\kappa+i\varepsilon}\,\Delta^{(+)}(q_n)S^{(+)}(p_n)\right] M_{\kappa 0}(p_nq_n;pq) + \dots \ .
\end{aligned} \tag{2.21}
$$

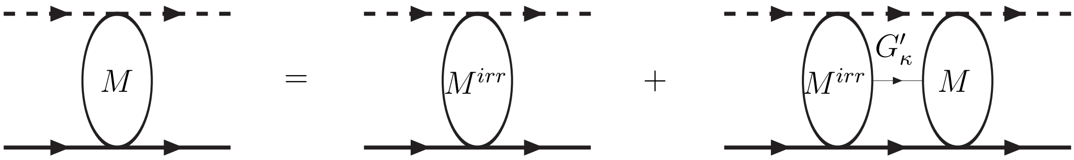


Figure 2.4: *Kadyshevsky integral equation*

As in the previous section we have the problem with the initial and final state spinors in the second term on the rhs of (2.21). In this situation the problem is easily cured because of the definition of $S^{(+)}(p_n)$ in (2.13) (and (2.12))

$$\begin{aligned} S^{(+)}(p_n) &= \Lambda^{(1/2)}(p_n)\ \theta(p_n^0)\delta(p_n^2 - M^2)\ , \\ &= \sum_{s_n} u(p_n s_n)\bar{u}(p_n s_n)\ \theta(p_n^0)\delta(p_n^2 - M^2)\ , \end{aligned} \tag{2.22}$$

where we include the spinors in $M_{0\kappa}(p'q'; p_n q_n)$ and $M_{\kappa 0}(p_n q_n; pq)$.

The step to the integral equation which generates all terms is similar to what is described before (text below (2.17)) making the Kadyshevsky integral equation also unsolvable by definition. Taking (2.22) into account we get

$$\begin{aligned} M(p'q'; pq) &= M^{irr}_{00}(p'q'; pq) \\ &\quad + \sum_n \int d^4P_n\, d\kappa\ M^{irr}_{0\kappa}(p'q'; P_n)\, G'_\kappa(P_n)\, M_{\kappa 0}(P_n; pq)\ , \\ G'_\kappa(P_n) &= \frac{1}{(2\pi)^3}\frac{1}{\kappa + i\varepsilon}\,\Delta^{(+)}_\pi(P_n)\Delta^{(+)}_N(P_n)\ . \end{aligned} \tag{2.23}$$

From (2.23) and figure 2.4 we see that the intermediate amplitude $M^{irr}_{0\kappa}$ contains an "external" quasi particle. This is the reason we have included external quasi particles in the Kadyshevsky rules (section 2.3).

To really get the three dimensional integral equation we write (2.23) as an integral over all internal momenta at the cost of a δ-function representing momentum conservation

$$\begin{aligned} M(p'q'; pq) &= M^{irr}_{00}(p'q'; pq) + \int d^4p_n d^4q_n\, d\kappa\ M^{irr}_{0\kappa}(p'q'; p_n q_n) \\ &\quad \times \left[\frac{1}{(2\pi)^3}\frac{1}{\kappa + i\varepsilon}\,\Delta^{(+)}_\pi(q_n)\Delta^{(+)}_N(p_n)\right] M_{\kappa 0}(p_n q_n; pq) \\ &\quad \times \delta^4(p_n + q_n + n\kappa - p - q)\ , \end{aligned} \tag{2.24}$$

Introducing the total and relative momenta as in (1.6) the integration variables are changed to $\int d^4p_n d^4q_n = \int d^4P_n d^4k_n$. Using also the CM system (see section 1.2) several integrals in (2.24) can be performed

$$\begin{aligned}\delta^4(p_n+q_n+n\kappa-p-q) &= \delta^4(P_n+n\kappa-P_i) \overset{cm}{\to} \delta(\vec{P}_n)\delta(\kappa-(P_i^0-P_n^0)) \ , \\ \Delta_\pi^{(+)}(q_n)\Delta_N^{(+)}(p_n) &= \theta(q_n^0)\delta(q_n^2-m^2)\theta(p_n^0)\delta(p_n^2-M^2) \\ &= \frac{1}{4\mathcal{E}_n E_n}\ \delta(q_n^0-\mathcal{E}_n)\delta(p_n^0-E_n) \\ &= \frac{1}{4\mathcal{E}_n E_n}\ \delta\left(P_n^0-(E_n+\mathcal{E}_n)\right)\delta\left(k_n^0\right) \ , \qquad (2.25)\end{aligned}$$

in such a way that (2.24) becomes

$$\begin{aligned}M(W'\,\mathbf{p}';W\,\mathbf{p}) &= M_{00}^{irr}(W'\,\mathbf{p}';W\,\mathbf{p}) + \int d^3k_n\, M_{0\kappa}^{irr}(W'\,\mathbf{p}';W_n\,\mathbf{k}_n) \\ &\quad \times\frac{1}{(2\pi)^3}\ \frac{1}{4\mathcal{E}_n E_n}\ \frac{1}{W-W_n+i\varepsilon}\ M_{\kappa 0}(W_n\,\mathbf{k}_n;W\,\mathbf{p}) \ . \qquad (2.26)\end{aligned}$$

Although there are still κ-labels in (2.26), obviously they are fixed by the κ-integration as a result of the first line of (2.25).

As can be seen from (2.20) and the text below it, we have called intermediate negative energy states ($\Delta^{(-)}(x-y;m_\pi^2)$ and $S^{(-)}(x-y;M_N^2)$) πN-irreducible and put them in $M_{\kappa\kappa'}^{irr}$, but in principle they could also participate in the integral equation in the same way as the second term on the rhs of (2.19). However, using pair suppression in the way we do in chapter 5, these terms vanish.

Having discussed both integral equations we can look at the difference between them. As far as the difference in dimensionality of both integral equations is concerned we consider the πN reducible part of $S^{(4)}$ in (2.16), again. The exposed TOPs can be decomposed in their Kadyshevsky components ($\theta(x-y)$, $\Delta_1^{(+)}(x-y)$, etc.). A contribution is

$$\begin{aligned}& T[\phi_1(x_1)\phi_1(x_2)]T[\phi_1(x_3)\phi_1(x_4)]T[\psi(x_1)\bar{\psi}(x_3)]T[\phi_c(x_2)\phi_d(x_4)] \\ =\ & \theta(x_1-x_2)\theta(x_2-x_4)\theta(x_4-x_3)\theta(x_1-x_3) \\ & \times\Delta_1^{(+)}(x_1-x_2)\Delta_{cd}^{(+)}(x_2-x_4)\Delta_1^{(+)}(x_4-x_3)S^{(+)}(x_1-x_3)+\dots \ . \qquad (2.27)\end{aligned}$$

Now, every TOP in (2.27) contains a four dimensional momentum integral. Since there is four momentum conservation at the vertices, only one four-dimensional integral will be left: the one over the loop momentum.

In Kadyshevsky formalism the product of a θ-function and a $\Delta^{(+)}$-function (or a $S^{(\pm)}$) also contains a four-dimensional integral: one for the θ-function (2.6) and three for the $\Delta^{(+)}$-function. By the same argument of momentum conservation only the integrals of one such product is left. In the above example (2.27) this is for instance the integral of $\theta(x_1 - x_3)S^{(+)}(x_1 - x_3)$. This θ-function, however is superfluous by means of the product of the other θ-functions in (2.27). Therefore there is only the three dimensional integral (the one of $S^{(+)}(x_1 - x_3)$) left. Although this is just a fourth order example, it is the main reason why the Kadyshevsky integral is a three dimensional integral equation.

When we consider the πN reducible part of $S^{(4)}$ in (2.16) again, and compare it with the one in (2.20) [4] we see that if we decompose the TOP of (2.16) in its Kadyshevsky components we get many more terms than the four exposed in (2.20). This means that in Kadyshevsky formalism more terms are incorporated in the driving term M^{irr} per order as compared to Feynman formalism or to put it in a different way: per order the reducible parts in both formalisms produce different terms .

2.4.3 n-independence of Kadyshevsky Integral Equation

When generating Kadyshevsky diagrams to random order using the Kadyshevsky integral equation as exposed in (2.23) the (full) amplitude is identical to the one obtained in Feynman formalism when the external quasi particle momenta are put to zero. It is therefore n-independent, i.e. frame independent.

Since an approximation is used to solve the Kadyshevsky integral equation, namely tree level diagrams as driving terms, it is not clear whether the full amplitude remains to be n-independent when the external quasi particle momenta are put to zero.

In examining the n-dependence of the amplitude we write (2.23) schematically as

$$M_{00} = M_{00}^{irr} + \int d\kappa \; M_{0\kappa}^{irr} \; G'_{\kappa} \; M_{\kappa 0} \; , \tag{2.28}$$

Since $n^2 = 1$, only variations in a space-like direction are unrestricted, i.e. $n \cdot \delta n = 0$ [20]. We therefore introduce the projection operator

$$P^{\alpha\beta} = g^{\alpha\beta} - n^{\alpha}n^{\beta} \; , \tag{2.29}$$

from which it follows that $n_{\alpha}P^{\alpha\beta} = 0$. The n-dependence of the amplitude

[4] As mentioned before πN reducibility has different meaning in both formalisms

can now be studied

$$P^{\alpha\beta}\frac{\partial}{\partial n^\beta}M_{00} = P^{\alpha\beta}\frac{\partial M_{00}^{irr}}{\partial n^\beta} + P^{\alpha\beta}\int d\kappa\left[\frac{\partial M_{0\kappa}^{irr}}{\partial n^\beta}\;G'_\kappa\;M_{\kappa 0} + M_{0\kappa}^{irr}\;G'_\kappa\;\frac{\partial M_{\kappa 0}}{\partial n^\beta}\right]\;. \quad (2.30)$$

If both Kadyshevsky contributions are considered at second order in M_{00}, then it is n-independent, since it yields the Feynman expression. As far as the second term in (2.30) is concerned we observe the following

$$\frac{\partial M_{0\kappa}^{irr}}{\partial n^\beta} \propto \kappa f(\kappa)\;\;,\;\;\frac{\partial M_{\kappa 0}}{\partial n^\beta} \propto \kappa g(\kappa)\;, \quad (2.31)$$

where $f(\kappa)$ and $g(\kappa)$ are functions that do not contain poles or zero's at $\kappa = 0$. Therefore, the integral in (2.30) is of the form

$$\int d\kappa\;\kappa\;h(\kappa)G'_\kappa\;. \quad (2.32)$$

When performing the integral we decompose the G'_κ as follows

$$G'_\kappa \propto \frac{1}{\kappa + i\varepsilon} = P\frac{1}{\kappa} - i\pi\delta(\kappa)\;. \quad (2.33)$$

As far as the $\delta(\kappa)$-part of (2.33) is concerned we immediately see that it gives zero when used in the integral (2.32). For the Principle valued integral, indicated in figure 2.5 by **I**, we close the integral by connecting the end point ($\kappa = \pm\infty$) via a (huge) semi-circle in the upper half, complex κ-plane (line **II** in figure 2.5) and by connecting the points around zero via a small semi circle also in the upper half plane (line **III** in figure 2.5). Since every single (tree level) amplitude is proportional to $1/(\kappa + A + i\varepsilon)$, where κ is related to the momentum of the incoming or outgoing quasi particle and A some positive or negative number, the poles will always be in the lower half plane and not within the contour. Therefore, the contour integral is zero.

Since we have added integrals (**II** and **III** in figure 2.5) we need to know what their contributions are. The easiest part is integral **III**. Its contribution is half the residue at $\kappa = 0$ and since the only remaining integrand part $h(\kappa)$ in (2.32) does not contain a pole at zero it is zero.

If we want the contribution of integral **II** to be zero, than the integrand should at least be of order $O(\frac{1}{\kappa^2})$. Unfortunately, this is not (always) the case as we will see in chapters 4 and 5. To this end we introduce a phenomenological "form factor"

$$F(\kappa) = \left(\frac{\Lambda_\kappa^2}{\Lambda_\kappa^2 - \kappa^2 - i\epsilon(\kappa)\varepsilon}\right)^{N_\kappa}\;, \quad (2.34)$$

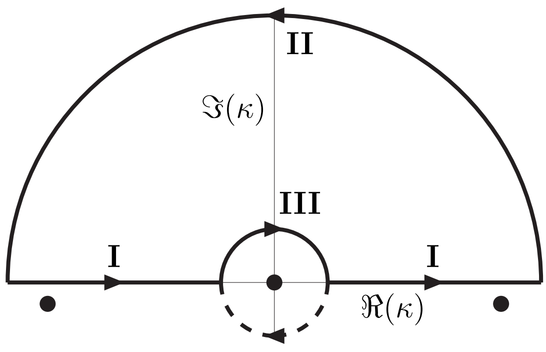


Figure 2.5: *Principle value integral*

where Λ_κ is large and N_κ is some positive integer. In (2.34) ε is real, positive, though small and $\epsilon(\kappa) = \theta(\kappa) - \theta(-\kappa)$.

The effect of the function $F(\kappa)$ (2.34) on the original integrand in (2.32) is little, since for large Λ_κ it is close to unity. However, including this function in the integrand makes sure that it is at least of order $O(\frac{1}{\kappa^2})$ so that integral **II** gives zero contribution. The $-i\epsilon(\kappa)\varepsilon$ part ensures that there are no poles on or within the closed contour, since they are always in the lower half plane (indicated by the dots in figure 2.5).

2.5 Second Quantization

When discussing the Kadyshevsky rules in section 2.3 and the Kadyshevsky integral equation in section 2.4.2 we allowed for quasi particles to occur in the initial and final state. In order to do this properly a new theory needs to be set up containing quasi particle creation and annihilation operators. It is set up in such a way that external quasi particles occur in the S-matrix as trivial exponentials so that when the external quasi momenta are taken to be zero the Feynman expression is obtained. We, therefore, require that the vacuum expectation value of the quasi particles is the θ-function

$$\langle 0|\chi(nx)\bar{\chi}(nx')|0\rangle = \theta[n(x-x')] \ , \tag{2.35}$$

and that a quasi field operator acting on a state with quasi momentum $(n)\kappa$ only yields a trivial exponential

$$\begin{aligned} \chi(nx)|\kappa\rangle &= e^{-i\kappa nx} \ , \\ \langle\kappa|\bar{\chi}(nx) &= e^{i\kappa nx} \ . \end{aligned} \tag{2.36}$$

Assuming that a state with quasi momentum $(n)\kappa$ is created in the usual way

$$
\begin{aligned}
a^\dagger(\kappa)|0\rangle &= |\kappa\rangle \ , \\
\langle 0|a(\kappa) &= \langle\kappa| \ ,
\end{aligned} \tag{2.37}
$$

we have from the requirements (2.35) and (2.36) the following momentum expansion of the fields

$$
\begin{aligned}
\chi(nx) &= \frac{i}{2\pi}\int \frac{d\kappa}{\kappa + i\varepsilon}\ e^{-i\kappa nx}a(\kappa) \ , \\
\bar{\chi}(nx') &= \frac{i}{2\pi}\int \frac{d\kappa}{\kappa + i\varepsilon}\ e^{i\kappa nx'}a^\dagger(\kappa) \ ,
\end{aligned} \tag{2.38}
$$

and the fundamental commutation relation of the creation and annihilation operators

$$
\left[a(\kappa), a^\dagger(\kappa')\right] = -i2\pi\kappa\delta(\kappa-\kappa') \ . \tag{2.39}
$$

From this commutator (2.39) it is clear that the quasi particle is not a physical particle nor a ghost.

Now that we have set up the second quantization for the quasi particles we need to include them in the S-matrix. This is done by redefining it

$$
S = 1 + \sum_{n=1}(-i)^n \int d^4x_1 \ldots d^4x_n \tilde{\mathcal{H}}_I(x_1)\ldots\tilde{\mathcal{H}}_I(x_n) \ , \tag{2.40}
$$

where

$$
\tilde{\mathcal{H}}_I(x) \equiv \mathcal{H}_I(x)\bar{\chi}(nx)\chi(nx) \ . \tag{2.41}
$$

In this sense contraction of the quasi fields causes propagation of this field between vertices, just as in the Feynman formalism. Those quasi particles that are not contracted are used to annihilate external quasi particles from the vacuum.

$$
\begin{aligned}
& S^{(2)}(p's'q'n\kappa'; psqn\kappa) = \\
=\ & (-i)^2 \int d^4x_1 d^4x_2 \langle \pi N\chi|\tilde{\mathcal{H}}_I(x_1)\tilde{\mathcal{H}}_I(x_2)|\pi N\chi\rangle \\
=\ & (-i)^2 \int d^4x_1 d^4x_2 \langle 0|b(p's')a(q')a(\kappa') \\
& \times \left[\bar{\chi}(nx_1)\mathcal{H}_I(x_1)\chi(nx_1)\bar{\chi}(nx_2)\mathcal{H}_I(x_2)\chi(nx_2)\right] a^\dagger(\kappa)a^\dagger(q)b^\dagger(ps)|0\rangle \\
=\ & (-i)^2 \int d^4x_1 d^4x_2\ e^{in\kappa' x_1}e^{-in\kappa x_2} \\
& \times\langle 0|b(p's')a(q')\mathcal{H}_I(x_1)\theta[n(x_1-x_2)]\mathcal{H}_I(x_2)a^\dagger(q)b^\dagger(ps)|0\rangle \ .
\end{aligned} \tag{2.42}
$$

For the π and N fields we use the well-known momentum expansion

$$\begin{aligned}
\phi(x) &= \int \frac{d^3l}{(2\pi)^3 2E_l}\left[a(l)e^{-ilx} + a^\dagger(l)e^{ilx}\right] , \\
\psi(x) &= \sum_r \int \frac{d^3k}{(2\pi)^3 2E_k}\left[b(k,r)u(k,r)e^{-ikx} + d^\dagger(k,r)v(k,r)e^{ikx}\right] , \quad (2.43)
\end{aligned}$$

where the creation and annihilation operators satisfy the following (anti-) commutation relations

$$\begin{aligned}
[a(k), a^\dagger(l)] &= (2\pi)^3\, 2E_k\, \delta^3(k-l) , \\
\{b(k,s), b^\dagger(l,r)\} &= (2\pi)^3\, 2E_k\, \delta_{sr}\delta^3(k-l) = \{d(k,s), d^\dagger(l,r)\} . \quad (2.44)
\end{aligned}$$

Putting $\kappa' = \kappa = 0$ in (2.42) we see that we get the second order in the S-matrix expansion for πN-scattering as in Feynman formalism. Of course this is what we required from the beginning: external quasi particle momenta only occur in the S-matrix as exponentials.

So, we know now how to include the external quasi particles in the S-matrix and therefore we also know what their effect is on amplitudes. For practical purposes we will not use the S-matrix as in (2.40), but keep the above in mind. In those cases where the (possible) inclusion of external quasi fields is less trivial we will make some comments.

Chapter 3

Treatment General Interactions: TU and GJ method

In the previous chapter we have discussed the Kadyshevsky rules (section 2.3) so we know now how to construct amplitudes. When we consider a general interaction Lagrangian containing for instance derivatives on fields or higher spin fields and apply the Kadyshevsky rules straightforward, it seems that there arise problems when comparing the Feynman and the Kadyshevsky results and when analyzing the n-dependence, i.e. the frame dependence. We illustrate this in section 3.1 with an example. In sections 3.2 and 3.4 we discuss two different methods how these problems can be overcome: the Takahashi and Umezawa (TU) method [21, 22, 23] and the Gross and Jackiw (GJ) method [20]. These methods are applied to the example in section 3.5 and we show that the final results in the Feynman formalism and in the Kadyshevsky formalism are not only the same, but also frame independent. We stress here that both methods (TU and GJ) yield the same result. In section 3.3 we make some remarks on the Haag theorem [24]. Since it is properly introduced in that specific section, there are no further comments at this point. The main results of this chapter are summarized in section 3.6.

3.1 Example: Part I

As mentioned in the introduction we are going show an example to illustrate seeming problems. In order to do so we take the vector extension of interaction Lagrangian (2.15)

$$\mathcal{L}_I \;=\; g\,\phi_a i\overleftrightarrow{\partial_\mu}\phi_b \cdot \phi^\mu + g\,\bar{\psi}\gamma_\mu\psi \cdot \phi^\mu \;, \qquad (3.1)$$

where ϕ^μ is a massive vector boson and the indices a and b indicate the outgoing and incoming scalars, again. For the derivative $\overleftrightarrow{\partial_\mu} = \overrightarrow{\partial_\mu} - \overleftarrow{\partial_\mu}$.

We consider vector meson exchange in the Feynman formalism (section 3.1.1) as well as in the Kadyshevsky formalism (section 3.1.2). Actually, the interaction Lagrangian in (3.1) is a simplified version of the one used in [11] (see also chapter 4). This, because it is merely used to illustrate some problems.

3.1.1 Feynman Approach

The Feynman diagram for (simplified) vector meson exchange is shown in figure 3.1 For the various components of the diagrams we take the following

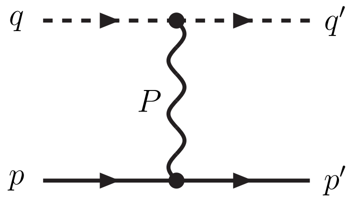


Figure 3.1: *Vector meson exchange in Feynman formalism.*

functions

$$
\begin{aligned}
D^{\mu\nu} &= \left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{M_V^2}\right) \frac{1}{P^2 - M_V^2 + i\varepsilon} \ , \\
\Gamma_\mu^{\bar\psi\psi} &= \gamma_\mu \ , \\
\Gamma_\mu^{\phi\phi} &= (q' + q)_\mu \ ,
\end{aligned} \tag{3.2}
$$

where we obtained the vertex functions via $\mathcal{L}_I = -\mathcal{H}_I \to -\Gamma$.

Following [12] for the definition of the Feynman rules we get the following amplitude

$$
\begin{aligned}
-iM_{fi} &= \bar u(p's') \left(-ig\,\Gamma_\mu^{\bar\psi\psi}\right) u(ps)\, iD^{\mu\nu}(P)\ \left(-ig\,\Gamma_\nu^{\phi\phi}\right) \ , \\
\Rightarrow M_{fi} &= -g^2\ [\bar u(p's')\gamma_\mu u(ps)] \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{M_V^2}\right) \frac{1}{P^2 - M_V^2 + i\varepsilon} (q' + q)_\nu \ ,
\end{aligned} \tag{3.3}
$$

where $P = \frac{1}{2}(p' - p - q' + q) = \Delta_t$. After some (Dirac) algebra we find

$$
M_{fi} = -g^2 \bar u(p's') \left[2\not{Q} + \frac{(M_f - M_i)}{M_V^2}(m_f^2 - m_i^2)\right] u(ps)\ \frac{1}{t - M_V^2 + i\varepsilon} \ , \tag{3.4}
$$

where $Q = \frac{1}{2}(q' + q)$ and t is defined in (1.4) with $\kappa' = \kappa = 0$.

3.1.2 Kadyshevsky Approach

The Kadyshevsky diagrams for the (simplified) vector meson exchange are shown in figure 3.2. The vertex functions are the same as in Feynman theory

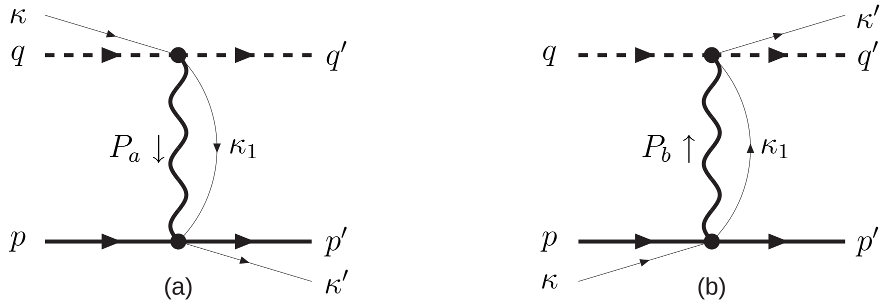


Figure 3.2: *Vector meson exchange in Kadyshevsky formalism.*

(3.2). Applying the Kadyshevsky rules as given in section 2.3 straightforward we get the following amplitudes

$$
\begin{aligned}
M^{(a,b)}_{\kappa'\kappa} &= -g^2 \int \frac{d\kappa_1}{\kappa_1 + i\varepsilon} \left[\bar{u}(p's')\gamma_\mu u(ps)\right] \left(g^{\mu\nu} - \frac{P^\mu_{a,b} P^\nu_{a,b}}{M_V^2} \right) \\
& \qquad \times \theta(P^0_{a,b}) \delta(P^2_{a,b} - M_V^2) \, (q' + q)_\nu \ , \qquad (3.5)
\end{aligned}
$$

where $P_{a,b} = \pm\Delta_t + \frac{1}{2}(\kappa' + \kappa)n - n\kappa_1$ (here a corresponds to the $+$ sign and b to the $-$ sign). For the κ_1 integration we consider the δ-function in (3.5)

$$
\begin{aligned}
(a): \quad \delta(P_a^2 - M_V^2) &= \frac{1}{|\kappa_1^+ - \kappa_1^-|} \left(\delta(\kappa_1 - \kappa_1^+) + \delta(\kappa_1 - \kappa_1^-) \right) \ , \\
\kappa_1^\pm &= \Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) \pm A_t \ , \\
(b): \quad \delta(P_b^2 - M_V^2) &= \frac{1}{|\kappa_1^+ - \kappa_1^-|} \left(\delta(\kappa_1 - \kappa_1^+) + \delta(\kappa_1 - \kappa_1^-) \right) \ , \\
\kappa_1^\pm &= -\Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) \pm A_t \ , \qquad (3.6)
\end{aligned}
$$

where $A_t = \sqrt{(n \cdot \Delta_t)^2 - \Delta_t^2 + M_V^2}$. In both cases $\theta(P^0_{a,b})$ selects the κ_1^- solution. Therefore,

$$
\begin{aligned}
P_a &= \Delta_t - (\Delta_t \cdot n)\, n + A_t n \ , \\
P_b &= -\Delta_t + (\Delta_t \cdot n)\, n + A_t n \ . \qquad (3.7)
\end{aligned}
$$

With these expressions we find for the amplitudes

$$
\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= -g^2\,\bar{u}(p's')\left[2\not{Q} - \frac{1}{M_V^2}\left((M_f-M_i)+\frac{1}{2}\not{n}(\kappa'-\kappa)-(\Delta_t\cdot n-A_t)\not{n}\right)\right.\\
&\qquad\times\left(\frac{1}{4}\left(s_{p'q'}-s_{pq}\right)+\frac{1}{4}\left(u_{pq'}-u_{p'q}\right)-(m_f^2-m_i^2)\right.\\
&\qquad\left.\left.-\,2(\Delta_t\cdot n-A_t)n\cdot Q\right)\right]u(ps)\\
&\quad\times\frac{1}{2A_t}\;\frac{1}{\Delta_t\cdot n+\frac{1}{2}\left(\kappa'+\kappa\right)-A_t+i\varepsilon}\;,\\
M^{(b)}_{\kappa'\kappa} &= -g^2\,\bar{u}(p's')\left[2\not{Q} - \frac{1}{M_V^2}\left((M_f-M_i)+\frac{1}{2}\not{n}(\kappa'-\kappa)-(\Delta_t\cdot n+A_t)\not{n}\right)\right.\\
&\qquad\times\left(\frac{1}{4}\left(s_{p'q'}-s_{pq}\right)+\frac{1}{4}\left(u_{pq'}-u_{p'q}\right)-(m_f^2-m_i^2)\right.\\
&\qquad\left.\left.-\,2(\Delta_t\cdot n+A_t)n\cdot Q\right)\right]u(ps)\\
&\quad\times\frac{1}{2A_t}\;\frac{1}{-\Delta_t\cdot n+\frac{1}{2}\left(\kappa'+\kappa\right)-A_t+i\varepsilon}\;.
\end{aligned}
\tag{3.8}
$$

Adding the two together and putting $\kappa' = \kappa = 0$ we should get back the Feynman expression (3.4)

$$
\begin{aligned}
M_{00} &= M^{(a)}_{00} + M^{(b)}_{00}\\
&= -g^2\bar{u}(p's')\left[2\not{Q} + \frac{(M_f-M_i)}{M_V^2}\left(m_f^2-m_i^2\right)\right]u(ps)\,\frac{1}{t-M_V^2+i\varepsilon}\\
&\quad -g^2\bar{u}(p's')\left[\not{n}\right]u(ps)\,\frac{2Q\cdot n}{M_V^2}\;.
\end{aligned}
\tag{3.9}
$$

Similar discrepancies are obtained when couplings containing higher spin fields ($s \geq 1$) are used. Therefore, it seems that the Kadyshevsky formalism does not yield the same results in these cases as the Feynman formalism when κ' and κ are put to zero. Since the real difference between Feynman formalism and Kadyshevsky formalism lies in the treatment of the TOP or θ-function also the difference in results should find its origin in this treatment.

In Feynman formalism derivatives are taken out of the TOP in order to get Feynman functions, which may yield extra terms. This is also the case in the above example [1]

$$
\begin{aligned}
T[\phi^\mu(x)\phi^\nu(y)] &= -\left[g^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{M_V^2}\right] i\Delta_F(x-y) - \frac{i\delta_0^\mu\delta_0^\nu}{M_V^2}\,\delta^4(x-y)\ , \\
S_{fi} &= (-i)^2 g^2 \int d^4x d^4y \left[\bar{\psi}\gamma_\mu\psi\right]_x T[\phi^\mu(x)\phi^\nu(y)] \left[\phi_a i\overleftrightarrow{\partial_\nu}\phi_b\right]_y\ , \\
\Rightarrow M_{extra} &= -g^2\bar{u}(p's')\,[\not{n}]\,u(ps)\,\frac{2Q\cdot n}{M_V^2}\ . \qquad (3.10)
\end{aligned}
$$

[2] If we include the extra term of (3.10) on the Feynman side we see that both formalisms yield the same result. So, that is cured.

Although we have exact equivalence between the two formalisms, the result, though covariant, is still n-dependent, i.e. frame-dependent. Of course this is not what we want. As it will turn out there is another source of extra terms exactly cancelling for instance the one that pops-up in our example ((3.9), (3.10)). There are two methods for getting these extra terms cancelling the one in (3.9) and (3.10): ones is more fundamental, which we will discuss in section 3.2 and one is more systematic and pragmatic, which we will discuss in section 3.4.

3.2 Takahashi & Umezawa Method

In order to find the second source of extra terms we deal with a set of local fields $\Phi_\alpha(x)$ in the Heisenberg and the Interaction representation, henceforth referred to as H.R. and I.R., respectively. In [12] (Ch 17) the relation between the fields in these two representations is, as in quantum mechanics, assumed to be

$$
\boldsymbol{\Phi}_\alpha(x) = U^{-1}(t)\ \Phi_\alpha(x)\ U(t)\ , \qquad (3.11)
$$

where the boldfaced fields are the fields in the H.R.

A covariant formulation of (3.11) was given by Tomonaga and Schwinger [25, 26]

$$
\boldsymbol{\Phi}_\alpha(x) = U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\ , \qquad (3.12)
$$

where σ is a space-like surface to which we will come back later.

[1] Of course that is why we have chosen such an example.

[2] If we include the n^μ-vector in the θ-function of the TOP, which would not make a difference as we have seen before, then we can make the replacement $\delta_0^\mu \to n^\mu$. This, to make the result more general.

According to the Haag theorem [24] such a unitary operator does not exist for theories with a non-trivial S-matrix. Therefore, we will *not* use (3.11) or (3.12). Also, in [12] it is explicitly mentioned that theories with couplings containing derivatives (and higher spin fields) are excluded and those theories are precisely the theories we are interested in. In order to be able to treat those theories we rely on the method of Takahashi and Umezawa [21, 22, 23], although it should be mentioned that a specific example of this theory was already given by Yang and Feldman [27]. We will describe this method in this section.

In doing so we start with the interaction Lagrangian, the fields of which are in the H.R.

$$\mathcal{L}_I = \mathcal{L}_I\Big(\mathbf{\Phi}_\alpha(x), \partial_\mu \mathbf{\Phi}_\alpha(x), \ldots\Big) \ . \tag{3.13}$$

From the interaction Lagrangian the equations of motion can be deduced

$$\begin{aligned} \Lambda_{\alpha\beta}(\partial)\ \mathbf{\Phi}_\beta(x) &= \mathbf{J}_\alpha(x) \ , \\ \textit{where} \qquad \mathbf{J}_\alpha(x) &= \frac{\partial \mathcal{L}_I}{\partial \mathbf{\Phi}_\alpha(x)} - \partial_\mu \frac{\partial \mathcal{L}_I}{\partial\left(\partial_\mu \mathbf{\Phi}_\alpha(x)\right)} + \ldots \ . \end{aligned} \tag{3.14}$$

The fields in the I.R. $\Phi_\alpha(x)$ are assumed to satisfy the free field equations

$$\Lambda_{\alpha\beta}(\partial)\ \Phi_\beta(x) = 0 \ , \tag{3.15}$$

and the (anti-) commutation relations

$$\Big[\Phi_\alpha(x), \Phi_\beta(y)\Big]_\pm = iR_{\alpha\beta}(\partial)\ \Delta(x-y) \ . \tag{3.16}$$

[3] Solutions to the equations (3.14) and (3.15) are the Yang-Feldman (YF) [27] equations

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha(x) + \int d^4y\ R_{\alpha\beta}(\partial)\ \Delta_G(x-y)\mathbf{J}_\beta(y) \ , \tag{3.17}$$

where $\Delta_G(x)$ satisfies

$$\left(\Box + m^2\right) \Delta_G(x-y) = \delta(x-y) \ . \tag{3.18}$$

[3] For scalars: $\Phi_\alpha(x) = \phi_\alpha(x)$, and $\Lambda_{\alpha\beta}(\partial) = \left(\Box + m^2\right)\delta_{\alpha\beta}$, $R_{\alpha\beta} = \delta_{\alpha\beta}$.
For spin-1/2 fermions: $\Phi_\alpha(x) = \psi_\alpha(x)$, and $\Lambda_{\alpha\beta}(\partial) = (i\partial\!\!\!/ - M)_{\alpha\beta}$, $R_{\alpha\beta}(\partial) = (i\partial\!\!\!/ + M)_{\alpha\beta}$. Etc. Unless mentioned otherwise ∂ means partial derivation with respect to x (∂_x).

It can taken to be a linear combination of Δ_{ret}, Δ_{adv}, $\bar{\Delta}$ and $-\Delta_F$, which are all solutions to (3.18). For the definitions of such propagators we refer to appendix D.

By introducing the vectors $D_a(x)$ and $\mathbf{j}_{\alpha;a}(x)$

$$\begin{aligned} D_a(x) &\equiv (1, \partial_{\mu_1}, \partial_{\mu_1}\partial_{\mu_2}, \ldots) \;, \\ \mathbf{j}_{\alpha;a}(x) &\equiv \left(-\frac{\partial \mathcal{L}_I}{\partial \mathbf{\Phi}_\alpha(x)} \;,\; -\frac{\partial \mathcal{L}_I}{\partial\left(\partial_{\mu_1} \mathbf{\Phi}_\alpha(x)\right)} \;,\; -\frac{\partial \mathcal{L}_I}{\partial\left(\partial_{\mu_1}\partial_{\mu_2} \mathbf{\Phi}_\alpha(x)\right)} \;,\; \ldots \right) \;, \end{aligned} \quad (3.19)$$

we can rewrite (3.17) as

$$\mathbf{\Phi}_\alpha(x) \;=\; \Phi_\alpha(x) - \int d^4y \; R_{\alpha\beta}(\partial) \; D_a(y) \; \Delta_{ret}(x-y) \cdot \mathbf{j}_{\beta;a}(y) \;. \quad (3.20)$$

Here, we have chosen $\Delta_G = \Delta_{ret}$.

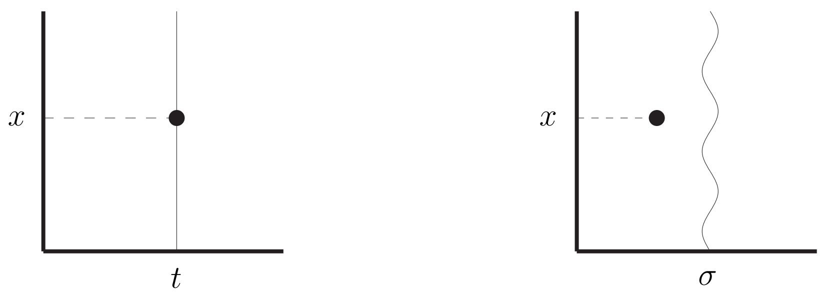


Figure 3.3: *In the left figure the spatial component x is a point on the surface t, forming the vector (t,x). In the right figure x is not a point on the surface σ*

Next, we introduce a free auxiliary field $\Phi_\alpha(x, \sigma)$, where σ is again a space-like surface and x does not necessarily lie on σ. This concept is illustrated in figure 3.3. We pose that it has the following form

$$\Phi_\alpha(x,\sigma) \equiv \Phi_\alpha(x) + \int_{-\infty}^{\sigma} d^4y \; R_{\alpha\beta}(\partial) D_a(y) \; \Delta(x-y) \cdot \mathbf{j}_{\beta;a}(y) \;, \quad (3.21)$$

[4] Although this equation (3.21) comes out of the blue, we are going to make a consistency check later.

[4]What is meant, here, is that the coordinates defining the surface σ form the upper bound of the integrals over y.

First, we combine (3.21) with (3.20) to come to

$$\mathbf{\Phi}_\alpha(x) \;=\; \Phi_\alpha(x/\sigma) + \frac{1}{2}\int d^4y \left[R_{\alpha\beta}(\partial)D_a(y), \epsilon(x-y)\right] \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ , \tag{3.22}$$

where x/σ means x on σ. This equation will be used to express the fields in the H.R. in terms of fields in the I.R.

From (3.21) we see that $\Phi_\alpha(x,-\infty) \equiv \Phi_\alpha(x)$. Furthermore, we impose that $\Phi_\alpha(x,\sigma)$ and $\Phi_\alpha(x)$ satisfy the same commutation relation, since they are both free. This means that there exists an unitary operator connecting the two in the following way

$$\Phi_\alpha(x,\sigma) = U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\ . \tag{3.23}$$

From this (3.23) it is easily proven that both fields indeed satisfy the same commutation relation

$$\begin{aligned}
[\Phi_\alpha(x,\sigma), \Phi_\beta(y,\sigma)] &= U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]U^{-1}[\sigma]\ \Phi_\beta(y)\ U[\sigma] \\
&\quad -U^{-1}[\sigma]\ \Phi_\beta(y)\ U[\sigma]U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma] \\
&= U^{-1}[\sigma]\ [\Phi_\alpha(x), \Phi_\beta(y)]\ U[\sigma] = iR_{\alpha\beta}(\partial)\ \Delta(x-y)\ ,
\end{aligned} \tag{3.24}$$

where the σ in the first line of (3.24) is for both $\Phi_\alpha(x,\sigma)$ and $\Phi_\beta(y,\sigma)$ the same.

Complementary to what is in [21, 22, 23] we explicitly show that the unitary operator mentioned in (3.23) is not any operator but the one connected to the S-matrix. We, therefore, consider *in*- and *out*-fields. Their relation to the fields in the H.R. is very similar to (3.17)

$$\begin{aligned}
\mathbf{\Phi}_\alpha(x) &= \Phi_{in,\alpha}(x) + \int d^4y\ R_{\alpha\beta}(\partial)\ \Delta_{ret}(x-y)\ \mathbf{J}_\beta(y) \\
&= \Phi_{out,\alpha}(x) + \int d^4y\ R_{\alpha\beta}(\partial)\ \Delta_{adv}(x-y)\ \mathbf{J}_\beta(y)\ ,
\end{aligned} \tag{3.25}$$

from which it can be deduced that

$$\begin{aligned}
\Phi_{out,\alpha}(x) - \Phi_{in,\alpha}(x) &= -\int_{-\infty}^{\infty} d^4y\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\ \mathbf{J}_\beta(y)\ , \\
&= \int_{-\infty}^{\infty} d^4y\ D_a(y)\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\ \mathbf{j}_{\beta,a}(y)\ .
\end{aligned} \tag{3.26}$$

Equation (3.25) makes clear that the choice of the Green function determines the choice of the free field (*in*- or *out*-field) to be used. In this light we make the following identification: $\Phi_\alpha(x,-\infty) \equiv \Phi_{in,\alpha}(x)$, since we have used the retarded Green function (text below (3.20)). With (3.25) and (3.26) we can also relate the *out*-field with the auxiliary field (3.21)

$$\begin{aligned} \Phi_\alpha(x,\sigma) &= \Phi_{in,\alpha}(x) + \int_{-\infty}^{\sigma} d^4y\ R_{\alpha\beta}(\partial)\ D_a(y)\ \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ , \\ \Phi_\alpha(x,\infty) &= \Phi_{in,\alpha}(x) + \int_{-\infty}^{\infty} d^4y\ R_{\alpha\beta}(\partial)\ D_a(y)\ \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ , \\ &= \Phi_{out,\alpha}(x)\ , \end{aligned} \tag{3.27}$$

These identifications we can use in (3.23) in order to relate $\Phi_{\alpha,in}(x)$ and $\Phi_{\alpha,out}(x)$

$$\begin{aligned} \Phi_{in,\alpha}(x) &= U^{-1}[-\infty]\ \Phi_\alpha(x)\ U[-\infty]\ , \\ \Phi_{out,\alpha}(x) &= U^{-1}[\infty]\ \Phi_\alpha(x)\ U[\infty]\ , \\ \Rightarrow \Phi_{\alpha,in}(x) &= U^{-1}[-\infty]U[\infty]\ \Phi_{\alpha,out}(x)\ U^{-1}[\infty]U[-\infty]\ . \end{aligned} \tag{3.28}$$

Obviously, the operator connecting the *in*- and *out*-fields is the S-matrix ($\Phi_{in,\alpha}(x) = S\Phi_{out,\alpha}S^{-1}$ [12]), from which we know its form (2.1). The connection between $U[\sigma]$ and the S-matrix is easily made

$$\begin{aligned} U[\sigma] &= T\left[exp\left(-i\int_{-\infty}^{\sigma} d^4x\mathcal{H}_I(x)\right)\right]\ , \\ U[\infty] &= S\ , \qquad U[-\infty] = 1\ . \end{aligned} \tag{3.29}$$

To make the connection with the interaction Hamiltonian we have to realize that the unitary operator in (3.24) is the time evolution operator and satisfies the Tomonaga-Schwinger equation

$$i\frac{\delta U[\sigma]}{\delta\sigma(x)} = \mathcal{H}_I(x;n)U[\sigma]\ . \tag{3.30}$$

Here, the interaction Hamiltonian will in general depend on the vector $n_\mu(x)$ locally normal to the surface $\sigma(x)$, i.e. $n^\mu(x)d\sigma_\mu = 0$. It is hermitean because of the unitarity of $U[\sigma]$. Then, from (3.23) and (3.30) one gets that

$$i\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(y)} = U^{-1}[\sigma]\left[\Phi_\alpha(x),\mathcal{H}_I(y;n)\right]\ U[\sigma]\ . \tag{3.31}$$

On the other hand, varying (3.21) with respect to $\sigma(y)$ gives

$$i\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(y)} = i\ D_a(y)\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ . \tag{3.32}$$

Comparing (3.31) and (3.32) gives the relation

$$\Big[\Phi_\alpha(x), \mathcal{H}_I(y;n)\Big] \;=\; i\, U[\sigma] \Big[D_a(y)\; R_{\alpha\beta}(\partial)\; \Delta(x-y) \cdot \mathbf{j}_{\beta;a}(y)\Big]\, U^{-1}[\sigma]\;. \tag{3.33}$$

This is the fundamental equation by which the interaction Hamiltonian must be determined.

In (3.21) we started with an equation that came out of the blue, though it had some nice features. In proceeding we posed that $\Phi_\alpha(x,\sigma)$ satisfies the same (anti-) commutation relation as $\Phi_\alpha(x)$. This is not so strange since they are both free fields. Having posed this we could show that the unitary operator $U[\sigma]$ connecting the two fields is on its turn connected to the S-matrix (3.29). Furthermore, we were able to construct the interaction Hamiltonian (3.33). Having obtained the interaction Hamiltonian we can use it in the unitary operator $U[\sigma]$ (3.29) and starting from (3.23) we proof in appendix A that equation (3.21) is indeed correct. In this way we have made a consistency check. The proof (appendix A) is not present in the original work of Takahashi and Umezawa.

In appendix B we also proof the relation between (3.21) and (3.23). There, the auxiliary field is introduced as (3.21) and we use the framework of Bogoliubov and collaborators [28, 29, 30], to which we refer to as BMP theory, to proof (3.23).

From (3.33) one can see that the interaction Hamiltonian will not only contain terms of order g, but also higher order terms. In our specific example of section 3.1, which continues in section 3.5, we will see that the g^2 terms in the interaction Hamiltonian is responsible for the cancellation. In this light we would also like to mention the specific example of scalar electrodynamics as described in [31], section 6-1-4. There the interaction Hamiltonian also contains a term of order g^2, which has the same purpose as in our case. The method described in [31] is not generally applicable, whereas the above described method is.

3.3 Remarks on the Haag Theorem

Here, we take a closer look at equation (3.12). This in light of the Haag theorem [24], which states that if there is an unitary operator connecting two representations at some time (as in (3.12)) both fields are free fields. This would lead to a triviality, which is not a preferable situation.

The question is whether we really have (3.12). In order to answer that question we look at (3.23) of the previous section (section 3.2). By assuming this equation we were in the end able to proof (3.21) (see appendix A)

$$\begin{aligned} \Phi_\alpha(x,\sigma) &= U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\ , \\ \Rightarrow \Phi_\alpha(x,\sigma) &= \Phi_\alpha(x) + \int_{-\infty}^{\sigma} d^4y\ R_{\alpha\beta}(\partial)\ D_a(y)\ \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ , \end{aligned} \quad (3.34)$$

that is, if $U[\sigma]$ satisfies that Tomonaga-Schwinger equation (3.30).

Now, we start with (3.20) and turn the argument around

$$\begin{aligned} \mathbf{\Phi}_\alpha(x) &= \Phi_\alpha(x) + \int_{-\infty}^{\infty} d^4y\ D_a(y)\ R_{\alpha\beta}(\partial)\ \theta[n(x-y)]\Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y) \\ &= \Phi_\alpha(x) + \int_{-\infty}^{\infty} d^4y\ \theta[n(x-y)]\ D_a(y)\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y) \\ &\quad + \int_{-\infty}^{\infty} d^4y\, [D_a(y)\ R_{\alpha\beta}(\partial), \theta[n(x-y)]]\, \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ , \\ \Rightarrow \mathbf{\Phi}_\alpha(x) &= U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]|_{x/\sigma} \\ &\quad + \frac{1}{2}\int_{-\infty}^{\infty} d^4y\, [D_a(y)\ R_{\alpha\beta}(\partial), \epsilon(x-y)]\, \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y)\ . \end{aligned} \quad (3.35)$$

[5] The above is different from what is exposed in [12] (ch 17.2). The difference is the commutator part of (3.35) and this term is non-zero for theories with couplings containing derivatives and higher spin fields, carefully excluded in the treatment of [12]. Therefore (3.35) could be seen as an extension of what is written in [12].

Returning to Haag's theorem we see that if the last term in (3.35) is absent there is an unitary operator connecting $\mathbf{\Phi}_\alpha(x)$ and $\Phi_\alpha(x)$ and therefore they are both free fields in the sense of the Haag theorem. Such theories can then be considered as trivial, although they can still be useful as effective theories.

In our application we use various interaction Lagrangians (for an overview see section 4.1) to be used in order to describe the various exchange and resonance processes. Whether or not the non-vanishing commutator part in (3.35) is present depends on the process under consideration. In the vector meson exchange diagrams (section 4.2.2) and in the spin-3/2 exchange and resonance diagrams (section 5.4 and 5.5.3) those commutator parts are non-vanishing. If we include pair suppression in the way we do in chapter 5 also in the spin-1/2 exchange and resonance diagrams the commutator parts will

[5] We have included the n^μ-vector in the first line of (3.35), which causes no effect in the same line of reasoning as in section 2.1 . The reason for this inclusion is that we can keep the surface σ general, though space-like.

be non-vanishing. So, if we take the model as a whole (all diagrams) then it is most certainly not trivial in the sense of the Haag theorem.

The already mentioned BMP theory is a Lehmann-Symanzik-Zimmermann (LSZ) [32] inspired S-matrix theory, constructed to avoid the use of an unitary operator as a mediator between fields in the H.R and the I.R.

3.4 Gross & Jackiw Method: Frame Dependence Analysis

As mentioned before we discuss in this section a more systematic and pragmatic way to find the second source of extra terms developed by Gross and Jackiw [20]. The main idea is to define the theory to be Lorentz invariant and n-independent. In practice this means: analyse the S-matrix for its n-dependence and, if necessary, introduce new contributions in order to make it n-independent.

In section 3.4.1 we describe and extend the original method of Gross and Jackiw and in section 3.4.2 we discuss its Kadyshevsky analog.

3.4.1 GJ Method in Feynman Formalism

In Feynman theory the S-matrix is defined as in (2.1). The main ingredient of this S-matrix is the TOP, which is then expanded using Wick's theorem in terms of TOPs of two fields only, although these TOPs may include (multiple) derivatives. Introducing the n^μ in the TOP in order to make it more general, it reads

$$T\left[A(x)B(x)\right] = \theta[n(x-y)]A(x)B(y) + \theta[n(y-x)]B(y)A(x) \ . \qquad (3.36)$$

[6] The essence of the Gross and Jackiw method [20] is to define a different TOP: the T^* product, which is by definition n-independent

$$T^*(x,y) = T(x,y;n) + \tau(x,y;n) \ , \qquad (3.37)$$

where $T(x,y;n)$ is defined in (3.36).

In analyzing the n-dependence we consider variations δn^μ in the same way as in section 2.4.3

$$P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T^*(x,y) \quad = \quad P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T(x,y;n) + P^{\alpha\beta}\frac{\delta}{\delta n^\beta}\tau(x,y;n) \equiv 0 \ . \qquad (3.38)$$

[6]Here, we assume that $A(x)$ and $B(y)$ are boson fields. This is not important for the following discussion.

In our applications we are interested in second order contributions to πN-scattering. Therefore, we analyze the n-dependence of the TOP of two interaction Hamiltonians, where we take it to be just $\mathcal{H}_I = -\mathcal{L}_I$

$$P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T(x,y;n) = P^{\alpha\beta}(x-y)_\beta \delta\left[n\cdot(x-y)\right]\left[\mathcal{H}_I(x),\mathcal{H}_I(y)\right] . \quad (3.39)$$

In general one has for equal time commutation relations

$$\delta(x^0-y^0)\left[\mathcal{H}_I(x),\mathcal{H}_I(y)\right] = \left[C + S^i\partial_i + Q^{ij}\partial_i\partial_j + \ldots\right]\delta^4(x-y) . \quad (3.40)$$

where the ellipsis stand for higher order derivatives. We will only consider (and encounter) up to quadratic derivatives. The S^i and Q^{ij} terms in (3.40) are known in the literature as *Schwinger terms*.

According to [20] equation (3.40) can be generalized to

$$\begin{aligned}\delta[n(x-y)]\left[\mathcal{H}_I(x),\mathcal{H}_I(y)\right] = &\left[C(n) + P^{\alpha\beta}S_\alpha(n)\partial_\beta\right. \\ &\left. + P^{\alpha\beta}P^{\mu\nu}Q_{\alpha\mu}(n)\partial_\beta\partial_\nu + \ldots\right]\delta^4(x-y) .\end{aligned} \quad (3.41)$$

It should be mentioned that in [20] only the first two terms on the rhs of (3.41) are considered.

Choosing $n^\mu = (1,\mathbf{0})$ we see that (3.40) and (3.41) indeed coincide. The expansion (3.41) we use via (3.39) in (3.38)

$$\begin{aligned}&P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T^*(x,y) = \\ = \;& P^{\alpha\beta}(x-y)_\beta\left[C(n) + P^{\mu\nu}S_\mu(n)\partial_\nu + P^{\mu\nu}P^{\rho\delta}Q_{\mu\rho}(n)\partial_\nu\partial_\delta\right]\delta^4(x-y) \\ & + P^{\alpha\beta}\frac{\delta}{\delta n^\beta}\tau(x,y;n) \\ = \;& -P^{\alpha\beta}S_\beta(n)\delta^4(x-y) - P^{\alpha\beta}P^{\mu\nu}\Big(Q_{\beta\mu}(n) + Q_{\mu\beta}(n)\Big)\partial_\nu\delta^4(x-y) \\ & + P^{\alpha\beta}\frac{\delta}{\delta n^\beta}\tau(x,y;n) = 0 .\end{aligned} \quad (3.42)$$

Here, we have used the fact that the TOP and therefore also the T^* product appears in the S-matrix as an integrand (2.1). We are therefore allowed to use partial integration for the $S_\alpha(n)$ and $Q_{\alpha\beta}(n)$ terms. As far as the $C(n)$ term is concerned, it disappears because $(x-y)_\beta\delta^4(x-y)$ is always zero. Furthermore, we have used the fact that $P^{\alpha\beta}$ is a projection operator.

From (3.42) we find the extra terms

$$\tau(x-y;n) = \int^n dn'^\beta\left[S_\beta(n') + P^{\mu\nu}\Big(Q_{\beta\mu}(n') + Q_{\mu\beta}(n')\Big)\partial_\nu\right]\delta^4(x-y) . \quad (3.43)$$

In principle the rhs of (3.43) can also contain a constant term, i.e. independent of n^μ. But since we are looking for n^μ-dependent terms only, this term is irrelevant.

3.4.2 GJ in Kadyshevsky formalism

Here, we discuss the Kadyshevsky analog of the Gross and Jackiw method. Before going into the details, a few points need to be taken into consideration. First of all, in Kadyshevsky formalism one may allow for external quasi particles, which are n^μ-dependent by definition. However, we are not looking for these terms. Therefore, we always have to take $\kappa' = \kappa = 0$.

A second drawback is that if look at the individual Kadyshevsky contributions, these contribution have different features then the sum of these contributions. As far as the sum is concerned we can use similar steps as in the previous section and on this basis we assign features to the individual contributions, which they in the strict sense do not have.

In Kadyshevsky formalism we use the S-matrix as exposed in (2.2). In this form the S-matrix consists of a product of θ-functions and fields. As in the previous section (section 3.4.1), the essence lies in the product of two fields. To this end we define the R-product

$$R\left[A(x)B(x)\right] = \theta[n(x-y)]A(x)B(y) \ . \tag{3.44}$$

Similar to before we introduce a new R-product: the R^*-product, which is n-independent

$$\begin{aligned} R^*(x,y) &= R(x,y;n) + \rho(x,y;n) \ , \\ P^{\alpha\beta}\frac{\delta}{\delta n^\beta}R^*(x,y) &= P^{\alpha\beta}\frac{\delta}{\delta n^\beta}R(x,y;n) + P^{\alpha\beta}\frac{\delta}{\delta n^\beta}\rho(x,y;n) \equiv 0 \ . \end{aligned} \tag{3.45}$$

Unfortunately, one can not expand the ordinary product of fields at equal times in a similar fashion as (3.40). This becomes clear when we look at the following example

$$\begin{aligned} \phi(x)\phi(y)|_0 &= N\left[\phi(x)\phi(y)\right]|_0 + \Delta^{(+)}(x-y)|_0 \ , \\ \dot{\phi}(x)\phi(y)|_0 &= N\left[\dot{\phi}(x)\phi(y)\right]|_0 - \frac{i}{2}\,\delta^3(x-y) \ . \end{aligned} \tag{3.46}$$

However, one should not forget that in Kadyshevsky formalism there are multiple contributions at a given order, which are added in the end (see section 2.3). Besides the contribution in (3.46) one should also consider the contribution

$$\begin{aligned} -\phi(y)\phi(x)|_0 &= -N\left[\phi(x)\phi(y)\right]|_0 - \Delta^{(-)}(x-y)|_0 \ , \\ -\phi(y)\dot{\phi}(x)|_0 &= -N\left[\dot{\phi}(x)\phi(y)\right]|_0 - \frac{i}{2}\,\delta^3(x-y) \ . \end{aligned} \tag{3.47}$$

So, if we take the sum then such an expansion is possible, because $\Delta^{(+)}(x - y)|_0 = \Delta^{(-)}(x - y)|_0$. Of course this is obvious since if we add (3.46) and (3.47) we exactly get (3.40), with $C = 0$ and $C = -i$, respectively.

The expansions for the product of two interaction Hamiltonians is

$$\begin{aligned} \delta(x^0 - y^0)\mathcal{H}_I(x)\mathcal{H}_I(y) &= \frac{1}{2}\left[C + S^i\partial_i + Q^{ij}\partial_i\partial_j\right]\delta^4(x - y) + \dots \ , \\ -\delta(x^0 - y^0)\mathcal{H}_I(y)\mathcal{H}_I(x) &= \frac{1}{2}\left[C + S^i\partial_i + Q^{ij}\partial_i\partial_j\right]\delta^4(x - y) + \dots \ , \end{aligned} \tag{3.48}$$

where the ellipsis indicate terms that can not be written as (derivatives acting on) δ-functions. However, these terms vanish when both contributions in (3.48) are added in the end, as mentioned before.

Just as in (3.41) we want to generalize (3.48) by including the vector n^μ. In (3.41) this was possible, because the commutator in (3.40) is a causal function. Unfortunately, the product of interaction Hamiltonians contains $\Delta^{(\pm)}$ propagators, as can be seen in the first lines of (3.46) and (3.47), which are non-causal functions. Therefore, a generalization as in (3.41) is not possible.

To solve this we call on the fact again that in the end we add both contributions, which does yield a causal function. Therefore, we pose the generalization of (3.48) to be

$$\begin{aligned} \delta[n(x - y)]\mathcal{H}_I(x)\mathcal{H}_I(y) &= \frac{1}{2}\Big[C(n) + P^{\alpha\beta}S_\alpha(n)\partial_\beta \\ &\qquad P^{\alpha\beta}P^{\mu\nu}Q_{\alpha\mu}\partial_\nu\partial_\beta\Big]\,\delta^4(x - y) \ , \\ -\delta[n(x - y)]\mathcal{H}_I(y)\mathcal{H}_I(x) &= \frac{1}{2}\Big[C(n) + P^{\alpha\beta}S_\alpha(n)\partial_\beta \\ &\qquad P^{\alpha\beta}P^{\mu\nu}Q_{\alpha\mu}\partial_\nu\partial_\beta\Big]\,\delta^4(x - y) \ . \end{aligned} \tag{3.49}$$

Following the same steps as in the previous section ((3.42) and the text below) we find for the summed ρ-functions exactly the same as we have found for τ-function (3.43).

Then, similarly as in the Feynman formalism, the introduction of the R^*-product in the Kadyshevsky formalism yields a covariant and frame independent S-matrix, and $S(Kadyshevky) = S(Feynman)$ for on-shell initial and final states.

3.5 Example: Part II

Having described two methods of getting the second source of extra terms (section 3.2 and 3.4) we are going to apply them here to the example of section 3.1. We start in section 3.5.1 by applying the Takahashi and Umezawa method and in section 3.5.2 we apply the Gross and Jackiw method.

3.5.1 Takahahsi & Umezawa Solution

Starting with the interaction Lagrangian (3.1) we get, according to (3.19), the following currents

$$\begin{aligned}
\boldsymbol{j}_{\phi_a,a} &= \left(-g\ i\partial_\mu \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu, ig\ \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu\right)\ , \\
\boldsymbol{j}_{\phi_b,a} &= \left(g\ i\partial_\mu \boldsymbol{\phi}_a \cdot \boldsymbol{\phi}^\mu, -ig\ \boldsymbol{\phi}_a \cdot \boldsymbol{\phi}^\mu\right)\ , \\
\boldsymbol{j}_{\psi,a} &= \left(-g\ \gamma_\mu \boldsymbol{\psi} \cdot \boldsymbol{\phi}^\mu, 0\right)\ , \\
\boldsymbol{j}_{\phi^\mu,a} &= \left(-g\ \boldsymbol{\phi}_a i\overleftrightarrow{\partial_\mu} \boldsymbol{\phi}_b - g\ \bar{\boldsymbol{\psi}}\gamma_\mu \boldsymbol{\psi}, 0\right)\ . \qquad (3.50)
\end{aligned}$$

Using (3.22) we can express the fields in the H.R. in terms of fields in the I.R., i.e. free fields

$$\begin{aligned}
\boldsymbol{\phi}_a(x) &= \phi_a(x/\sigma)\ , \\
\boldsymbol{\phi}_b(x) &= \phi_b(x/\sigma)\ , \\
\partial_\mu \boldsymbol{\phi}_a(x) &= \left[\partial_\mu \phi_a(x,\sigma)\right]_{x/\sigma} + \frac{1}{2}\int d^4y \left[\partial^x_\mu \partial^y_\nu, \epsilon(x-y)\right] \Delta(x-y) \left(ig\phi_b \cdot \phi^\nu\right)_y \\
&= \left[\partial_\mu \phi_a(x,\sigma)\right]_{x/\sigma} + ign_\mu \phi_b\ n\cdot\phi\ , \\
\partial_\mu \boldsymbol{\phi}_b(x) &= \left[\partial_\mu \phi_b(x,\sigma)\right]_{x/\sigma} + \frac{1}{2}\int d^4y \left[\partial^x_\mu \partial^y_\nu, \epsilon(x-y)\right] \Delta(x-y) \left(-ig\phi_a \cdot \phi^\nu\right)_y \\
&= \left[\partial_\mu \phi_b(x,\sigma)\right]_{x/\sigma} - ign_\mu \phi_a\ n\cdot\phi\ , \\
\boldsymbol{\psi}(x) &= \psi(x/\sigma)\ , \\
\boldsymbol{\phi}^\mu(x) &= \phi^\mu(x/\sigma) + \frac{1}{2}\int d^4y \left[\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right), \epsilon(x-y)\right] \Delta(x-y) \\
&\qquad \times \left(-g\phi_a i\overleftrightarrow{\partial_\nu}\phi_b - g\bar{\psi}\gamma_\nu\psi\right)_y \\
&= \phi^\mu(x/\sigma) - \frac{g\,n^\mu}{M_V^2}\left(\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b + \bar{\psi}\not{n}\psi\right)\ . \qquad (3.51)
\end{aligned}$$

As can be seen from (3.22) the first term on the rhs is a free field and the second term contains the current expressed in terms of fields in the H.R., which on their turn are expanded similarly. Therefore, one gets coupled equations. In solving these equations we assumed that the coupling constant is small and therefore considered only terms up to first order in the coupling constant in the expansion of the fields in the H.R. Practically speaking, the currents on the rhs of (3.51) are expressed in terms of free fields.

These expansions (3.51) are used in the commutation relation of the fields with the interaction Hamiltonian (3.33)

$$\begin{aligned}
[\phi_a(x), \mathcal{H}_I(y)] &= iU[\sigma]\Delta(x-y)\left[-g\ i\partial_\mu \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu + g\ \overleftarrow{i\partial_\mu} \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu\right]_y U^{-1}[\sigma] \\
&= i\Delta(x-y)\left[-g\ \overleftrightarrow{i\partial_\mu}\phi_b \cdot \phi^\mu \right.\\
&\quad \left. + \frac{g^2}{M_V^2}\ n\cdot \overleftrightarrow{i\partial}\,\phi_b \left(\phi_a n \cdot \overleftrightarrow{i\partial}\,\phi_b + \bar{\psi}\not{n}\psi\right) - g^2\ \phi_a (n\cdot\phi)^2\right]_y \\
[\psi(x), \mathcal{H}_I(y)] &= iU[\sigma](i\not{\partial} + M)\Delta(x-y)\left[-g\ \gamma_\mu \boldsymbol{\psi}\cdot\boldsymbol{\phi}^\mu\right]_y U^{-1}[\sigma] \\
&= i(i\not{\partial}+M)\Delta(x-y) \\
&\quad \times \left[-g\ \gamma_\mu\psi\cdot\phi^\mu + \frac{g^2}{M_V^2}\ \not{n}\psi\left(\phi_a n\cdot\overleftrightarrow{i\partial}\,\phi_b + \bar{\psi}\not{n}\psi\right)\right]_y , \\
[\phi^\mu(x), \mathcal{H}_I(y)] &= iU[\sigma]\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right)\Delta(x-y) \\
&\quad \times \left[-g\ \boldsymbol{\phi}_a \overleftrightarrow{i\partial_\nu}\boldsymbol{\phi}_b - g\ \bar{\boldsymbol{\psi}}\gamma_\nu\boldsymbol{\psi}\right]_y U^{-1}[\sigma] \\
&= i\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right)\Delta(x-y)\left[-g\ \phi_a\overleftrightarrow{i\partial_\nu}\phi_b - g\ \bar{\psi}\gamma_\nu\psi\right. \\
&\quad \left. - g^2\, n_\nu\, \phi_a^2 n\cdot\phi - g^2\, n_\nu\, \phi_b^2 n\cdot\phi\right]_y . \qquad (3.52)
\end{aligned}$$

As stated below (3.33) these are the fundamental equations from which the interaction Hamiltonian can be determined

$$\begin{aligned}
\mathcal{H}_I &= -g\ \phi_a\overleftrightarrow{i\partial_\mu}\phi_b\cdot\phi^\mu - g\ \bar{\psi}\gamma_\mu\psi\cdot\phi^\mu - \frac{g^2}{2}\ \phi_a^2(n\cdot\phi)^2 - \frac{g^2}{2}\ \phi_b^2(n\cdot\phi)^2 \\
&\quad + \frac{g^2}{2M_V^2}\ \left[\bar{\psi}\not{n}\psi\right]^2 + \frac{g^2}{M_V^2}\ \left[\bar{\psi}\not{n}\psi\right]\left[\phi_a n\cdot\overleftrightarrow{i\partial}\,\phi_b\right] + \frac{g^2}{2M_V^2}\ \left[\phi_a n\cdot\overleftrightarrow{i\partial}\,\phi_b\right]^2 \\
&\quad + O(g^3)\ldots . \qquad (3.53)
\end{aligned}$$

If equation (3.51) was solved completely, then the rhs of (3.51) would contain higher orders in the coupling constant and therefore also the interaction Hamiltonian (3.53). These terms are indicated by the ellipsis.

If we want to include the external quasi fields as in section 2.5, then the easy way to do this is to apply (2.41) straightforwardly. However, since we want to derive the interaction Hamiltonian from the interaction Lagrangian we would have to include a $\bar{\chi}(x)\chi(x)$ pair in (3.1) similar to (2.41). This would mean that the terms of order g^2 in (3.53) are quartic in the quasi field, where two of them can be contracted

$$\bar{\chi}(x)\chi(x)\bar{\chi}(x)\chi(x) = \bar{\chi}(x)\theta[n(x-x)]\chi(x) \; . \tag{3.54}$$

Defining the θ-function to be 1 in its origin we assure that all terms in the interaction Hamiltonian (3.53) relevant to πN-scattering are quadratic in the external quasi fields, even higher order terms in the coupling constant.

The only term of order g^2 in (3.53) that gives a contribution to the first order in the S-matrix describing πN-scattering is the second term on the second line in the rhs of (3.53). Its contribution to the first order in the S-matrix is

$$\begin{aligned} S_{fi}^{(1)} &= -i\int d^4x\mathcal{H}_I(x) = \frac{-ig^2}{M_V^2}\int d^4x\,[\bar{\psi}\not{n}\psi]\left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]_x \\ &= \frac{-ig^2}{M_V^2}\,\bar{u}(p's')\not{n}u(ps)n\cdot(q'+q)\;, \\ \Rightarrow M_{canc} &= g^2\,\bar{u}(p's')\not{n}u(ps)\frac{2n\cdot Q}{M_V^2}\;. \end{aligned} \tag{3.55}$$

Indeed we see that this term (3.55) cancels the extra term in (3.9).

3.5.2 Gross & Jackiw Solution

Here we apply the method of Gross and Jackiw as discussed in section 3.4 (or section 3.4.1, to be more specific).

As section 3.4.1 makes clear we need to determine the "covariantized" equal time commutator of interaction Hamiltonians

$$\begin{aligned} &\delta[n(x-y)]\,[\mathcal{H}_I(x),\mathcal{H}_I(y)] = \\ =\;& g^2\,\delta[n(x-y)]\Big[\phi_a(x)i\overleftrightarrow{\partial_\mu}\phi_b(x)\cdot\phi^\mu(x)+\bar{\psi}(x)\gamma_\mu\psi(x)\cdot\phi^\mu(x), \\ &\qquad \phi_a(y)i\overleftrightarrow{\partial_\nu}\phi_b(y)\cdot\phi^\nu(y)+\bar{\psi}(y)\gamma_\nu\psi(y)\cdot\phi^\nu(y)\Big]\;, \end{aligned} \tag{3.56}$$

where the different elements are calculated to be

$$\begin{aligned} \delta[n(x-y)]\,[\phi^\mu(x),\phi^\nu(y)] &= \frac{1}{M_V^2}\left(n^\mu P^{\nu\alpha}+n^\nu P^{\mu\alpha}\right)i\partial_\alpha\delta^4(x-y)\;, \\ \delta[n(x-y)]\left[i\partial_\mu\phi(x),i\partial_\nu\phi^\dagger(y)\right] &= -\left(n_\mu P_{\nu\alpha}+n_\nu P_{\mu\alpha}\right)i\partial^\alpha\delta^4(x-y)\;, \\ \delta[n(x-y)]\left\{\psi(x),\bar{\psi}(y)\right\} &= \not{n}\delta^4(x-y)\;, \\ \delta[n(x-y)]\left[i\partial_\mu\phi(x),\phi^\dagger(y)\right] &= n_\mu\delta^4(x-y)\;. \end{aligned} \tag{3.57}$$

Using these elements (3.56) becomes

$$
\begin{aligned}
&\delta[n(x-y)]\,[\mathcal{H}_I(x),\mathcal{H}_I(y)] \\
= &\left\{\frac{1}{M_V^2}\left([\psi\not{n}\psi]_x\left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_y + [\psi n_\mu\psi]_x\left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b\right]_y\right.\right.\\
&\quad + \left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b\right]_x [\psi n_\mu \psi]_y + \left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_x [\psi\not{n}\psi]_y \\
&\quad + [\psi\not{n}\psi]_y\,[\psi\gamma_\mu\psi]_x + [\psi\gamma_\mu\psi]_y\,[\psi\not{n}\psi]_x \\
&\quad \left. + \left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b\right]_y \left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_x + \left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_y \left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b\right]_x\right) \\
&+ \phi_a(y) n\cdot\phi(x)\phi_a(x)\phi_\mu(y) + \phi_a(y)\phi_\mu(x)\phi_a(x) n\cdot\phi(y) \\
&\left. + [\phi_b n\cdot\phi]_x\,[\phi_b\phi_\mu]_y + [\phi_b\phi_\mu]_x\,[\phi_b n\cdot\phi]_y\right\} P^{\mu\rho} i\partial_\rho \delta^4(x-y)\ . \qquad (3.58)
\end{aligned}
$$

Comparing this with (3.41) we see that the terms between curly brackets coincide with $-iS_\alpha(n)$. In calculating (3.58) we have neglected the $C(n)$ terms, since they do not give a contribution (see (3.42)) and as far as the $Q_{\alpha\beta}(n)$ terms are concerned, they are absent. Therefore, the τ-function, representing the compensating terms, becomes by means of (3.43) and (3.58)

$$
\begin{aligned}
\tau(x-y;n) &= ig^2\left[\frac{1}{M_V^2}\left(2\,[\psi\not{n}\psi]\left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b\right] + [\psi\not{n}\psi]^2 + \left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b\right]^2\right)\right.\\
&\qquad \left. + \phi_a^2 (n\cdot\phi)^2 + \phi_b^2(n\cdot\phi)^2\right]\delta^4(x-y)\ . \qquad (3.59)
\end{aligned}
$$

Its contribution to πN-scattering S-matrix and amplitude is

$$
\begin{aligned}
S^{(2)}_{canc} &= \frac{(-i)^2}{2!}\int d^4x d^4y\ \frac{2ig^2}{M_V^2}\ [\psi\not{n}\psi]\left[\phi_a n\cdot \overleftrightarrow{i\partial}\,\phi_b)\right]\delta^4(x-y)\ ,\\
M_{canc} &= g^2\ \bar{u}(p's')\not{n}u(ps)\frac{2n\cdot Q}{M_V^2}\ , \qquad (3.60)
\end{aligned}
$$

which is the same expression as the amplitude derived from the Takahashi-Umezawa scheme in (3.55).

3.5.3 $\bar{P}$ Approach

From the forgoing sections we have seen that if we add all contributions, results in the Feynman formalism and in the Kadyshevsky formalism are the same (of course we need to put $\kappa' = \kappa = 0$). Also, section 3.1 taught us that if we bring out the derivatives out of the TOP in Feynman formalism not only do we get Feynman functions, but also the n-dependent contact terms cancel out. Unfortunately, this is not the case in Kadyshevsky formalism. There, all n-dependent contact terms cancel out after adding up the amplitudes. So, when calculating an amplitude according to the Kadyshevsky rules in section 2.3 one always has to keep in mind the contributions as described in section 3.2 and 3.4. For practical purposes this is not very convenient.

Inspired by the Feynman procedure we could also do the same in Kadyshevsky formalism, namely let the derivatives not only act on the vector meson propagator [7] but also on the quasi particle propagator (θ-function). In doing so, we know that all contact terms cancel out; just as in Feynman formalism.

We show the above in formula form.

$$\begin{aligned}
& \theta[n(x-y)]\partial_x^\mu\partial_x^\nu\Delta^{(+)}(x-y) + \theta[n(y-x)]\partial_x^\mu\partial_x^\nu\Delta^{(+)}(y-x) \\
= \; & \partial_x^\mu\partial_x^\nu\theta[n(x-y)]\Delta^{(+)}(x-y) + \partial_x^\mu\partial_x^\nu\theta[n(y-x)]\Delta^{(+)}(y-x) \\
& +in^\mu n^\nu\delta^4(x-y) \\
= \; & \frac{i}{2\pi}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\int\frac{d^4P}{(2\pi)^3}\,\theta(P^0)\delta(P^2-M_V^2)\partial_x^\mu\partial_x^\nu \\
& \times\left(e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}\right) + in^\mu n^\nu\delta^4(x-y) \\
= \; & \frac{i}{2\pi}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\int\frac{d^4P}{(2\pi)^3}\,\theta(P^0)\delta(P^2-M_V^2)\left(-\bar{P}_\mu\bar{P}_\nu\right) \\
& \times\left(e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}\right) \\
& +in^\mu n^\nu\delta^4(x-y)\ , \qquad (3.61)
\end{aligned}$$

where $\bar{P} = P + n\kappa_1$. In this way the second order in the S-matrix becomes

$$\begin{aligned}
S_{fi}^{(2)} \; = \; & -g^2\int d^4x d^4y\,[\bar{u}(p's')\gamma_\mu u(ps)]\,(q'+q)_\nu\, e^{-ix(q-q')}e^{iy(p'-p)} \\
& \times\frac{i}{2\pi}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\int\frac{d^4P}{(2\pi)^3}\theta(P^0)\delta(P^2-M_V^2)\left(-g^{\mu\nu}+\frac{\bar{P}^\mu\bar{P}^\nu}{M_V^2}\right) \\
& \times\left(e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)}e^{in\kappa' x-in\kappa y} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}e^{-in\kappa x+in\kappa' y}\right) \\
& +ig^2\int d^4x\,[\bar{u}(p's')\not{n}u(ps)]\,n\cdot(q'+q)\,e^{-ix(q-q'-p'+p-n\kappa'+n\kappa)}\ . \qquad (3.62)
\end{aligned}$$

We see that the second term on the rhs of (3.62) brings about an amplitude, which is exactly the same as in (3.9) and (3.10) and is to be cancelled by (3.55) and (3.60).

[7] With 'propagator' we mean the $\Delta^+(x-y)$ and not the Feynman propagator $\Delta_F(x-y)$.

Performing the various integrals correctly we get

$$
\begin{aligned}
(a) \;\Rightarrow\; &\left\{ \begin{array}{lcl} \kappa_1 &=& \Delta_t \cdot n - A_t + \frac{1}{2}(\kappa' + \kappa) \\ \bar{P} &=& \Delta_t + \frac{1}{2}(\kappa' + \kappa)\, n \end{array} \right. \\
(b) \;\Rightarrow\; &\left\{ \begin{array}{lcl} \kappa_1 &=& -\Delta_t \cdot n - A_t + \frac{1}{2}(\kappa' + \kappa) \\ \bar{P} &=& -\Delta_t + \frac{1}{2}(\kappa' + \kappa)\, n \end{array} \right. .
\end{aligned}
\tag{3.63}
$$

This yields for the invariant amplitudes

$$
\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= -g^2\; \bar{u}(p's') \left[2\not{Q} + \frac{1}{M_V^2} \left((M_f - M_i) + \frac{1}{2}(\kappa' - \kappa)\not{n} + \not{n}\bar{\kappa} \right) \right. \\
&\quad \times \left. \left(\left(m_f^2 - m_i^2\right) + \frac{1}{4}\left(s_{pq} - s_{p'q'} + u_{p'q} - u_{pq'}\right) - 2\bar{\kappa} Q \cdot n \right) \right] u(ps) \\
&\quad \times \frac{1}{2A_t}\; \frac{1}{\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon} , \\
M^{(b)}_{\kappa'\kappa} &= -g^2\; \bar{u}(p's') \left[2\not{Q} + \frac{1}{M_V^2} \left((M_f - M_i) + \frac{1}{2}(\kappa' - \kappa)\not{n} - \not{n}\bar{\kappa} \right) \right. \\
&\quad \times \left. \left(\left(m_f^2 - m_i^2\right) + \frac{1}{4}\left(s_{pq} - s_{p'q'} + u_{p'q} - u_{pq'}\right) + 2\bar{\kappa} Q \cdot n \right) \right] u(ps) \\
&\quad \times \frac{1}{2A_t}\; \frac{1}{-\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon} \\
M &= M^{(a)}_{00} + M^{(b)}_{00} \\
&= -g^2 \bar{u}(p's') \left[2\not{Q} + \frac{(M_f - M_i)}{M_V^2} \left(m_f^2 - m_i^2\right) \right] u(ps) \frac{1}{t - M_V^2 + i\varepsilon} ,
\end{aligned}
\tag{3.64}
$$

where $\bar{\kappa} = \frac{1}{2}(\kappa' + \kappa)$. As before we get back the Feynman expression for the amplitude if we add both amplitudes obtained in Kadyshevsky formalism and put $\kappa' = \kappa = 0$. The big advantage of this procedure is that we do not need to worry about the contribution n-dependent contact terms because they cancelled out when introducing $\bar{P}$.

It should be noticed however that the $\bar{P}$-method is only possible when both Kadyshevsky contributions at second order are added. This becomes clear when looking at the first two lines of (3.61): Letting the derivatives also act on the θ-function gives compensating terms for the $\Delta^{(+)}(x-y)$-part and for the $\Delta^{(-)}(x-y)$-part. Only when added together they combine to the $\delta^4(x-y)$-part.

Also it becomes clear from (3.61) that at least two derivatives are needed to generate the $\delta^4(x-y)$-part. Therefore, when there is only one derivative, for instance in the case of baryon exchange (so, no derivatives in coupling only in the propagator) at second order, the $\delta^4(x-y)$-part is not present and it is not necessary to use the $\bar{P}$-method. In these cases it does not matter for the summed diagrams whether or not the $\bar{P}$-method is used, however for the individual diagrams it does make a difference. This ambiguity is absent in Feynman theory, there derivatives are always taken out of the TOP (which is similar to the $\bar{P}$-method, as discussed above) in order to come to Feynman propagators.

3.6 Conclusion

We end this chapter by summarizing the main results. In section 3.1 we have shown that it seems that the Kadyshevsky formalism gives different results than the Feynman formalism, particularly for couplings containing derivatives and/or higher spin fields. This seems very odd since both formalisms can be deduced from the same S-matrix ((2.1), (2.2)).

When taking a closer look it turns out that extra terms in Kadyshevsky formalism are also present in Feynman formalism. Hence, both formalisms yield the same result. Unfortunately, this result, though covariant, is frame-dependent. After a systematic analysis of this n-dependence, the n-dependent terms can be removed pragmatically by using the method of Gross & Jackiw, described in section 3.4. The important idea behind this is that a covariant and frame-independent theory is defined and therefore starting point.

A more fundamental method to remove the extra, n-dependent terms is developed by Takahashi and Umezawa, which is described in section 3.2. Here, the interaction Hamiltonian contains orders of g^2, which gives a non-vanishing contribution in the first order of the S-matrix. This contribution cancels exactly the unwanted n-dependent terms. Also, we have introduced and discussed the $\bar{P}$-method and we have shown in appendix B the use of BMP theory in light of the TU method.

We stress that both methods: GJ and TU give the same results. In the Kadyshevsky and the Feynman formalism the final results are therefore not only the same, but also covariant and frame-independent. This is shown in section 3.5.

Chapter 4

Application: Pion-Nucleon Scattering

In the previous chapters (chapter 2 and 3) we have presented the Kadyshevsky formalism in great detail. Now, we are going to apply it to the pion-nucleon system, although we present it in such a way that it can easily be extended to other meson-baryon systems. The isospin factors are not included in our treatment; we are only concerned about the Lorentz and Dirac structure. For the details about the isospin factors we refer to [11].

In section 4.1 we describe the ingredients of the model by discussing the exchanged particles at tree level and the interaction Lagrangian densities that describe the vertices. The meson exchange processes are discussed in section 4.2. The discussion of the baryon exchange processes (including pair suppression) is postponed to chapter 5.

4.1 Ingredients

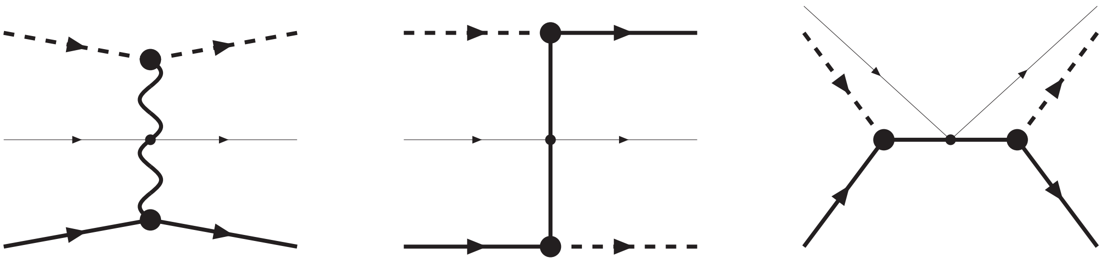
The ingredients of the model are tree level, exchange amplitudes as mentioned before. These amplitudes serve as input for the integral equation. Very similar to what is done in [11] we consider for the amplitudes the exchanged particles as in table 4.1. Graphically, this is shown in figure 4.1.

Contrary to [11] we do not consider the exchange of the tensor mesons, since their contribution is small. The inclusion of them can be regarded as an extension of this work.

For the description of the amplitudes we need the interaction Lagrangians

Channel	Exchanged particle
t	f_0, σ, P, ρ
u	$N, N^*, S_{11}, \Delta_{33}$
s	$N, N^*, S_{11}, \Delta_{33}$

Table 4.1: *Exchanged particles in the various channels*



t : f_0, σ, P, ρ u : N, N^*, Δ s : N, N^*, Δ

Figure 4.1: *Tree level amplitudes as input for integral equation. The inclusion of the quasi particle lines is schematically. Therefore, the diagrams represent either the (a) or the (b) diagram.*

- Triple meson vertices

$$\mathcal{L}_{SPP} = g_{PPS}\,\phi_{P,a}\phi_{P,b}\cdot\phi_S\ , \tag{4.1a}$$

$$\mathcal{L}_{VPP} = g_{VPP}\left(\phi_a i\overleftrightarrow{\partial}_\mu\phi_b\right)\phi^\mu\ , \tag{4.1b}$$

where S, V, P stand for scalar, vector and pseudo scalar to indicate the various mesons.

- Meson-baryon vertices

$$\mathcal{L}_{SNN} = g_S\,\bar{\psi}\psi\cdot\phi_S\ , \tag{4.2a}$$

$$\mathcal{L}_{VNN} = g_V\,\bar{\psi}\gamma_\mu\psi\ \phi^\mu - \frac{f_V}{2M_V}\ i\partial^\mu\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)\cdot\phi^\nu\ , \tag{4.2b}$$

$$\mathcal{L}_{PV} = \frac{f_{PV}}{m_\pi}\,\bar{\psi}\gamma_5\gamma_\mu\psi\cdot\partial^\mu\phi_P\ , \tag{4.2c}$$

$$\mathcal{L}_{V} = \frac{f_{V}}{m_\pi}\,\bar{\psi}\gamma_\mu\psi\cdot\partial^\mu\phi_P\ , \tag{4.2d}$$

where $\sigma_{\mu\nu} = \frac{1}{2}\left[\gamma_\mu, \gamma_\nu\right]$. The coupling constants f_V of (4.2b) and (4.2d) do not necessarily coincide.

We have chosen (4.2b) in such a way that the vector meson couples to a current, which may contain a derivative. This is a bit different from [11, 33], where the derivative acts on the vector meson. In Feynman theory this does not make a difference, however it does in Kadyshevsky formalism, because of the presence of the quasi particles.

Equation (4.2c) is used to describe the exchange (u, s-channel) of the nucleon and Roper (N^*) and (4.2d) is used for the S_{11} exchange. This, because of their intrinsic parities. Note, that we could also have chosen the pseudo scalar and scalar couplings for these exchanges. However, since the interactions (4.2c) and (4.2d) are also used in [11] and in chiral symmetry based models, we use these interactions.

- $\pi N \Delta_{33}$ vertex

$$\mathcal{L}_{\pi N\Delta} = g_{gi}\, \epsilon^{\mu\nu\alpha\beta} \left(\partial_\mu \bar{\Psi}_\nu\right) \gamma_5 \gamma_\alpha \psi \left(\partial_\beta \phi\right) + g_{gi}\, \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_5 \gamma_\alpha \left(\partial_\mu \Psi_\nu\right) \left(\partial_\beta \phi\right) \ , \tag{4.3}$$

The use of this interaction Lagrangian differs from the one used in [11]. We will come back to this in section 5.4.

An other important ingredient of the model is the use of form factors. We postpone the discussion of them to chapter 6.

4.2 Meson Exchange

Here, we proceed with the discussion of the meson exchange processes. In this section we give the amplitudes for meson-baryon scattering or pion-nucleon scattering, specifically, meaning that we take equal initial and final states ($M_f = M_i = M$ and $m_f = m_i = m$, where M and m are the masses of the nucleon and pion, respectively). The results for general meson-baryon initial and final states are presented in appendix C.

4.2.1 Scalar Meson Exchange

For the description of the scalar meson exchange processes at tree level, graphically shown in figure 4.2, we use the interaction Lagrangians (4.1a) and (4.2a), which lead to the vertices

$$\begin{aligned} \Gamma_{PPS} &= g_{PSS} \ , \\ \Gamma_S &= g_S \ . \end{aligned} \tag{4.4}$$

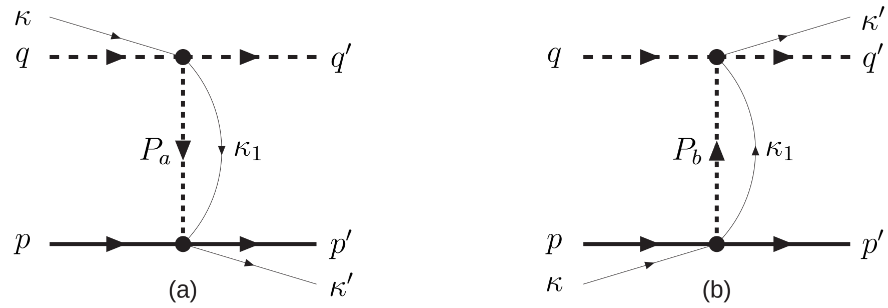


Figure 4.2: *Scalar meson exchange*

Applying the Kadyshevsky rules as discussed in section 2.3, the amplitudes read

$$M^{a,b}_{\kappa'\kappa} = g_{PSS}g_S \int \frac{d\kappa_1}{\kappa_1 + i\varepsilon} \left[\bar{u}(p's')u(ps)\right] \theta(P^0)\delta(P^2 - M^2) \ , \qquad (4.5)$$

The κ_1-integral is discussed in (3.6) and (3.7). We, therefore, give the results immediately

$$\begin{aligned} M^{(a)}_{\kappa'\kappa} &= g_{PSS}g_S \left[\bar{u}(p's')u(ps)\right] \frac{1}{2A_t} \cdot \frac{1}{\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon} \ , \\ M^{(b)}_{\kappa'\kappa} &= g_{PSS}g_S \left[\bar{u}(p's')u(ps)\right] \frac{1}{2A_t} \cdot \frac{1}{-\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon} \ , \qquad (4.6) \end{aligned}$$

where $\bar{\kappa}$ (text below (3.64)), Δ_t (text below (3.3)) and A_t (text below (3.6)) are already defined.

Adding the two together and putting $\kappa' = \kappa = 0$ we get

$$M_{00} = g_{PSS}g_S \left[\bar{u}(p's')u(ps)\right] \frac{1}{t - M_S^2 + i\varepsilon} \ , \qquad (4.7)$$

which is Feynman result [11].

In section 2.4.3 we discussed the n-dependence of the Kadyshevsky integral equation. In order to do that we need to know the n-dependence of the amplitude (2.30)

$$\begin{aligned} M^{(a+b)}_{0\kappa} &= M^{(a)}_{0\kappa} + M^{(b)}_{0\kappa}, \\ &= g_{PSS}g_S \left[\bar{u}(p's')u(ps)\right] \frac{A_t - \frac{\kappa}{2}}{A_t} \frac{1}{(\Delta_t \cdot n)^2 - (\frac{\kappa}{2} - A_t)^2 + i\varepsilon} \ , \\ \frac{\partial M^{a+b}_{0\kappa}}{\partial n^\beta} &= \kappa \ g_{PSS}g_S \left[\bar{u}(p's')u(ps)\right] \\ &\quad \times \frac{n \cdot \Delta_t (\Delta_t)_\beta}{2A_t^3} \frac{(n \cdot \Delta_t)^2 - 3A_t^2 - \frac{\kappa^2}{4} + 2\kappa A_t}{\left((n \cdot \Delta_t)^2 - \left(A_t - \frac{\kappa}{2}\right)^2 + i\varepsilon\right)^2} \ . \qquad (4.8) \end{aligned}$$

If we would only consider scalar meson exchange in the Kadyshevsky integral equation (2.23) the integrand would be of the form (2.32), where $h(\kappa)$ would by itself be of order $O(\frac{1}{\kappa^2})$ as can be seen from (4.8). Therefore, the phenomenological "form factor" (2.34) would not be needed.

Since there is no propagator as far as Pomeron exchange is concerned, the Kadyshevsky amplitude is the same as the Feynman amplitude for Pomeron exchange [11]

$$M_{\kappa'\kappa} \;=\; \frac{g_{PPP}g_P}{M}\left[\bar{u}(p's')u(p)\right] \;. \tag{4.9}$$

4.2.2 Vector Meson Exchange

In order to describe vector meson exchange at tree level we use the interaction Lagrangians as in (4.1b) and (4.2b). From these interaction Lagrangians we distillate the vertices

$$\begin{aligned}\Gamma^{\mu}_{VPP} &= g_{VPP}\,(q'+q)^{\mu}\;,\\ \Gamma^{\mu}_{VNN} &= g_V\,\gamma^{\mu} + \frac{f_V}{2M_V}\,(p'-p)_{\alpha}\,\sigma^{\alpha\mu}\;.\end{aligned} \tag{4.10}$$

The Kadyshevsky diagrams representing vector meson exchange are already exposed in figure 3.2. Applying the Kadyshevsky rules of section 2.3 and chapter 3 we obtain the following amplitudes

$$\begin{aligned}M^{(a)}_{\kappa'\kappa} &= -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V\not{Q} - \frac{2g_V}{M_V^2}\bar{\not{P}}_a\bar{P}_a\cdot Q + \frac{f_V}{2M_V}\Bigg(\left(\not{p}'-\not{p}\right)\left(\not{q}'+\not{q}\right)\\ &\quad -(p'-p)\cdot(q'+q) - \frac{1}{M_V^2}\Bigg(\left(\not{p}'-\not{p}\right)\bar{\not{P}}_a - (p'-p)\cdot\bar{P}_a\Bigg)\\ &\quad \times\bar{P}_a\cdot(q'+q)\Bigg)\Bigg]\,u(ps)\,\frac{1}{2A_t}\,\frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\;,\\ M^{(b)}_{\kappa'\kappa} &= -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V\not{Q} - \frac{2g_V}{M_V^2}\bar{\not{P}}_b\bar{P}_b\cdot Q + \frac{f_V}{2M_V}\Bigg(\left(\not{p}'-\not{p}\right)\left(\not{q}'+\not{q}\right)\\ &\quad -(p'-p)\cdot(q'+q) - \frac{1}{M_V^2}\Bigg(\left(\not{p}'-\not{p}\right)\bar{\not{P}}_b - (p'-p)\cdot\bar{P}_b\Bigg)\\ &\quad \times\bar{P}_b\cdot(q'+q)\Bigg)\Bigg]\,u(ps)\,\frac{1}{2A_t}\,\frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\;,\end{aligned} \tag{4.11}$$

which lead, after some (Dirac) algebra, to

$$
\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V \not{Q} \\
&\quad -\frac{g_V}{M_V^2}\,\kappa'\not{n}\left(\frac{1}{4}\,(s_{p'q'}-s_{pq}+u_{pq'}-u_{p'q})+2\bar{\kappa}Q\cdot n\right) \\
&\quad +\frac{f_V}{2M_V}\Bigg(4M\not{Q}+\frac{1}{2}\,(u_{pq'}+u_{p'q})-\frac{1}{2}\,(s_{p'q'}+s_{pq}) \\
&\qquad -\frac{1}{M_V^2}\Bigg(M^2+m^2-\frac{1}{2}\left(\frac{1}{2}(t_{p'p}+t_{q'q})+u_{pq'}+s_{pq}\right) \\
&\qquad\qquad +2M\not{n}\kappa'+\frac{1}{4}\,(\kappa'-\kappa)^2-(p'+p)\cdot n\bar{\kappa}\Bigg) \\
&\qquad\quad \times\left(\frac{1}{4}(s_{p'q'}-s_{pq})+\frac{1}{4}(u_{pq'}-u_{p'q})+2\bar{\kappa}n\cdot Q\right)\Bigg)\Bigg]\,u(ps) \\
&\quad \times\frac{1}{2A_t}\,\frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\,, \\
M^{(b)}_{\kappa'\kappa} &= -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V \not{Q} \\
&\quad +\frac{g_V}{M_V^2}\,\kappa\not{n}\left(\frac{1}{4}\,(s_{p'q'}-s_{pq}+u_{pq'}-u_{p'q})-2\bar{\kappa}Q\cdot n\right) \\
&\quad +\frac{f_V}{2M_V}\Bigg(4M\not{Q}+\frac{1}{2}\,(u_{pq'}+u_{p'q})-\frac{1}{2}\,(s_{p'q'}+s_{pq}) \\
&\qquad -\frac{1}{M_V^2}\Bigg(M^2+m^2-\frac{1}{2}\left(\frac{1}{2}(t_{p'p}+t_{q'q})+u_{pq'}+s_{pq}\right) \\
&\qquad\qquad -2M\not{n}\kappa+\frac{1}{4}\,(\kappa'-\kappa)^2+(p'+p)\cdot n\bar{\kappa}\Bigg) \\
&\qquad\quad \times\left(\frac{1}{4}(s_{p'q'}-s_{pq})+\frac{1}{4}(u_{pq'}-u_{p'q})-2\bar{\kappa}n\cdot Q\right)\Bigg)\Bigg]\,u(ps) \\
&\quad \times\frac{1}{2A_t}\,\frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\,.
\end{aligned}
\tag{4.12}
$$

The sum of the two in the limit of $\kappa'=\kappa=0$ yields

$$
\begin{aligned}
M_{00} &= -g_{VPP}\;\bar{u}(p's')\left[2g_V\not{Q}+\frac{f_V}{2M_V}\left((u-s)+4M\not{Q}\right)\right]u(ps) \\
&\quad\times\frac{1}{t-M_V^2+i\varepsilon}\,,
\end{aligned}
\tag{4.13}
$$

which is, again, the Feynman result [11].

Just as in the previous section (section 4.2.1) we study the n-dependence of the amplitude. This, in light of the n-dependence of the Kadyshevsky integral equation (see section 2.4.3).

$$
\begin{aligned}
M_{0\kappa}^{(a+b)} &= M_{0\kappa}^{(a)} + M_{0\kappa}^{(b)}\,, \\
&= -g_{VPP}\;\bar{u}(ps)\left[2g_V \not{Q} + \frac{f_V}{2M_V}\left(4M\not{Q} + \frac{1}{2}\left(u_{pq'} + u_{p'q}\right)\right.\right. \\
&\qquad \left.\left. -\frac{1}{2}\left(s_{p'q'} + s_{pq}\right)\right)\right] u(ps)\frac{A_t - \frac{\kappa}{2}}{A_t}\frac{1}{(\Delta_t\cdot n)^2 - \left(A_t - \frac{\kappa}{2}\right)^2 + i\varepsilon} \\
&\quad -\frac{g_V f_V \kappa}{2M_V^3}\;\bar{u}(ps)\left[\frac{1}{2}(p'+p)\cdot n(Q\cdot n)\kappa\left(A_t - \frac{\kappa}{2}\right)\right. \\
&\qquad +\frac{1}{8}(p'+p)\cdot n\left(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}\right)\Delta_t\cdot n \\
&\qquad -n\cdot Q\left(M^2 + m^2 - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\right) + \frac{\kappa^2}{4}\right) \\
&\qquad \left.\times\Delta_t\cdot n\right] u(ps)\,\frac{1}{A_t}\,\frac{1}{(\Delta_t\cdot n)^2 - (\frac{\kappa}{2} - A_t)^2 + i\varepsilon} \\
&\quad +\frac{g_{VPP}\kappa}{M_V^2}\;\bar{u}(p's')\left[\not{n}\left(g_V + \frac{f_V M}{M_V}\right)\left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q})\right.\right. \\
&\qquad \left.\left. + \kappa n\cdot Q\right)\right] u(ps)\,\frac{1}{2A_t}\;\frac{1}{\Delta_t\cdot n + \frac{\kappa}{2} - A_t + i\varepsilon}\;. \qquad (4.14)
\end{aligned}
$$

Differentiating this with respect to n^α in the same way as in (4.8) we know that the result will contain an overall factor of κ. This can be seen as follows: The first term in (4.14) is very similar to $M_{0\kappa}^{(a+b)}$ in (4.8). Therefore, the overall factor of κ when differentiating with respect to n^α is obvious. All other terms in (4.14) contain already an overall factor of κ, which does not change when differentiating.

As can be seen from (4.14) the numerator is of higher degree in κ then the denominator. Therefore, the function $h(\kappa)$ in (2.32) will not be of order $O(\frac{1}{\kappa^2})$ and the "form factor" (2.34) is necessary.

In (4.12) as well as in (4.6) we have taken u and $\bar{u}$ spinors. The reason behind this is pair suppression which we will discuss in the next chapter (chapter 5).

Chapter 5

Baryon Exchange and Pair Suppression

In this chapter we deal with the baryon exchange sector of the model. We construct tree level amplitudes for baryon exchange and resonance or, to put it in other words, u- and s-channel baryon exchange diagrams.

In [11] pair suppression is assumed by considering positive states in the integral equation only (see text below (2.19)). Here, we implement pair suppression formally. This is done by discussing the formalism in section 5.1 and applying it in sections 5.2, 5.3 and 5.4, where we have distinguished for various couplings. The amplitudes are calculated in 5.5.

5.1 Pair Suppression Formalism

To understand the idea of pair suppression at low energy, picture a general meson-baryon (MB) vertex in terms of their constituent quarks as in the QPC model (see figure 5.1). As stated in [34] every time a quark - anti-quark ($q\bar{q}$) pair is created from the vacuum the vertex is damped. This idea is supported by [35] whose author considers a vertex creating a baryon - anti-baryon ($B\bar{B}$) pair in a large N, $SU(N)$ theory [1]. Such a vertex is comparable to figure 5.1(b), but now $N-1$ pairs need to be created. It is claimed in [35] that such vertices are indeed suppressed. Although it is questionable whether $N = 3$ is really large, we assume that pair suppression holds for $SU_{(F)}(3)$ theories at low energy.

Now, one could imagine that this principle should also apply for the creation of a meson - anti-meson ($M\bar{M}$) pair and therefore pair suppression

[1] In a $SU(N)$ theory a baryon is represented as a q^N state, whereas a meson is always a $q\bar{q}$ state, independent of N.

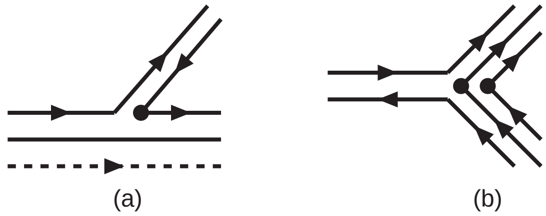


Figure 5.1: *(a) MMM (MBB) vertex and (b) $MB\bar{B}$ vertex*

should be implemented in the meson exchange sector (chapter 4). For the reason why we have not done this one should look again at figure 5.1 and consider the large N, $SU(N)$ theory again. For the creation of a $M\bar{M}$ pair at the vertex only one extra $q\bar{q}$ pair needs to be created instead of the $N-1$ pairs in the $B\bar{B}$ case and is therefore much likelier to happen. Going back to the real $SU_{(F)}(3)$ the difference is only one $q\bar{q}$ pair, nevertheless we assume that a $M\bar{M}$ pair creation is not suppressed.

Also from physical point of view it is nonsense to imply pair suppression in the meson sector. In order to see this one has to realize that an anti-meson is also a meson. So, assuming pair suppression in the meson sector means that a triple meson (MMM) vertex is suppressed, which makes it impossible to consider meson exchange in meson-baryon scattering as we did in chapter 4. From figure 5.1(a) we see that the MMM vertex is of the same order (in number of $q\bar{q}$ creations, as compared to figure 5.1(b)) as the meson-baryon-baryon (MBB) vertex in $SU_{(F)}(3)$. So, suppressing the MMM vertex means that we should also suppress the MBB vertex and no description of MB-scattering in terms of MB vertices is possible at all!

This does not mean, however, that there is no pair suppression what so ever in the meson sector of chapter 4. As can be seen from the amplitudes (4.6) and (4.12) we only considered MBB vertices in figure 4.2 and 3.2, whereas also $MB\bar{B}$ vertices could have been included. The latter vertices are suppressed as discussed above. We will come back to this later.

Since we suppressed the $MB\bar{B}$ vertex it means that pair suppression should also be active in the Vector Meson Dominance (VMD) [36] model describing nucleon Compton scattering ($\gamma N \to \gamma N$). From electron Compton scattering it is well-know that the Thomson limit is exclusively due to the negative energy electron states (see for instance section 3-9 of [37]). However, since the nucleon is composite it may well be that the negative energy contribution is produced by only one of the constituents [38] and it is not necessary to create an entire anti-baryon.

The suppression of negative energy states may harm the causality and

Lorentz invariance condition. Therefore, the question may arise whether it is possible to include pair suppression and still maintain causality and Lorentz invariance. The following example shows that it should in principle be possible: Imagine an infinitely dense medium where all anti-nucleon states are filled, i.e. the Fermi energy of the anti-nucleons $\bar{p}_F = \infty$, and that for nucleons $p_F = 0$. An example would be an anti-neutron star of infinite density. Then, in such an example pair production in πN-scattering is Pauli-blocked, because all anti-nucleon states are filled. Denoting the ground-state by $|\Omega\rangle$, one has, see e.g. [39],

$$S_F(x-y) \;=\; -i\langle\Omega|T[\psi(x)\bar{\psi}(y)]|\Omega\rangle \;,$$

which gives in momentum space [39]

$$\begin{aligned} S_F(p;p_F,\bar{p}_F) \;=\; \frac{\not{p}+M}{2E_p}\Bigg\{ & \frac{1-n_F(p)}{p_0-E_p+i\varepsilon} + \frac{n_F(p)}{p_0-E_p-i\varepsilon} \\ & -\frac{1-\bar{n}_F(p)}{p_0+E_p-i\varepsilon} - \frac{\bar{n}_F(p)}{p_0+E_p+i\varepsilon}\Bigg\} \;. \end{aligned}$$

At zero temperature $T=0$ the non-interacting fermion functions $n_F, \bar{n}_F$ are defined by

$$n_F \;=\; \begin{cases} 1, & |\mathbf{p}| < p_F \\ 0, & |\mathbf{p}| > p_F \end{cases} \quad , \quad \bar{n}_F = \begin{cases} 1, & |\mathbf{p}| < \bar{p}_F \\ 0, & |\mathbf{p}| > \bar{p}_F \end{cases} \quad .$$

In the medium sketched above, clearly $n_F(p) = 0$ and $\bar{n}_F(p) = 1$, which leads to a propagator $S_{ret}(p;0,\infty)$. This propagator is causal and Lorentz invariant.

The above (academic) example may perhaps convince a sceptical reader that a perfect relativistic model with 'absolute pair suppression' is feasible indeed.

As far as our results are concerned we refer to section 5.5, where we will see that intermediate baryon states are represented by retarded (-like) propagators, which have the nice feature to be causal and n-independent. We, therefore, have a theory that is relativistic and yet it does contain (absolute) pair suppression.

5.1.1 Equations of Motion

Consider a Lagrangian containing not only the free fermion part, but also a (simple) coupling between fermions and a scalar

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{free} + \mathcal{L}_I \\ &= \bar{\psi}\left(\frac{i}{2}\overrightarrow{\not{\partial}} - \frac{i}{2}\overleftarrow{\not{\partial}} - M\right)\psi + g\,\bar{\psi}\Gamma\psi\cdot\phi \end{aligned} \tag{5.1}$$

The Euler-Lagrange equation for the fermion part reads

$$(i\partial\!\!\!/ - M)\,\psi = -g\Gamma\psi\cdot\phi \tag{5.2}$$

In order to incorporate pair suppression we pose that the transitions between positive and negative energy fermion states vanish in the interaction part of (5.1), i.e. $\overline{\psi^{(+)}}\Gamma\psi^{(-)} = \overline{\psi^{(-)}}\Gamma\psi^{(+)} = 0$. So, we impose *absolute* pair suppression. From now on, when we speak of pair suppression we mean absolute pair suppression, unless it is mentioned otherwise. Of course it is in principle possible to allow for some pair production. This can be done for instance by not eliminating the terms $\overline{\psi^{(+)}}\Gamma\psi^{(-)}$ and $\overline{\psi^{(-)}}\Gamma\psi^{(+)}$ in (5.1), but allowing them with some small coupling $g' \ll g$. This, however, makes the situation much more difficult.

Since half of the term on the rhs of (5.2) finds its origin in such vanished terms, it is reduced by a factor 2 by the pair suppression condition.

Making the split up $\psi = \psi^{(+)}+\psi^{(-)}$, which is invariant under orthochronous Lorentz transformations, in (5.2) we assume both parts are independent, so that we have

$$(i\partial\!\!\!/ - M)\,\psi^{(+)} = -\frac{g}{2}\,\Gamma\psi^{(+)}\cdot\phi\ , \tag{5.3a}$$

$$(i\partial\!\!\!/ - M)\,\psi^{(-)} = -\frac{g}{2}\,\Gamma\psi^{(-)}\cdot\phi\ . \tag{5.3b}$$

One might wonder why we did not consider independent positive and negative energy fields from the start in (5.1). Although this would not cause any trouble in the interaction part ($\mathcal{L}_I$) it will in the free part. The quantum condition in such a situation would be $\left\{\psi^{(\pm)}(x), \pi^{(\pm)}(y)\right\} = i\delta^3(x-y)$. This is in conflict with the important relations between the positive and negative energy components

$$\begin{aligned}\left\{\psi^{(+)}(x), \overline{\psi^{(+)}}(y)\right\} &= (i\partial\!\!\!/ + M)\,\Delta^+(x-y)\ ,\\ \left\{\psi^{(-)}(x), \overline{\psi^{(-)}}(y)\right\} &= -\,(i\partial\!\!\!/ + M)\,\Delta^-(x-y)\ ,\end{aligned} \tag{5.4}$$

which we do need. Therefore we do not make the split up in the Lagrangian, but in the equations of motion.

The assumption that both parts $\psi^{(+)}$ and $\psi^{(-)}$ are independent means that besides the anti-commutation relations in (5.4) all others are zero.

In order to incorporate pair suppression in the meson sector (see chapter 4) the only thing to do is to exclude the transitions $\overline{\psi^{(+)}}\Gamma\psi^{(-)}$ and $\overline{\psi^{(-)}}\Gamma\psi^{(+)}$ in the interaction Lagrangians (4.4) and (4.10). By doing so, only u and $\bar{u}$ spinors will contribute. Therefore, only these spinors are present in (4.6) and (4.12).

For baryon exchange and resonance diagrams the implications of pair suppression are less trivial. We, therefore, discuss how pair suppression can be implemented in these situation in the following sections.

5.1.2 Takahashi Umezawa Scheme for Pair Suppression

In order to obtain the interaction Hamiltonian in case of pair suppression we set up the theory very similar to the Takahashi-Umezawa scheme presented in section 3.2. Since we only make the split-up in the fermion fields, the scalar fields are unaffected and therefore not included in this section.

We start with defining the currents

$$\boldsymbol{j}_{\psi^{(\pm)},a}(x) = \left(-\frac{\partial \mathcal{L}_I}{\partial \overline{\boldsymbol{\psi}^{(\pm)}}(x)}, -\frac{\partial \mathcal{L}_I}{\partial \left(\partial_\mu \overline{\boldsymbol{\psi}^{(\pm)}}\right)(x)} \right) . \tag{5.5}$$

Solutions to the equations of motion resulting from a general (interaction) Lagrangian are the YF equations

$$\begin{aligned} \boldsymbol{\psi}^{(\pm)}(x) &= \psi^{(\pm)}(x) + \frac{1}{2}\int d^4y\; D_a(y)\,(i\partial\!\!\!/ + M)\,\theta[n(x-y)] \\ &\qquad \times \Delta(x-y)\cdot \boldsymbol{j}_{\psi^{(\pm)},a}(y) . \end{aligned} \tag{5.6}$$

Here, we have chosen to use the retarded Green functions again, this, in order to be close to the treatment in section 3.2.

Furthermore, we introduce the auxiliary fields

$$\begin{aligned} \psi^{(\pm)}(x,\sigma) &= \psi^{(\pm)}(x) \\ &\mp i\int_{-\infty}^{\sigma} d^4y D_a(y)\,(i\partial\!\!\!/ + M)\,\Delta^{\pm}(x-y)\cdot \boldsymbol{j}_{\psi^{(\pm)},a}(y) . \end{aligned} \tag{5.7}$$

Combining these two equations ((5.6) and (5.7)) we get

$$\begin{aligned} \boldsymbol{\psi}^{(\pm)}(x) &= \psi^{(\pm)}(x/\sigma) \\ &+\frac{1}{4}\int d^4y \left[D_a(y)\,(i\partial\!\!\!/ + M)\,, \epsilon(x-y)\right]\Delta(x-y)\cdot \boldsymbol{j}_{\psi^{(\pm)},a}(y) \\ &\pm\frac{i}{2}\int d^4y\; \theta[n(x-y)] D_a(y)\,(i\partial\!\!\!/ + M)\,\Delta^{(1)}(x-y)\cdot \boldsymbol{j}_{\psi^{(\pm)},a}(y) . \end{aligned} \tag{5.8}$$

The factor 1/2 in (5.6) is essential. This becomes clear when we decompose $\Delta^{\pm}(x-y) = \frac{\pm i}{2}\,\Delta(x-y) + \frac{1}{2}\,\Delta^{(1)}(x-y)$ in (5.7). The first part (Δ) combines with (5.6) to the second term on the rhs of (5.8) and the second part ($\Delta^{(1)}$) gives a new contribution to $\boldsymbol{\psi}^{(\pm)}$ as compared to (3.22). We see that if we add $\boldsymbol{\psi}^{(+)}$ and $\boldsymbol{\psi}^{(-)}$ we get back (3.22), again. This makes the factor 1/2 difference in the first part of (5.8) as compared to (3.22) easier to understand.

As in section 3.2 we pose that $\psi^{(\pm)}(x)$ and $\psi^{(\pm)}(x,\sigma)$ satisfy the same commutation relation, since they satisfy the same EoM. The unitary operator connecting the two is related to the S-matrix by the same arguments as used in (3.27)-(3.29) and therefore satisfies the Tomanaga-Schwinger equation (3.30). Similar to the steps (3.31)-(3.33) we get the commutators of the different fields with the interaction Hamiltonian

$$\begin{aligned} &\left[\psi^{(\pm)}(x), \mathcal{H}_I(y;n)\right] = \\ &= U[\sigma]\left[D_a(y)(\pm)\,(i\partial\!\!\!/ + M)\,\Delta^{\pm}(x-y)\cdot \boldsymbol{j}_{\psi^{(\pm)},a}(y)\right]U^{-1}[\sigma]\ , \end{aligned} \tag{5.9}$$

from which the interaction Hamiltonian can be deduced. In section 3.2 we were able, once the interaction Hamiltonian was known, to proof that (3.21) was indeed correct (see appendix A). Since the main ingredient of the proof are the commutation relations of the free fields with the interaction Hamiltonian (in terms of free fields) (A.5), it is not hard to realize that $\Delta^{\pm}$ appears in (5.7).

Having discussed the formalism to implement pair suppression, now, we are going to apply it.

5.2 (Pseudo) Scalar Coupling

In the (pseudo) scalar sector of the theory including pair suppression we start with the following interaction Lagrangian

$$\mathcal{L}_I = g\,\overline{\boldsymbol{\psi}^{(+)}}\Gamma\boldsymbol{\psi}^{(+)}\cdot\boldsymbol{\phi} + g\,\overline{\boldsymbol{\psi}^{(-)}}\Gamma\boldsymbol{\psi}^{(-)}\cdot\boldsymbol{\phi}\ , \tag{5.10}$$

[2] where $\Gamma = 1$ or $\Gamma = i\gamma^5$. We will not use the specific forms for Γ until the discussion of the amplitudes in section 5.5. This, in order to be as general as possible.

From (5.10) we deduce the currents according to (5.5)

$$\begin{aligned} \boldsymbol{j}_{\psi^{(\pm)},a} &= \left(-g\,\Gamma\boldsymbol{\psi}^{(\pm)}\cdot\boldsymbol{\phi}, 0\right)\ , \\ \boldsymbol{j}_{\phi,a} &= \left(-g\,\overline{\boldsymbol{\psi}^{(+)}}\Gamma\boldsymbol{\psi}^{(+)} - g\,\overline{\boldsymbol{\psi}^{(-)}}\Gamma\boldsymbol{\psi}^{(-)}, 0\right)\ . \end{aligned} \tag{5.11}$$

[2] We note that this interaction Lagrangian (5.10) is charge invariant.

The fields in the H.R. can be expressed in terms of fields in the I.R. using (5.8)

$$\begin{aligned}\boldsymbol{\psi}^{(\pm)}(x) &= \psi^{(\pm)}(x/\sigma) \mp \frac{ig}{2}\int d^4y\;\theta[n(x-y)]\,(i\partial\!\!\!/ + M)\,\Delta^{(1)}(x-y) \\ &\qquad \times \Gamma\psi^{(\pm)}(y)\cdot\phi(y)\;, \qquad (5.12a)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\phi}(x) &= \phi(x/\sigma) + \frac{1}{2}\int d^4y\,[D_a(y), \epsilon(x-y)]\,\Delta(x-y)\cdot \boldsymbol{j}_{\phi,a}(y) \\ &= \phi(x/\sigma)\;. \qquad (5.12b)\end{aligned}$$

Equation (5.12a) was found by assuming that the coupling constant is small and considering only contributions up to order g, just as in (3.51) and the text below it.

With the expressions (5.12a) and (5.12b) and the definition of the commutator of the (fermion) fields with the interaction Hamiltonian (5.9) we get

$$\begin{aligned}\left[\psi^{(+)}(x), \mathcal{H}_I(y;n)\right] &= -g\,(i\partial\!\!\!/ + M)\,\Delta^{+}(x-y)\Gamma\psi^{(+)}(y)\cdot\phi(y) \\ &\quad + \frac{ig^2}{2}\,(i\partial\!\!\!/ + M)\,\Delta^{+}(x-y)\int d^4z\,\Gamma\,\theta[n(y-z)] \\ &\quad \times \left(i\partial\!\!\!/_y + M\right)\Delta^{(1)}(y-z)\,\Gamma\,\psi^{(+)}(z)\cdot\phi(z)\phi(y)\;, \\ \left[\psi^{(-)}(x), \mathcal{H}_I(y;n)\right] &= g\,(i\partial\!\!\!/ + M)\,\Delta^{-}(x-y)\Gamma\psi^{(-)}(y)\cdot\phi(y) \\ &\quad + \frac{ig^2}{2}\,(i\partial\!\!\!/ + M)\,\Delta^{-}(x-y)\int d^4z\,\Gamma\,\theta[n(y-z)] \\ &\quad \times \left(i\partial\!\!\!/_y + M\right)\Delta^{(1)}(y-z)\,\Gamma\,\psi^{(-)}(z)\cdot\phi(z)\phi(y)\;. \end{aligned} \qquad (5.13)$$

Here, we have not included the commutator of the scalar field ϕ with the interaction Hamiltonian, because (5.13) already contains enough information to get the interaction Hamiltonian

$$\begin{aligned} &\mathcal{H}_I(x;n) = \\ &= -g\,\overline{\psi^{(+)}}\Gamma\psi^{(+)}\cdot\phi - g\,\overline{\psi^{(-)}}\Gamma\psi^{(-)}\cdot\phi \\ &\quad + \frac{ig^2}{2}\int d^4y\left[\overline{\psi^{(+)}}\,\Gamma\,\phi\right]_x \theta[n(x-y)]\,(i\partial\!\!\!/_x + M)\,\Delta^{(1)}(x-y)\left[\Gamma\psi^{(+)}\phi\right]_y \\ &\quad - \frac{ig^2}{2}\int d^4y\left[\overline{\psi^{(-)}}\,\Gamma\,\phi\right]_x \theta[n(x-y)]\,(i\partial\!\!\!/_x + M)\,\Delta^{(1)}(x-y)\left[\Gamma\psi^{(-)}\phi\right]_y\;. \end{aligned} \qquad (5.14)$$

In (5.14) we see that the interaction Hamiltonian contains terms proportional to $\Delta^{(1)}(x-y)$ which are of order $O(g^2)$. These terms will be essential to get covariant and n-independent S-matrix elements and amplitudes at order $O(g^2)$.

If we would include external quasi fields in interaction Lagrangian (5.10), then the terms of order g^2 in the interaction Hamiltonian (5.14) would be quartic in the quasi field. As in (3.54) two quasi fields can be contracted

$$\bar{\chi}(x)\underbrace{\chi(x)\bar{\chi}}(y)\chi(y) = \bar{\chi}(x)\theta[n(x-y)]\chi(y) \ . \tag{5.15}$$

So, the terms of order g^2 get an additional factor $\theta[n(x-y)]$. However, since these terms already contain such a factor, we make the identification $\theta[n(x-y)]\theta[n(x-y)] \to \theta[n(x-y)]$. Therefore, all relevant πN terms in (5.14) are quadratic in the external quasi field, just as we want. This argument is valid for all couplings.

5.3 (Pseudo) Vector Coupling

Here, we repeat the steps of the previous section (section 5.2) but now in the case of (pseudo) vector coupling. The interaction Lagrangian reads

$$\mathcal{L}_I = \frac{f}{m_\pi}\,\overline{\boldsymbol{\psi}^{(+)}}\Gamma_\mu\boldsymbol{\psi}^{(+)}\cdot\partial^\mu\boldsymbol{\phi} + \frac{f}{m_\pi}\,\overline{\boldsymbol{\psi}^{(-)}}\Gamma_\mu\boldsymbol{\psi}^{(-)}\cdot\partial^\mu\boldsymbol{\phi} \ , \tag{5.16}$$

where $\Gamma_\mu = \gamma_\mu$ or $\Gamma_\mu = \gamma_5\gamma_\mu$. From (5.16) we deduce the currents

$$\begin{aligned} \boldsymbol{j}_{\psi^{(\pm)},a} &= \left(-\frac{f}{m_\pi}\,\Gamma_\mu\boldsymbol{\psi}^{(\pm)}\cdot\partial^\mu\boldsymbol{\phi}, 0\right) \ , \\ \boldsymbol{j}_{\phi,a} &= \left(0, -\frac{f}{m_\pi}\,\overline{\boldsymbol{\psi}^{(+)}}\Gamma_\mu\boldsymbol{\psi}^{(+)} - \frac{f}{m_\pi}\,\overline{\boldsymbol{\psi}^{(-)}}\Gamma_\mu\boldsymbol{\psi}^{(-)}\right) \ . \end{aligned} \tag{5.17}$$

The fields in the H.R. are expressed in terms of fields in the I.R. as follows

$$\begin{aligned} \boldsymbol{\psi}^{(\pm)}(x) &= \psi^{(\pm)}(x/\sigma) \mp \frac{if}{2m_\pi}\int d^4y\theta[n(x-y)]\left(i\partial\!\!\!/ + M\right)\Delta^{(1)}(x-y) \\ &\qquad \times \Gamma_\mu\psi^{(\pm)}(y)\cdot\partial^\mu\phi(y) \ , \end{aligned} \tag{5.18a}$$

$$\boldsymbol{\phi}(x) = \phi(x/\sigma) \ , \tag{5.18b}$$

$$\begin{aligned} \partial^\mu\boldsymbol{\phi}(x) &= \left[\partial^\mu\phi(x,\sigma)\right]_{x/\sigma} - \frac{f}{m_\pi}\,n^\mu\,\overline{\psi^{(+)}}(x)n\cdot\Gamma\psi^{(+)}(x) \\ &\quad -\frac{f}{m_\pi}\,n^\mu\,\overline{\psi^{(-)}}(x)n\cdot\Gamma\psi^{(-)}(x) \ . \end{aligned} \tag{5.18c}$$

The commutators of the different fields with the interaction Hamiltonian are

$$
\begin{aligned}
&\left[\psi^{(+)}(x), \mathcal{H}_I(y;n)\right] = \\
= \; & \frac{f}{m_\pi}\,(i\partial\!\!\!/ + M)\,\Delta^+(x-y)\,\Bigg[-\Gamma_\mu \psi^{(+)}\cdot\partial^\mu\phi \\
& +\frac{f}{m_\pi}\; n\cdot\Gamma\psi^{(+)}\,\overline{\psi^{(+)}}n\cdot\Gamma\psi^{(+)} + \frac{f}{m_\pi}\; n\cdot\Gamma\psi^{(+)}\,\overline{\psi^{(-)}}n\cdot\Gamma\psi^{(-)}\Bigg]_y \\
& +\frac{if^2}{2m_\pi^2}\,(i\partial\!\!\!/ + M)\,\Delta^+(x-y)\int d^4z\,\Gamma_\mu\,\theta[n(y-z)] \\
& \qquad \times \left(i\partial\!\!\!/_y + M\right)\Delta^{(1)}(y-z)\,\Gamma_\nu\,\psi^{(+)}(z)\cdot\partial^\nu\phi(z)\partial^\mu\phi(y)\;, \\
&\left[\psi^{(-)}(x), \mathcal{H}_I(y;n)\right] = \\
= \; & -\frac{f}{m_\pi}\,(i\partial\!\!\!/ + M)\,\Delta^-(x-y)\,\Bigg[-\Gamma_\mu \psi^{(-)}\cdot\partial^\mu\phi \\
& +\frac{f}{m_\pi}\; n\cdot\Gamma\psi^{(-)}\,\overline{\psi^{(+)}}n\cdot\Gamma\psi^{(+)} + \frac{f}{m_\pi}\; n\cdot\Gamma\psi^{(-)}\,\overline{\psi^{(-)}}n\cdot\Gamma\psi^{(-)}\Bigg]_y \\
& -\frac{if^2}{2m_\pi^2}\,(i\partial\!\!\!/ + M)\,\Delta^-(x-y)\int d^4z\,\Gamma_\mu\,\theta[n(y-z)] \\
& \qquad \times \left(i\partial\!\!\!/_y + M\right)\Delta^{(1)}(y-z)\,\Gamma_\nu\,\psi^{(-)}(z)\cdot\partial^\nu\phi(z)\partial^\mu\phi(y)\;,
\end{aligned}
\tag{5.19}
$$

from these equations we deduce the interaction Hamiltonian

$$
\begin{aligned}
\mathcal{H}_I(x;n) \;=\; & -\frac{f}{m_\pi}\,\overline{\psi^{(+)}}\Gamma_\mu\psi^{(+)}\cdot\partial^\mu\phi - \frac{f}{m_\pi}\,\overline{\psi^{(-)}}\Gamma_\mu\psi^{(-)}\cdot\partial^\mu\phi \\
& +\frac{f^2}{2m_\pi^2}\left[\overline{\psi^{(+)}}\,n\cdot\Gamma\,\psi^{(+)}\right]^2 + \frac{f^2}{2m_\pi^2}\left[\overline{\psi^{(-)}}\,n\cdot\Gamma\,\psi^{(-)}\right]^2 \\
& +\frac{f^2}{m_\pi^2}\left[\overline{\psi^{(+)}}\,n\cdot\Gamma\,\psi^{(+)}\right]\left[\overline{\psi^{(-)}}\,n\cdot\Gamma\,\psi^{(-)}\right] \\
& +\frac{if^2}{2m_\pi^2}\int d^4y\left[\overline{\psi^{(+)}}\Gamma_\mu\partial^\mu\phi\right]_x\theta[n(x-y)]\,(i\partial\!\!\!/ + M) \\
& \qquad \times \Delta^{(1)}(x-y)\left[\Gamma_\nu\psi^{(+)}\partial^\nu\phi\right]_y \\
& -\frac{if^2}{2m_\pi^2}\int d^4y\left[\overline{\psi^{(-)}}\Gamma_\mu\partial^\mu\phi\right]_x\theta[n(x-y)]\,(i\partial\!\!\!/ + M) \\
& \qquad \times \Delta^{(1)}(x-y)\left[\Gamma_\nu\psi^{(-)}\partial^\nu\phi\right]_y\;.
\end{aligned}
\tag{5.20}
$$

As in (5.14) there are also terms proportional to $\Delta^{(1)}(x-y)$ quadratic in the coupling constant. Also, (5.20) contains contact terms, but they do not contribute to πN-scattering.

5.4 $\pi N \Delta_{33}$ Coupling

At this point we deviated from [11] as far as the interaction Lagrangian is concerned. For the description of the coupling of the Δ_{33}, which is a spin-3/2 field, to πN we follow [40] by using the interaction Lagrangian

$$\begin{aligned}
\mathcal{L}_I &= g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu \overline{\boldsymbol{\Psi}_\nu^{(+)}}\right)\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(+)}\left(\partial_\beta \boldsymbol{\phi}\right) \\
&\quad + g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(+)}}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}_\nu^{(+)}\right)\left(\partial_\beta \boldsymbol{\phi}\right) \\
&\quad + g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu \overline{\boldsymbol{\Psi}_\nu^{(-)}}\right)\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(-)}\left(\partial_\beta \boldsymbol{\phi}\right) \\
&\quad + g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(-)}}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}_\nu^{(-)}\right)\left(\partial_\beta \boldsymbol{\phi}\right) \ .
\end{aligned} \tag{5.21}$$

Here, Ψ_μ represents the spin-3/2 Δ_{33} field. As is mentioned in [40, 41] the Ψ_μ field does not only contain spin-3/2 components but also spin-1/2 components. By using the interaction Lagrangian as in (5.21) it is assured that only the spin-3/2 components of the Δ_{33} field couple.

From (5.21) we deduce the currents

$$\begin{aligned}
\boldsymbol{j}_{\phi,a}(x) &= \Big[0,\ -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu \overline{\boldsymbol{\Psi}_\nu^{(+)}}\right)\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(+)} \\
&\quad - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(+)}}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}_\nu^{(+)}\right) - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu \overline{\boldsymbol{\Psi}_\nu^{(-)}}\right)\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(-)} \\
&\quad - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(-)}}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}_\nu^{(-)}\right)\Big] \\
\boldsymbol{j}_{\psi^{(\pm)},a}(x) &= \Big[-g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}_\nu^{(+)}\right)\left(\partial_\beta \boldsymbol{\phi}\right),0\Big] \\
\boldsymbol{j}_{\Psi_\nu^{(\pm)},a}(x) &= \Big[0,-g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(+)}\left(\partial_\beta \boldsymbol{\phi}\right)\Big] \ .
\end{aligned} \tag{5.22}$$

To avoid lengthy equations we express the commutators of the various fields with the interaction Hamiltonian in terms of fields in the H.R. (5.9)

$$\begin{aligned}
&\Big[\phi(x), \mathcal{H}_I(y;n)\Big] = \\
= \;& U[\sigma] i\Delta(x-y)\overleftarrow{\partial}^y_\beta \Big[-g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu \overline{\boldsymbol{\Psi}^{(+)}_\nu}\right)\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(+)} \\
& -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(+)}}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}^{(+)}_\nu\right) - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu \overline{\boldsymbol{\Psi}^{(-)}_\nu}\right)\gamma_5\gamma_\alpha \boldsymbol{\psi}^{(-)} \\
& -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(-)}}\gamma_5\gamma_\alpha\left(\partial_\mu \boldsymbol{\Psi}^{(-)}_\nu\right)\Big]_y U^{-1}[\sigma]\ ,
\end{aligned}$$

$$\begin{aligned}
&\Big[\psi^{\pm}(x), \mathcal{H}_I(y;n)\Big] = \\
= \;& U[\sigma](\pm)\,(i\partial\!\!\!/_x + M)\,\Delta^{\pm}(x-y)\,\Big[-g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(+)}\,(\partial_\beta\boldsymbol{\phi})\Big]_y U^{-1}[\sigma]\ ,
\end{aligned}$$

$$\begin{aligned}
&\Big[\Psi^{\pm}_\mu(x), \mathcal{H}_I(y;n)\Big] = \\
= \;& U[\sigma](\pm)\,(i\partial\!\!\!/_x + M_\Delta)\,(-) \\
& \times\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2\partial_\mu\partial_\nu}{3M_\Delta^2} - \frac{1}{3M_\Delta^2}\,(\gamma_\mu i\partial_\nu - i\partial_\mu\gamma_\nu)\right)\Delta^{\pm}(x-y)\overleftarrow{\partial}^y_\rho \\
& \times\left(-g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(+)}\,(\partial_\beta\boldsymbol{\phi})\right)_y U^{-1}[\sigma]\ ,
\end{aligned} \tag{5.23}$$

where the fields in the H.R. are expressed in terms of fields in the I.R. using (5.8)

$$\begin{aligned}
\boldsymbol{\psi}^{(\pm)}(x) \;=\;& \psi^{(\pm)}(x/\sigma) \pm \frac{i}{2}\int d^4y\;\theta[n(x-y)](i\partial\!\!\!/ + M)\Delta^{(1)}(x-y) \\
& \times g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\Big[\left(\partial_\mu\Psi^{(\pm)}_\nu\right)(\partial_\beta\phi)\Big]_y\ ,
\end{aligned}$$

$$\begin{aligned}
&\partial_\rho\boldsymbol{\phi}(x) = \\
= \;& [\partial_\rho\phi(x,\sigma)]_{x/\sigma} \\
& -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}n_\rho\left(\partial_\mu\overline{\Psi^{(+)}_\nu}\right)\gamma_5\gamma_\alpha\psi^{(+)}n_\beta - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}n_\rho\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\left(\partial_\mu\Psi^{(+)}_\nu\right)n_\beta \\
& -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}n_\rho\left(\partial_\mu\overline{\Psi^{(-)}_\nu}\right)\gamma_5\gamma_\alpha\psi^{(-)}n_\beta - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}n_\rho\overline{\psi^{(-)}}\gamma_5\gamma_\alpha\left(\partial_\mu\Psi^{(-)}_\nu\right)n_\beta\ ,
\end{aligned}$$

$$
\begin{aligned}
\partial_\rho \mathbf{\Psi}^{(\pm)}_\mu(x) &= \left[\partial_\rho \Psi^{(\pm)}_\mu(x,\sigma)\right]_{x/\sigma} \\
&+\frac{g_{gi}}{2}\left[(i\partial\!\!\!/_x + M_\Delta)\, n_\rho n_\gamma + n\!\!\!/\,(i\partial_\rho n_\gamma + n_\rho i\partial_\gamma) - 2n\!\!\!/ n_\rho n_\gamma n\cdot i\partial\right] \\
&\qquad \times \left(g_{\mu\nu} - \frac{1}{3}\,\gamma_\mu\gamma_\nu\right)\epsilon^{\rho\nu\alpha\beta}\gamma_5\gamma_\alpha\psi^{(\pm)}\,(\partial_\beta\phi) \\
&\mp\frac{ig_{gi}}{2}\int d^4y\,\theta[n(x-y)]\,(i\partial\!\!\!/_x + M_\Delta)\left[g_{\mu\nu} - \frac{1}{3}\,\gamma_\mu\gamma_\nu\right] \\
&\qquad \times \partial_\rho\partial_\gamma\Delta^{(1)}(x-y)\left[\epsilon^{\rho\nu\alpha\beta}\gamma_5\gamma_\alpha\psi^{(\pm)}\,(\partial_\beta\phi)\right]_y\ . \qquad (5.24)
\end{aligned}
$$

Here, we have already used that $\partial_\rho \mathbf{\Psi}^{(\pm)}_\mu(x)$ always appears in combination with $\epsilon^{\rho\mu\alpha\beta}$. Therefore, we have eliminated terms that are symmetric in ρ and μ.

With these ingredients we can construct the interaction Hamiltonian. Because it contains a lot of terms we only focus on those terms that contribute to πN-scattering

$$
\begin{aligned}
&\mathcal{H}_I(x;n) = \\
= \;& -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu\overline{\Psi^{(+)}_\nu}\right)\gamma_5\gamma_\alpha\psi^{(+)}\,(\partial_\beta\phi) - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\left(\partial_\mu\Psi^{(+)}_\nu\right)(\partial_\beta\phi) \\
& -g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu\overline{\Psi^{(-)}_\nu}\right)\gamma_5\gamma_\alpha\psi^{(-)}\,(\partial_\beta\phi) - g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(-)}}\gamma_5\gamma_\alpha\left(\partial_\mu\Psi^{(-)}_\nu\right)(\partial_\beta\phi) \\
& -\frac{g_{gi}^2}{2}\,\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\,(\partial_\beta\phi)\left[(i\partial\!\!\!/_x + M_\Delta)\, n_\mu n_{\mu'} + n\!\!\!/\,(i\partial_\mu n_{\mu'} + n_\mu i\partial_{\mu'})\right. \\
&\qquad \left. - 2n\!\!\!/ n_\mu n_{\mu'} n\cdot i\partial\right]\left(g_{\nu\nu'} - \frac{1}{3}\,\gamma_\nu\gamma_{\nu'}\right)\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\,(\partial_{\beta'}\phi) \\
& -\frac{g_{gi}^2}{2}\,\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(-)}}\gamma_5\gamma_\alpha\,(\partial_\beta\phi)\left[(i\partial\!\!\!/_x + M_\Delta)\, n_\mu n_{\mu'} + n\!\!\!/\,(i\partial_\mu n_{\mu'} + n_\mu i\partial_{\mu'})\right. \\
&\qquad \left. - 2n\!\!\!/ n_\mu n_{\mu'} n\cdot i\partial\right]\left(g_{\nu\nu'} - \frac{1}{3}\,\gamma_\nu\gamma_{\nu'}\right)\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(-)}\,(\partial_{\beta'}\phi) \\
& +\frac{ig_{gi}^2}{2}\int d^4y\left[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\,(\partial_\beta\phi)\right]_x\theta[n(x-y)]\,(i\partial\!\!\!/_x + M_\Delta) \\
&\qquad \times\left(g_{\nu\nu'} - \frac{1}{3}\,\gamma_\nu\gamma_{\nu'}\right)\partial_\mu\partial_{\mu'}\Delta^{(1)}(x-y)\left[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\,(\partial_{\beta'}\phi)\right]_y
\end{aligned}
$$

$$-\frac{ig_{gi}^2}{2}\int d^4y\left[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(-)}}\gamma_5\gamma_\alpha\left(\partial_\beta\phi\right)\right]_x\theta[n(x-y)]\,(i\partial\!\!\!/_x+M_\Delta)$$
$$\times\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\partial_\mu\partial_{\mu'}\Delta^{(1)}(x-y)\left[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(-)}\left(\partial_{\beta'}\phi\right)\right]_y\ . \tag{5.25}$$

5.5 S-Matrix Elements and Amplitudes

Since the Kadyshevsky rules as presented in section 2.3 do not contain pair suppression, we are going to derive the amplitudes from the S-matrix (2.2). The basic ingredients, namely the interaction Hamiltonians, we have constructed in the previous sections (section 5.2, 5.3 and 5.4) for different couplings. As in chapter 4 we also consider in this section equal initial and final states, i.e. πN (MB) scattering. For the results for general MB initial and final states we refer to appendix C

5.5.1 (Pseudo) Scalar Coupling

For the pseudo scalar coupling case we collect all g^2 contributions to the S-matrix (see (5.14))

$$\begin{aligned}
S^{(2)} &= (-i)^2\int d^4xd^4y\,\theta[n(x-y)]\mathcal{H}_I(x)\mathcal{H}_I(y)\\
&= -g^2\int d^4xd^4y\,\theta[n(x-y)]\left[\overline{\psi^{(+)}}\,\Gamma\phi\right]_x(i\partial\!\!\!/+M)\\
&\qquad\times\Delta^+(x-y)\left[\Gamma\psi^{(+)}\phi\right]_y\ ,\\
S^{(1)} &= (-i)\int d^4x\,\mathcal{H}_I(x)\\
&= \frac{g^2}{2}\int d^4xd^4y\left[\overline{\psi^{(+)}}\,\Gamma\phi\right]_x\theta[n(x-y)]\,(i\partial\!\!\!/_x+M)\\
&\qquad\times\Delta^{(1)}(x-y)\left[\Gamma\psi^{(+)}\phi\right]_y\ ,
\end{aligned}\tag{5.26}$$

which need to be added

$$\begin{aligned}
S^{(2)}+S^{(1)} &= -\frac{ig^2}{2}\int d^4xd^4y\left[\overline{\psi^{(+)}}\,\Gamma\phi\right]_x\theta[n(x-y)]\,(i\partial\!\!\!/+M)\\
&\qquad\times\Delta(x-y)\left[\Gamma\psi^{(+)}\phi\right]_y\ .
\end{aligned}\tag{5.27}$$

We see here that indeed the $\Delta^{(1)}(x-y)$ propagator in the interaction Hamiltonian (5.14) is crucial, since it combines with the $\Delta^{(+)}(x-y)$ propagator (5.26) to form a $\Delta(x-y)$ propagator (5.27). Together with the $\theta[n(x-y)]$ in (5.27) we recognize the causal retarded (-like) character as we already mentioned in the section 5.1. The S-matrix element is therefore covariant and if we analyze its n-dependence as in section 2.4.3 and section 3.4 we would see that it is n-independent (for vanishing external quasi momenta, of course).

Also we notice that the initial and final states are still positive energy states. We started with a separation of positive and negative energy states in section 5.1 and after the whole procedure this is still valid for the end-states. However, we have to notice that in an intermediate state, negative energy propagates via the $\Delta(x-y)$ propagator, but this is also the case in our example of the infinite dense anti-nucleon star of section 5.1. Moreover, in [11] pair suppression is assumed by only considering positive energy end-states, and this is what we have achieved formally.

All the above observations are also valid in the case of (pseudo) vector coupling and the $\pi N \Delta_{33}$ coupling of section 5.5.2 and section 5.5.3, respectively as we will see.

The last important observation is that in (5.27) it does not matter whether the derivative just acts on the $\Delta(x-y)$ propagator or also on the $\theta[n(x-y)]$ function. Therefore, the $\bar{P}$-method of section 3.5.3 can be applied, although it is not really necessary. This situation is contrary to ordinary baryon exchange, where the $\bar{P}$ method can only be applied for the summed diagrams, as explained in section 3.5.3.

The summed S-matrix elements (5.27) lead to baryon exchange and resonance Kadyshevsky diagrams, which are exposed in figure 5.2. We are going to treat them separately.

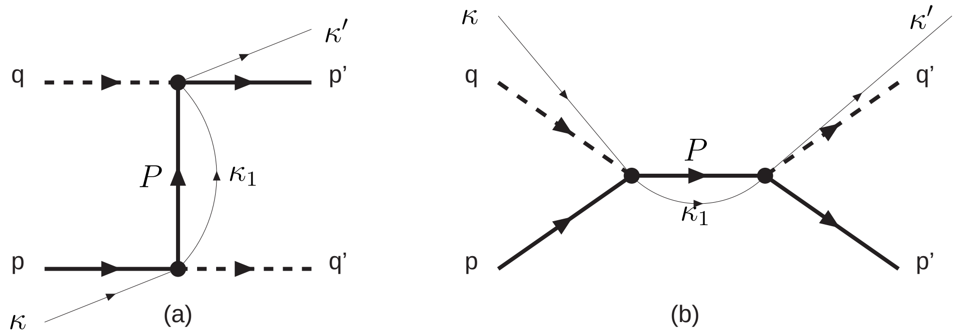


Figure 5.2: *Baryon exchange (a) and resonance (b) diagrams*

The amplitude for the (pseudo) scalar baryon exchange and resonance resulting from the S-matrix in (5.27) are

$$
\begin{aligned}
M_{\kappa'\kappa}(u) &= \frac{g^2}{2}\int \frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\bar{u}(p's')\left[\Gamma\,(\not{P}_u+M_B)\,\Gamma\right]u(ps)\Delta(P_u)\ , \\
M_{\kappa'\kappa}(s) &= \frac{g^2}{2}\int \frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\bar{u}(p's')\left[\Gamma\,(\not{P}_s+M_B)\,\Gamma\right]u(ps)\Delta(P_s)\ . \qquad (5.28)
\end{aligned}
$$

Here $P_i = \Delta_i + n\bar{\kappa} - n\kappa_1$ and $\Delta(P_i) = \epsilon(P_i^0)\delta(P_i^2 - M_B^2)$ $(i = u, s)$. The Δ_i stand for

$$
\begin{aligned}
\Delta_u &= \frac{1}{2}\,(p' + p - q' - q)\ , \\
\Delta_s &= \frac{1}{2}\,(p' + p + q' + q)\ . \qquad (5.29)
\end{aligned}
$$

After expanding the $\delta(P_i^2 - M_B^2)$-function the κ_1 integral can be performed, just as in (3.6)

$$
\begin{aligned}
\delta(P_i^2 - M_B^2) &= \frac{1}{|\kappa_1^+ - \kappa_1^-|}\left(\delta(\kappa_1-\kappa_1^+) + \delta(\kappa_1-\kappa_1^-)\right)\ , \\
\kappa_1^{\pm} &= \Delta_i\cdot n + \bar{\kappa} \pm A_i\ . \qquad (5.30)
\end{aligned}
$$

The $\epsilon(P_i^0)$ selects both solutions with a relative minus sign

$$
\begin{aligned}
&\frac{1}{2A_i}\left[\frac{\not{\Delta}_i - (\Delta_i\cdot n - A_i)\not{n} + M_B}{\Delta_i\cdot n + \bar{\kappa} - A_i + i\varepsilon} - \frac{\not{\Delta}_i - (\Delta_i\cdot n + A_i)\not{n} + M_B}{\Delta_i\cdot n + \bar{\kappa} + A_i + i\varepsilon}\right] \\
&= (\not{\Delta}_i + M_B + \bar{\kappa}\not{n})\,\frac{1}{2A_i}\left[\frac{1}{\Delta_i\cdot n + \bar{\kappa} - A_i + i\varepsilon} - \frac{1}{\Delta_i\cdot n + \bar{\kappa} + A_i + i\varepsilon}\right] \\
&= \frac{\not{\Delta}_i + M_B + \bar{\kappa}\not{n}}{(\Delta_i\cdot n + \bar{\kappa})^2 - A_i^2 + i\varepsilon}\ . \qquad (5.31)
\end{aligned}
$$

This yields for the amplitudes

$$
\begin{aligned}
M^S_{\kappa'\kappa}(u) &= \frac{g_S^2}{2}\,\bar{u}(p's')\,[M + M_B - \not{Q} + \bar{\kappa}\not{n}]\,u(ps)\,\frac{1}{(\Delta_u\cdot n + \bar{\kappa})^2 - A_u^2 + i\varepsilon}\ , \\
M^{PS}_{\kappa'\kappa}(u) &= \frac{g_{PS}^2}{2}\,\bar{u}(p's')\,[M - M_B - \not{Q} + \bar{\kappa}\not{n}]\,u(ps)\,\frac{1}{(\Delta_u\cdot n + \bar{\kappa})^2 - A_u^2 + i\varepsilon}\ , \\
M^S_{\kappa'\kappa}(s) &= \frac{g_S^2}{2}\,\bar{u}(p's')\,[M + M_B + \not{Q} + \bar{\kappa}\not{n}]\,u(ps)\,\frac{1}{(\Delta_s\cdot n + \bar{\kappa})^2 - A_s^2 + i\varepsilon}\ , \\
M^{PS}_{\kappa'\kappa}(s) &= \frac{g_{PS}^2}{2}\,\bar{u}(p's')\,[M - M_B + \not{Q} + \bar{\kappa}\not{n}]\,u(ps)\,\frac{1}{(\Delta_s\cdot n + \bar{\kappa})^2 - A_s^2 + i\varepsilon}\ , \qquad (5.32)
\end{aligned}
$$

where S and PS stand for *scalar* and *pseudo scalar*, respectively. Taking the limit of $\kappa' = \kappa = 0$ in (5.32) we get

$$\begin{aligned}
M^S_{00}(u) &= \frac{g_S^2}{2}\,\bar{u}(p's')\,[M + M_B - \not{Q}]\,u(ps)\,\frac{1}{u - M_B^2 + i\varepsilon}\;, \\
M^{PS}_{00}(u) &= \frac{g_{PS}^2}{2}\,\bar{u}(p's')\,[M - M_B - \not{Q}]\,u(ps)\,\frac{1}{u - M_B^2 + i\varepsilon}\;, \\
M^S_{00}(s) &= \frac{g_S^2}{2}\,\bar{u}(p's')\,[M + M_B + \not{Q}]\,u(ps)\,\frac{1}{s - M_B^2 + i\varepsilon}\;, \\
M^{PS}_{00}(s) &= \frac{g_{PS}^2}{2}\,\bar{u}(p's')\,[M - M_B + \not{Q}]\,u(ps)\,\frac{1}{s - M_B^2 + i\varepsilon}\;, \qquad (5.33)
\end{aligned}$$

which is a factor 1/2 of the result in [11]. This factor is because of the fact that we only took the positive energy contribution. This difference can easily be intercepted by considering an interaction Lagrangian as in (5.10) scaled by a factor of $\sqrt{2}$. We stress here that although we have included absolute pair suppression formally, we still get a factor 1/2 of the usual Feynman expression.

In order to study the n-dependence of the amplitudes (see section 2.4.3) we take a closer look at the denominators in (5.32)

$$(\Delta_i \cdot n + \bar{\kappa})^2 - A_s^2 = \Delta_i^2 - M_B^2 + 2\Delta_i \cdot n\bar{\kappa} + \bar{\kappa}^2\;. \qquad (5.34)$$

From this we conclude that all n-dependent terms in (5.32) are proportional to $\bar{\kappa}$, therefore differentiating (5.32) with respect to n^α will yield a result linear proportional to κ. If we would only consider (P)S baryon exchange or resonance in the Kadyshevsky integral equation, then we indeed would have a situation as in (2.32). Looking at the powers of κ, κ' in (5.32) we see that $h(\kappa)$ in (2.32) will be of the order $O(\frac{1}{\kappa^2})$ and the phenomenological "form factor" (2.34) would not be necessary.

5.5.2 (Pseudo) Vector Coupling

The g^2 contributions of (pseudo) vector coupling in the second and first order of the S-matrix are

$$\begin{aligned}
S^{(2)} &= (-i)^2 \int d^4x d^4y\, \theta[n(x-y)]\mathcal{H}_I(x)\mathcal{H}_I(y) \\
&= -\frac{f^2}{m_\pi^2}\int d^4x d^4y\, \theta[n(x-y)]\left[\overline{\psi^{(+)}}\Gamma_\mu\,(\partial^\mu\phi)\right]_x (i\not{\partial} + M) \\
&\qquad \times \Delta^+(x-y)\left[\Gamma_\nu\psi^{(+)}\,(\partial^\nu\phi)\right]_y\;,
\end{aligned}$$

$$\begin{aligned}
S^{(1)} &= (-i)\int d^4x\,\mathcal{H}_I(x) \\
&= \frac{f^2}{2m_\pi^2}\int d^4x d^4y\,\left[\overline{\psi^{(+)}}\Gamma_\mu\left(\partial^\mu\phi\right)\right]_x \theta[n(x-y)]\left(i\partial\!\!\!/+M\right) \\
&\qquad \times\Delta^{(1)}(x-y)\left[\Gamma_\nu\psi^{(+)}\left(\partial^\nu\phi\right)\right]_y\ . \qquad (5.35)
\end{aligned}$$

Adding the two together

$$\begin{aligned}
S^{(2)}+S^{(1)} &= -\frac{if^2}{2m_\pi^2}\int d^4x d^4y\,\theta[n(x-y)]\left[\overline{\psi^{(+)}}\Gamma_\mu\left(\partial^\mu\phi\right)\right]_x\left(i\partial\!\!\!/+M\right) \\
&\qquad \times\Delta(x-y)\left[\Gamma_\nu\psi^{(+)}\left(\partial^\nu\phi\right)\right]_y\ , \qquad (5.36)
\end{aligned}$$

leads again to a covariant, n-independent result ($\kappa'=\kappa=0$). See the text below (5.27) about this issue and other important observations.

The two Kadyshevsky diagrams resulting from (5.36) are the same as shown in figure 5.2. The amplitudes that go with them, in case of (pseudo) vector coupling, are

$$\begin{aligned}
M_{\kappa'\kappa}(u) &= \frac{f^2}{2m_\pi^2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\bar{u}(p's')\left[\left(\Gamma\cdot q\right)\left(P\!\!\!/_u+M_B\right)\left(\Gamma\cdot q'\right)\right]u(ps)\Delta(P_u)\ , \\
M_{\kappa'\kappa}(s) &= \frac{f^2}{2m_\pi^2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\bar{u}(p's')\left[\left(\Gamma\cdot q'\right)\left(P\!\!\!/_s+M_B\right)\left(\Gamma\cdot q\right)\right]u(ps)\Delta(P_s)\ ,
\end{aligned} \qquad (5.37)$$

where P_i and $\Delta(P_i)$ are defined below (5.28). As far as the κ_1 integration is concerned we take similar steps as in (5.30) and (5.31).

After some (Dirac) algebra the amplitudes in (5.37) become

$$\begin{aligned}
M^V_{\kappa'\kappa}(u) &= \frac{f_V^2}{2m_\pi^2}\,\bar{u}(p's')\Bigg[-\left(M-M_B\right)\Bigg(-M^2+\frac{1}{2}\left(u_{p'q}+u_{pq'}\right)+2MQ\!\!\!/ \\
&\quad -\frac{1}{2}\left(\kappa'-\kappa\right)\left(p'-p\right)\cdot n+\frac{1}{2}\left(\kappa'-\kappa\right)[n\!\!\!/,Q\!\!\!/]-\frac{1}{2}\left(\kappa'-\kappa\right)^2\Bigg) \\
&\quad -\frac{1}{2}\left(u_{pq'}-M^2\right)\left(Q\!\!\!/+\frac{1}{2}\left(\kappa'-\kappa\right)n\!\!\!/\right) \\
&\quad -\frac{1}{2}\left(u_{p'q}-M^2\right)\left(Q\!\!\!/-\frac{1}{2}\left(\kappa'-\kappa\right)n\!\!\!/\right) \\
&\quad +\bar{\kappa}\left(-(p'-p)\cdot n\,Q\!\!\!/+2Q\cdot n\,Q\!\!\!/+M^2n\!\!\!/-\frac{n\!\!\!/}{2}\left(u_{p'q}+u_{pq'}\right)\right)\Bigg]u(p) \\
&\quad \times\frac{1}{\left(\Delta_u\cdot n+\bar{\kappa}\right)^2-A_u^2+i\varepsilon}\ ,
\end{aligned}$$

$$
\begin{aligned}
M^{PV}_{\kappa'\kappa}(u) &= \frac{f^2_{PV}}{2m^2_\pi}\,\bar{u}(p's')\,\Bigg[-(M+M_B)\Bigg(-M^2+\frac{1}{2}\left(u_{p'q}+u_{pq'}\right)+2M\not{Q} \\
&\quad -\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n+\frac{1}{2}(\kappa'-\kappa)[\not{n},\not{Q}]-\frac{1}{2}(\kappa'-\kappa)^2\Bigg) \\
&\quad -\frac{1}{2}\left(u_{pq'}-M^2\right)\left(\not{Q}+\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
&\quad -\frac{1}{2}\left(u_{p'q}-M^2\right)\left(\not{Q}-\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
&\quad +\bar{\kappa}\left(-(p'-p)\cdot n\,\not{Q}+2Q\cdot n\,\not{Q}+M^2\not{n}-\frac{\not{n}}{2}\left(u_{p'q}+u_{pq'}\right)\right)\Bigg]\,u(p) \\
&\quad \times\frac{1}{(\Delta_u\cdot n+\bar{\kappa})^2-A^2_u+i\varepsilon}\ , \\
M^{V}_{\kappa'\kappa}(s) &= \frac{f^2_{V}}{2m^2_\pi}\,\bar{u}(p's')\,\Bigg[-(M-M_B)\Bigg(-M^2+\frac{1}{2}\left(s_{p'q'}+s_{pq}\right)-2M\not{Q} \\
&\quad -\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)[\not{n},\not{Q}]-\frac{1}{2}(\kappa'-\kappa)^2\Bigg) \\
&\quad +\frac{1}{2}\left(s_{p'q'}-M^2\right)\left(\not{Q}+\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
&\quad +\frac{1}{2}\left(s_{pq}-M^2\right)\left(\not{Q}-\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
&\quad +\bar{\kappa}\left((p'-p)\cdot n\,\not{Q}+2Q\cdot n\,\not{Q}+M^2\,\not{n}\ -\frac{\not{n}}{2}\left(s_{p'q'}+s_{pq}\right)\right)\Bigg]\,u(p) \\
&\quad \times\frac{1}{(\Delta_s\cdot n+\bar{\kappa})^2-A^2_s+i\varepsilon}\ , \\
M^{PV}_{\kappa'\kappa}(s) &= \frac{f^2_{PV}}{2m^2_\pi}\,\bar{u}(p's')\,\Bigg[-(M+M_B)\Bigg(-M^2+\frac{1}{2}\left(s_{p'q'}+s_{pq}\right)-2M\not{Q} \\
&\quad -\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)[\not{n},\not{Q}]-\frac{1}{2}(\kappa'-\kappa)^2\Bigg) \\
&\quad +\frac{1}{2}\left(s_{p'q'}-M^2\right)\left(\not{Q}+\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
&\quad +\frac{1}{2}\left(s_{pq}-M^2\right)\left(\not{Q}-\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
&\quad +\bar{\kappa}\left((p'-p)\cdot n\,\not{Q}+2Q\cdot n\,\not{Q}+M^2\,\not{n}\ -\frac{\not{n}}{2}\left(s_{p'q'}+s_{pq}\right)\right)\Bigg]\,u(p) \\
&\quad \times\frac{1}{(\Delta_s\cdot n+\bar{\kappa})^2-A^2_s+i\varepsilon}\ .
\end{aligned}
\tag{5.38}
$$

Here, (P)V stands for *(pseudo) vector*. Taking the limit $\kappa' = \kappa = 0$

$$
\begin{aligned}
M_{00}^{V}(u) &= \frac{f_V^2}{2m_\pi^2}\,\bar{u}(p's')\Big[-(M-M_B)\left(-M^2+u+2M\not{Q}\right) \\
&\qquad -\left(u-M^2\right)\not{Q}\Big]\,u(p)\,\frac{1}{u-M_B^2+i\varepsilon}\ , \\
M_{00}^{PV}(u) &= \frac{f_{PV}^2}{2m_\pi^2}\,\bar{u}(p's')\Big[-(M+M_B)\left(-M^2+u+2M\not{Q}\right) \\
&\qquad -\left(u-M^2\right)\not{Q}\Big]\,u(p)\,\frac{1}{u-M_B^2+i\varepsilon}\ , \\
M_{00}^{V}(s) &= \frac{f_V^2}{2m_\pi^2}\,\bar{u}(p's')\Big[-(M-M_B)\left(-M^2+s-2M\not{Q}\right) \\
&\qquad +\left(s-M^2\right)\not{Q}\Big]\,u(p)\,\frac{1}{s-M_B^2+i\varepsilon}\ , \\
M_{00}^{PV}(s) &= \frac{f_{PV}^2}{2m_\pi^2}\,\bar{u}(p's')\Big[-(M+M_B)\left(-M^2+s-2M\not{Q}\right) \\
&\qquad +\left(s-M^2\right)\not{Q}\Big]\,u(p)\,\frac{1}{s-M_B^2+i\varepsilon}\ , \qquad (5.39)
\end{aligned}
$$

where we, again, get factor 1/2 from the result in [11] for the same reason as mentioned in section 5.5.1.

Studying the n-dependence of the amplitudes (5.38) in light of the n-dependence of the Kadyshevsky integral equation as before (see section 2.4.3), we see that, again, all n-dependent terms in (5.38) are linear proportional to either κ or κ'. Therefore, when we would only consider (P)V baryon exchange or resonance in the Kadyshevsky integral equation, we would, again, find ourself in a similar situation as in (2.32), when studying the n-dependence. However, looking at the powers of κ and κ' in (5.38) we notice that the function $h(\kappa)$ in (2.32) is of higher order then $O(\frac{1}{\kappa^2})$. Therefore, the phenomenological "form factor" (2.34) would be necessary.

5.5.3 $\pi N \Delta_{33}$ Coupling

As far as the $\pi N\Delta_{33}$ coupling is concerned we find the following g_{gi}^2 contribution in the second and first order of the S-matrix from (5.25)

$$
S^{(2)} = (-i)^2 \int d^4x d^4y\, \theta[n(x-y)]\mathcal{H}_I(x)\mathcal{H}_I(y)
$$

$$
\begin{aligned}
&= -g_{gi}^2 \int d^4x d^4y\, \theta[n(x-y)] \left[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi\right]_x \partial^x_\mu \partial^y_{\mu'} \left(i\partial\!\!\!/ + M_\Delta\right) \\
&\qquad \times \Lambda_{\nu\nu'}\Delta^+(x-y) \left[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi\right]_y \,, \\
S^{(1)} &= (-i)\int d^4x \mathcal{H}_I(x) \\
&= \frac{g_{gi}^2}{2} \int d^4x d^4y \left[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi\right]_x \theta[n(x-y)]\partial_\mu\partial_{\mu'} \left(i\partial\!\!\!/ + M_\Delta\right) \\
&\qquad \times \left(g_{\nu\nu'} - \frac{1}{3}\gamma_\nu\gamma_{\nu'}\right) \Delta^{(1)}(x-y) \left[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi\right]_y \\
&\quad + \frac{ig_{gi}^2}{2}\left[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi\right] \Big[\left(i\partial\!\!\!/ + M_\Delta\right) n_\mu n_{\mu'} + n\!\!\!/ \left(n_\mu i\partial_{\mu'} + i\partial_\mu n_{\mu'}\right) \\
&\qquad - 2n\!\!\!/ n_\mu n_{\mu'} n\cdot i\partial \Big] \left(g_{\nu\nu'} - \frac{1}{3}\gamma_\nu\gamma_{\nu'}\right) \left[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi\right] \,,
\end{aligned}
\tag{5.40}
$$

where

$$
\Lambda_{\mu\nu} = -\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2\partial_\mu\partial_\nu}{3M^2} - \frac{1}{3M_\Delta}\left(\gamma_\mu i\partial_\nu - \gamma_\nu i\partial_\mu\right)\right] \,. \tag{5.41}
$$

Because of the anti-symmetric property of the epsilon tensor all derivative terms in (5.41) do not contribute.

Upon addition of the two contributions in (5.40) we find

$$
\begin{aligned}
&S^{(2)} + S^{(1)} = \\
&= -\frac{ig_{gi}^2}{2} \int d^4x d^4y \left[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi\right]_x \partial_\mu\partial_{\mu'} \left(i\partial\!\!\!/ + M_\Delta\right) \left(g_{\nu\nu'} - \frac{1}{3}\gamma_\nu\gamma_{\nu'}\right) \\
&\qquad \times \theta[n(x-y)]\Delta(x-y) \left[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi\right]_y \,.
\end{aligned}
\tag{5.42}
$$

Again, we have a similar situation for the S-matrix element as in section 5.5.1. Therefore, we refer for the discussion of (5.42) to the text below (5.27).

A difference of this S-matrix element as compared of those of the forgoing sections (sections 5.5.1 and 5.5.2) is that the derivatives do not only act on the $\Delta(x-y)$ propagator in (5.42), but also on the $\theta[n(x-y)]$. Therefore, the $\bar{P}$ method of section 3.5.3 can be applied. Of course this is obvious since this method was introduced in order to incorporate terms like the second term on the rhs of $S^{(1)}$ in (5.40).

As in the previous sections (sections 5.5.1 and 5.5.2) two amplitudes arise from this S-matrix: Δ_{33} exchange and resonance, whose the Kadyshevsky diagrams are shown in figure 5.2. The amplitudes are

$$\begin{aligned}
M_{\kappa'\kappa}(u) &= -\frac{g_{gi}^2}{2}\int \frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\epsilon^{\mu\nu\alpha\beta}\bar{u}(p's')\gamma_\alpha\gamma_5 q_\beta \left(\bar{P}_u\right)_\mu \left(\bar{P}_u\right)_{\mu'} \left(\bar{\not{P}}_u + M_\Delta\right) \\
&\quad \times \left(g_{\nu\nu'} - \frac{1}{3}\,\gamma_\nu\gamma_{\nu'}\right)\Delta(P_u)\,\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_{\alpha'}\gamma_5 q'_{\beta'} u(ps)\ , \\
M_{\kappa'\kappa}(s) &= -\frac{g_{gi}^2}{2}\int \frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\epsilon^{\mu\nu\alpha\beta}\bar{u}(p's')\gamma_\alpha\gamma_5 q'_\beta \left(\bar{P}_s\right)_\mu \left(\bar{P}_s\right)_{\mu'} \left(\bar{\not{P}}_s + M_\Delta\right) \\
&\quad \times \left(g_{\nu\nu'} - \frac{1}{3}\,\gamma_\nu\gamma_{\nu'}\right)\Delta(P_s)\,\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_{\alpha'}\gamma_5 q_{\beta'} u(ps)\ , \qquad (5.43)
\end{aligned}$$

where $\bar{P}_i = P_i + n\kappa_1$, $i = u, s$ (see section 3.5.3). P_i and $\Delta(P_i)$ are as before.

Performing the κ_1 integral is in this situation even simpler then in the previous cases (section 5.5.1 and 5.5.2). As can be seen from (5.30) the $\Delta(P_i)$ in (5.43) selects two solutions for κ_1 (with a relative minus sign, due to $\epsilon(P_i^0)$), which only need to be applied to the quasi scalar propagator $1/(\kappa_1+i\varepsilon)$. This, because the $\bar{P}_i$ is κ_1-independent

$$\begin{aligned}
&\frac{1}{2A_i}\left[\frac{1}{\Delta_i\cdot n + \bar{\kappa} - A_i + i\varepsilon} - \frac{1}{\Delta_i\cdot n + \bar{\kappa} + A_i + i\varepsilon}\right] \\
&= \frac{1}{\left(\Delta_i\cdot n + \bar{\kappa}\right)^2 - A_i^2 + i\varepsilon}\ . \qquad (5.44)
\end{aligned}$$

Contracting all the indices in (5.43) the amplitudes become

$$\begin{aligned}
M_{\kappa'\kappa}(u) &= -\frac{g_{gi}^2}{2}\,\bar{u}(p's')\Big[\left(\bar{\not{P}}_u + M_\Delta\right)\Big(\bar{P}_u^2\,(q'\cdot q) - \frac{1}{3}\,\bar{P}_u^2\not{q}\not{q}' - \frac{1}{3}\,\bar{\not{P}}_u\not{q}\,\left(\bar{P}_u\cdot q'\right) \\
&\quad + \frac{1}{3}\,\bar{\not{P}}_u\not{q}'\,\left(\bar{P}_u\cdot q\right) - \frac{2}{3}\,\left(\bar{P}_u\cdot q'\right)\left(\bar{P}_u\cdot q\right)\Big)\Big]\,u(ps) \\
&\quad \times\frac{1}{\left(\Delta_u\cdot n + \bar{\kappa}\right)^2 - A_u^2 + i\varepsilon}\ , \\
M_{\kappa'\kappa}(s) &= -\frac{g_{gi}^2}{2}\,\bar{u}(p's')\Big[\left(\bar{\not{P}}_s + M_\Delta\right)\Big(\bar{P}_s^2\,(q'\cdot q) - \frac{1}{3}\,\bar{P}_s^2\not{q}'\not{q} - \frac{1}{3}\,\bar{\not{P}}_s\not{q}'\,\left(\bar{P}_s\cdot q\right) \\
&\quad + \frac{1}{3}\,\bar{\not{P}}_s\not{q}\,\left(\bar{P}_s\cdot q'\right) - \frac{2}{3}\,\left(\bar{P}_s\cdot q'\right)\left(\bar{P}_s\cdot q\right)\Big)\Big]\,u(ps) \\
&\quad \times\frac{1}{\left(\Delta_s\cdot n + \bar{\kappa}\right)^2 - A_s^2 + i\varepsilon}\ , \qquad (5.45)
\end{aligned}$$

which leads, after some (Dirac) algebra, to

$$
\begin{aligned}
M_{\kappa'\kappa}(u) = & -\frac{g_{gi}^2}{2}\,\bar{u}(p's')\Bigg[\frac{1}{2}\,\bar{P}_u^2\,\left(M+M_\Delta-\not{Q}+\bar{\kappa}\not{n}\right)\left(2m^2-t_{q'q}\right) \\
& -\frac{1}{3}\,\bar{P}_u^2\,\Bigg(\left(M+M_\Delta\right)\not{q}\not{q}'+\frac{1}{2}\,\left(u_{pq'}-M^2\right)\not{q} \\
& \qquad +\frac{1}{2}\,\left(s_{pq}+t_{q'q}-M^2-4m^2\right)\not{q}'+\bar{\kappa}\not{n}\not{q}\not{q}'\Bigg) \\
& -\frac{1}{12}\left(\bar{P}_u^2\not{q}+\frac{M_\Delta}{2}\left(s_{pq}-M^2-2m^2\right)-\frac{M_\Delta}{2}\not{q}'\not{q}+M_\Delta\bar{\kappa}\not{n}\not{q}\right)\Bigg(-4m^2 \\
& \qquad +s_{p'q'}-u_{pq'}+t_{q'q}-2\bar{\kappa}(p'-p)\cdot n+4\bar{\kappa}n\cdot Q-\left(\kappa'^2-\kappa^2\right)\Bigg) \\
& +\frac{1}{12}\left(\bar{P}_u^2\not{q}'+\frac{M_\Delta}{2}\left(M^2-u_{pq'}\right)-\frac{M_\Delta}{2}\not{q}\not{q}'+M_\Delta\bar{\kappa}\not{n}\not{q}'\right)\Bigg(-4m^2 \\
& \qquad +s_{pq}-u_{p'q}+t_{q'q}+2\bar{\kappa}(p'-p)\cdot n+4\bar{\kappa}n\cdot Q+\left(\kappa'^2-\kappa^2\right)\Bigg) \\
& -\frac{1}{24}\left(M+M_\Delta-\not{Q}+\bar{\kappa}\not{n}\right)\Bigg(-4m^2+s_{p'q'}-u_{pq'}+t_{q'q} \\
& \qquad -2\bar{\kappa}(p'-p)\cdot n+4\bar{\kappa}n\cdot Q-\left(\kappa'^2-\kappa^2\right)\Bigg)\Bigg(-4m^2+s_{pq} \\
& \qquad -u_{p'q}+t_{q'q}+2\bar{\kappa}(p'-p)\cdot n+4\bar{\kappa}n\cdot Q+\left(\kappa'^2-\kappa^2\right)\Bigg)\Bigg]\,u(ps) \\
& \times\frac{1}{\left(\Delta_u\cdot n+\bar{\kappa}\right)^2-A_u^2+i\varepsilon}\ ,
\end{aligned}
$$

$$
\begin{aligned}
M_{\kappa'\kappa}(s) = & -\frac{g_{gi}^2}{2}\,\bar{u}(p's')\Bigg[\frac{1}{2}\,\bar{P}_s^2\,\left(M+M_\Delta+\not{Q}+\bar{\kappa}\not{n}\right)\left(2m^2-t_{q'q}\right) \\
& -\frac{1}{3}\,\bar{P}_s^2\,\Bigg(\left(M+M_\Delta\right)\not{q}'\not{q}-\frac{1}{2}\,\left(s_{pq}-M^2\right)\not{q}' \\
& \qquad -\frac{1}{2}\,\left(u_{pq'}+t_{q'q}-M^2-4m^2\right)\not{q}+\bar{\kappa}\not{n}\not{q}'\not{q}\Bigg) \\
& -\frac{1}{12}\left(\bar{P}_s^2\not{q}'+\frac{M_\Delta}{2}\left(M^2+2m^2-u_{pq'}\right)+\frac{M_\Delta}{2}\not{q}\not{q}'+M_\Delta\bar{\kappa}\not{n}\not{q}'\right)\Bigg(4m^2 \\
& \qquad +s_{pq}-u_{p'q}-t_{q'q}+2\bar{\kappa}(p'-p)\cdot n+4\bar{\kappa}n\cdot Q+\left(\kappa'^2-\kappa^2\right)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{12}\left(\bar{P}_s^2 \not{q} + \frac{M_\Delta}{2}\left(s_{pq}-M^2\right) + \frac{M_\Delta}{2}\not{q}'\not{q} + M_\Delta \bar{\kappa}\not{n}\not{q}\right)\Big(4m^2 \\
&\qquad + s_{p'q'} - u_{pq'} - t_{q'q} - 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa} n\cdot Q - \left(\kappa'^2-\kappa^2\right)\Big) \\
&-\frac{1}{24}\left(M + M_\Delta + \not{Q} + \bar{\kappa}\not{n}\right)\Big(4m^2 + s_{p'q'} - u_{pq'} - t_{q'q} \\
&\qquad - 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa} n\cdot Q - \left(\kappa'^2-\kappa^2\right)\Big)\Big(4m^2 + s_{pq} \\
&\qquad - u_{p'q} - t_{q'q} + 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa} n\cdot Q + \left(\kappa'^2-\kappa^2\right)\Big)\Big]\, u(ps) \\
&\times \frac{1}{\left(\Delta_s\cdot n + \bar{\kappa}\right)^2 - A_s^2 + i\varepsilon}\ , \qquad (5.46)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{P}_u^2 &= \frac{1}{2}\left(u_{p'q} + u_{pq'}\right) - \frac{1}{4}(\kappa'-\kappa)^2 + 2\bar{\kappa}\Delta_u\cdot n + \bar{\kappa}^2\ , \\
\bar{P}_s^2 &= \frac{1}{2}\left(s_{p'q'} + s_{pq}\right) - \frac{1}{4}(\kappa'-\kappa)^2 + 2\bar{\kappa}\Delta_s\cdot n + \bar{\kappa}^2\ , \qquad (5.47)
\end{aligned}
$$

and

$$
\begin{aligned}
\not{q}' &= \not{Q} - \frac{1}{2}\not{n}(\kappa'-\kappa)\ , \\
\not{q} &= \not{Q} + \frac{1}{2}\not{n}(\kappa'-\kappa)\ , \\
\not{q}'\not{q} &= -2M\not{Q} + \frac{1}{2}\left(s_{p'q'} + s_{pq}\right) - M^2 - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n \\
&\quad + \frac{1}{2}(\kappa'-\kappa)\left[\not{Q},\not{n}\right] - \frac{1}{2}(\kappa'-\kappa)^2\ , \\
\not{q}\not{q}' &= 2M\not{Q} + \frac{1}{2}\left(u_{p'q} + u_{pq'}\right) - M^2 - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n \\
&\quad - \frac{1}{2}(\kappa'-\kappa)\left[\not{Q},\not{n}\right] - \frac{1}{2}(\kappa'-\kappa)^2\ , \\
\not{n}\not{q}' &= M\not{n} - (n\cdot p') - \frac{1}{2}\left[\not{Q},\not{n}\right] + n\cdot Q - \frac{1}{2}(\kappa'-\kappa)\ , \\
\not{n}\not{q} &= -M\not{n} + (n\cdot p') - \frac{1}{2}\left[\not{Q},\not{n}\right] + n\cdot Q + \frac{1}{2}(\kappa'-\kappa)\ , \\
\not{n}\not{q}'\not{q} &= -M^2\not{n} + \frac{1}{2}\left(s_{p'q'} + s_{pq}\right)\not{n} - \frac{1}{2}(\kappa'-\kappa)n\cdot(p'-p)\not{n} \\
&\quad + (\kappa'-\kappa)(n\cdot Q)\not{n} - (\kappa'-\kappa)\not{Q} - 2n\cdot(p'-p)\not{Q} - \frac{1}{2}(\kappa'-\kappa)^2\not{n}\ ,
\end{aligned}
$$

$$\begin{aligned} \not{n}\not{q}\not{q}' &= -M^2\not{n} + \frac{1}{2}\left(u_{p'q} + u_{pq'}\right)\not{n} - \frac{1}{2}(\kappa'-\kappa)n\cdot(p'-p)\not{n} \\ &\quad - (\kappa'-\kappa)(n\cdot Q)\not{n} + (\kappa'-\kappa)\not{Q} + 2n\cdot(p'-p)\not{Q} - \frac{1}{2}(\kappa'-\kappa)^2\not{n} \ . \end{aligned} \tag{5.48}$$

Taking the limit $\kappa' = \kappa = 0$ yields

$$\begin{aligned} M_{00}(u) &= -\frac{g_{gi}^2}{2}\bar{u}(p's')\left[\frac{u}{2}\left(M + M_\Delta - \not{Q}\right)\left(2m^2 - t\right)\right. \\ &\quad - \frac{u}{3}\left(\left(M + M_\Delta\right)\left(2M\not{Q} + u - M^2\right) - m^2\not{Q}\right) \\ &\quad - \frac{1}{6}\left(u\not{Q} + M_\Delta\left(M\not{Q} - m^2\right)\right)\left(M^2 - m^2 - u\right) \\ &\quad + \frac{1}{6}\left(u\not{Q} + M_\Delta\left(M^2 - u - M\not{Q}\right)\right)\left(M^2 - m^2 - u\right) \\ &\quad \left. -\frac{1}{6}\left(M + M_\Delta - \not{Q}\right)\left(M^2 - m^2 - u\right)^2\right] u(ps) \\ &\quad \times \frac{1}{u - M_\Delta^2 + i\varepsilon} \ , \\ M_{00}(s) &= -\frac{g_{gi}^2}{2}\bar{u}(p's')\left[\frac{s}{2}\left(M + M_\Delta + \not{Q}\right)\left(2m^2 - t\right)\right. \\ &\quad - \frac{s}{3}\left(\left(M + M_\Delta\right)\left(-2M\not{Q} + s - M^2\right) + m^2\not{Q}\right) \\ &\quad - \frac{1}{6}\left(s\not{Q} + M_\Delta\left(M\not{Q} + m^2\right)\right)\left(s - M^2 + m^2\right) \\ &\quad + \frac{1}{6}\left(s\not{Q} + M_\Delta\left(s - M^2 - M\not{Q}\right)\right)\left(s - M^2 + m^2\right) \\ &\quad \left. - \frac{1}{6}\left(M + M_\Delta + \not{Q}\right)\left(s - M^2 + m^2\right)^2\right] u(ps) \\ &\quad \times \frac{1}{s - M_\Delta^2 + i\varepsilon} \ . \end{aligned} \tag{5.49}$$

Considering only the Δ_{33} exchange and resonance in the Kadyshevsky integral equation and study its n-dependence, we see from (5.46) and (5.48) that we have a similar situation as in the previous section (section 5.5.2): all n-dependent terms in (5.46) and (5.48) are either proportional to κ or to κ' and therefore (2.32) applies. The function $h(\kappa)$ is such that the phenomenological "form factor" (2.34) is necessary.

5.6 Conclusion and Discussion

At the end of this chapter we conclude and discuss the main results. We started with formally implementing absolute pair suppression by excluding $\overline{\psi^{(\pm)}}\Gamma\psi^{(\mp)}$ transitions in the interaction Lagrangian. After a whole procedure of getting the interaction Hamiltonian this feature is still present in the amplitudes, where the $\bar{u}v, \bar{v}u$ contributions are zero. This is a particularly nice for the Kadyshevsky integral equation as we have discussed in section 2.4.2.

It should be noticed that still negative energy propagates inside an amplitude via the Δ-propagator. However, this is also the case in [11] and in the example of the infinite dense anti-neutron star.

From the S-matrix elements and the amplitudes we see that they are causal, covariant and n-independent. Moreover, the amplitudes are just a factor 1/2 of the usual Feynman expressions. This could be intercepted by rescaling the coupling constant in the interaction Lagrangian.

We have seen that it is particularly convenient to use the Kadyshevsky formalism. Since positive and negative energy contributions are separated it is much easier to implement pair suppression and to analyze the n-dependence.

Chapter 6

Partial Wave Expansion

In elastic scattering processes important observables are the phase-shifts. In this chapter we introduce the phase-shifts by introducing the partial wave expansion, which is particularly convenient for solving the Kadyshevsky integral equation (2.26). By also using the helicity basis we're able to link the amplitudes obtained in the previous sections (section 4 and 5) to the phase-shifts.

6.1 Amplitudes and Invariants

In Feynman formalism the most general form of the parity-conserving amplitude describing πN-scattering is [42, 43]

$$M_{fi} \quad = \quad \bar{u}(p's') \left[A + B\not{Q} \right] u(ps) \ , \tag{6.1}$$

where the invariants A and B are functions of the Mandelstam variable t, u and s. However, in Kadyshevsky formalism there's an extra variable n^μ. Therefore the number of invariants is doubled. Following the procedure of [43] we can construct an extra vector and tensor term

$$M_{\kappa'\kappa} \quad = \quad \bar{u}(p's') \left[A + B\not{Q} + A'\not{n} + B' \left[\not{n}, \not{Q}\right] \right] u(ps) \ . \tag{6.2}$$

In Kadyshevsky formalism the invariants A, B, A' and B' are not only functions of the Mandelstam variables (1.4), but also of κ and κ'. The contribution of the invariants to the various exchange processes is given in appendix C.

In proceeding we don't keep n^μ general, but choose it to be [15, 17]

$$n^\mu = \frac{(p+q)^\mu}{\sqrt{s_{pq}}} = \frac{(p'+q')^\mu}{\sqrt{s_{p'q'}}} \ . \tag{6.3}$$

With this choice, n^μ is not an independent variable anymore and the number of invariants is reduced to two, again. This is made explicit as follows

$$\begin{aligned}\bar{u}(p's')\,[\not{n}]\,u(ps) &= \frac{1}{\sqrt{s_{p'q'}}+\sqrt{s_{pq}}}\bar{u}(p's')\,[M_f+M_i+2\not{Q}]\,u(ps)\ ,\\ \bar{u}(p's')\left[[\not{n},\not{Q}]\right]u(ps) &= 0\ .\end{aligned} \tag{6.4}$$

As a result of the choice (6.3) the invariants A and B in (6.2) receive contributions from the invariant A'. We, therefore, redefine the amplitude

$$\begin{aligned}M_{\kappa'\kappa} &= \bar{u}(p's')\left[A''+B''\not{Q}\right]u(ps)\ ,\\ A'' &= A+\frac{1}{\sqrt{s_{p'q'}}+\sqrt{s_{pq}}}\,(M_f+M_i)\,A'\ ,\\ B'' &= B+\frac{2}{\sqrt{s_{p'q'}}+\sqrt{s_{pq}}}\,A'\ .\end{aligned} \tag{6.5}$$

Besides the invariants A'' and B'', we also introduce the invariants F and G very similar to [42] [1]

$$M_{\kappa'\kappa} = \chi^\dagger(s')\left[F+G\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}')\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{p}})\right]\chi(s)\ , \tag{6.6}$$

since we will use the helicity basis. Here, $\chi(s)$ is a helicity state vector. In [11] this expansion was used in combination with the expansion of the amplitude in Pauli spinor space. The connection between the two are also given there.

In order to see the connection between the invariants A'', B'' and F, G we express the operators 1 and $\not{Q}$ sandwiched between initial and final state u spinors in terms of initial and final state χ vectors

$$\begin{aligned}\bar{u}(p's')\ u(ps) &= \sqrt{(E'+M_f)(E+M_i)}\\ &\qquad \chi'^{\,\dagger}(s')\left[1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\ \boldsymbol{\sigma}\cdot\mathbf{p}}{(E'+M_f)(E+M_i)}\right]\chi(s)\ ,\\ \bar{u}(p's')\ \not{Q}\,u(ps) &= \sqrt{(E'+M_f)(E+M_i)}\ \chi'^{\,\dagger}(s')\left[\frac{1}{2}\left[(W'-M_f)+(W-M_i)\right]\right.\\ &\qquad \left.+\frac{1}{2}\left[(W'+M_f)+(W+M_i)\right]\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\ \boldsymbol{\sigma}\cdot\mathbf{p}}{(E'+M_f)(E+M_i)}\right]\chi(s)\ ,\end{aligned} \tag{6.7}$$

[1] The difference is a normalization factor.

from this we deduce that

$$\begin{aligned} F &= \sqrt{(E'+M_f)(E+M_i)}\ \left\{ A'' + \frac{1}{2}\left[(W'-M_f)+(W-M_i)\right]\ B'' \right\}\ , \\ G &= \sqrt{(E'-M_f)(E-M_i)}\ \left\{ -A'' + \frac{1}{2}\left[(W'+M_f)+(W+M_i)\right]\ B'' \right\}\ . \end{aligned} \tag{6.8}$$

6.2 Helicity Amplitudes and Partial Waves

In this section we want to link the invariants A'' and B'' to experimental observable phase-shifts. This is done by using the helicity basis and the partial wave expansion. The procedure is based on [44] and similar to [33].

The helicity amplitude in terms of the invariants F and G (see (6.6)) is

$$M_{\kappa'\kappa}(\lambda_f,\lambda_i) = C_{\lambda_f,\lambda_i}(\theta,\phi)\left[F + 4\lambda_f\lambda_i G\right]\ , \tag{6.9}$$

where

$$\begin{aligned} C_{\lambda_f,\lambda_i}(\theta,\phi) &= \chi^\dagger_{\lambda_f}(\hat{\mathbf{p}}')\cdot\chi_{\lambda_i}(\hat{\mathbf{p}}) = D^{1/2*}_{\lambda_i\lambda_f}(\phi,\theta,-\phi)\ , \\ C_{\pm 1/2,\pm 1/2}(\theta,\phi) &= \cos\theta/2\ , \\ C_{\pm 1/2,\mp 1/2}(\theta,\phi) &= \mp e^{\pm i\phi}\sin\theta/2\ . \end{aligned} \tag{6.10}$$

Here, $D^J_{mm'}(\alpha,\beta,\gamma)$ are the Wigner D-matrices [44] and the angles θ and ϕ are defined as the polar angles of the CM-momentum $\mathbf{p}'$ in a coordinate system that has $\mathbf{p}$ along the positive z-axis. In the following we take as the scattering plane the xz-plane, i.e. $\phi = 0$ (see figure 1.1). Furthermore, we introduce the functions $f_{1,2}$ by

$$F = \frac{f_1}{4\pi}\ \ ,\ \ G = \frac{f_2}{4\pi}\ . \tag{6.11}$$

Then, with these settings the helicity amplitude (6.9) is

$$M_{\kappa'\kappa}(\lambda_f,\lambda_i) = \frac{1}{4\pi}\, d^{1/2}_{\lambda_i\lambda_f}(\theta)\left(f_1 + 4\lambda_f\lambda_i f_2\right)\ , \tag{6.12}$$

Using the explicit forms of the Wigner d-matrices as in (6.10) we see that we have the following relations between the various helicity amplitudes

$$\begin{aligned} M_{\kappa'\kappa}(1/2,1/2) &= M_{\kappa'\kappa}(-1/2,-1/2)\ , \\ M_{\kappa'\kappa}(-1/2,1/2) &= -M_{\kappa'\kappa}(1/2,-1/2)\ . \end{aligned} \tag{6.13}$$

Next, we make the partial wave expansion of the helicity amplitudes in the CM-frame very similar to [42] [2]

$$\begin{aligned} M_{\kappa'\kappa}(\lambda_f\lambda_i) &= (4\pi)^{-1}\sum_J(2J+1)M^J_{\kappa'\kappa}(\lambda_f\lambda_i)\ D^{J*}_{\lambda_i,\lambda_f}(\phi,\theta,-\phi)\ , \\ &= (4\pi)^{-1}e^{i(\lambda_i-\lambda_f)\phi}\sum_J(2J+1)M^J_{\kappa'\kappa}(\lambda_f\lambda_i)\ d^J_{\lambda_i,\lambda_f}(\theta)\ , \end{aligned} \tag{6.14}$$

Because of the properties of the Wigner d-matrices the partial wave equivalent of (6.13) is

$$\begin{aligned} M^J_{\kappa'\kappa}(1/2,1/2) &= M^J_{\kappa'\kappa}(-1/2,-1/2)\ , \\ M^J_{\kappa'\kappa}(-1/2,1/2) &= M^J_{\kappa'\kappa}(1/2,-1/2)\ . \end{aligned} \tag{6.15}$$

Using the partial wave expansion as in (6.14) we obtain the Kadyshevsky integral equation (2.26) in the partial wave basis. Here, we just show the result; for the details we refer to [33]

$$\begin{aligned} M^J_{00}(\lambda_f\lambda_i) &= M^{irr\ J}_{00}(\lambda_f\lambda_i)+\sum_{\lambda_n}\int_0^\infty k_n^2dk_n\ M^{irr\ J}_{0\kappa}(\lambda_f\lambda_n) \\ &\times G'_\kappa\left(W_n;W\right)\ M^J_{\kappa 0}(\lambda_n\lambda_i)\ . \end{aligned} \tag{6.16}$$

As mentioned in the text below (2.26), the κ-label is fixed after integration.

Because of the summation over the intermediate helicity states the partial wave Kadyshevsky integral equation (6.16) is a coupled integral equation. It can be decoupled using the combinations $f_{(J-1/2)+}$ and $f_{(J+1/2)-}$ defined by

$$\begin{pmatrix} f_{L+} \\ f_{(L+1)-} \end{pmatrix} = \begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} \begin{pmatrix} M^J(+1/2\ 1/2) \\ M^J(-1/2\ 1/2) \end{pmatrix}\ , \tag{6.17}$$

here we introduced $L \equiv J-1/2$ [3].

In (6.17) and in the following we omit the subscript 00 for the final amplitudes where κ and κ' are put to zero.

A similar expansion as (6.17) holds for $M^{irr\ J}_{\kappa'\kappa}(\lambda_f\lambda_i)$, therefore the decoupling can easily be seen by adding and subtracting (6.16) for $M^J(1/2\ 1/2)$ and $M^J(-1/2\ 1/2)$, and using (6.15). What one gets is

$$\begin{aligned} f_{L\pm}(W',W) &= f^{irr}_{L\pm}(W',W)+\int_0^\infty k_n^2dk_n\ f^{irr}_{L\pm}(W',W_n) \\ &\times G\left(W',W_n\right)\ f_{L\pm}(W_n,W)\ . \end{aligned} \tag{6.18}$$

[2]The difference is again a normalization factor. We use the same normalization as [11] and [33].

[3]The labels $L+$ and $(L+1)-$ in (6.17) and their relation to total angular momentum J come from parity arguments as is best explained in [44].

The two-particle unitarity relation for the partial-wave helicity states reads [42]

$$i\left[M^J(\lambda_f\lambda_i) - M^{J*}(\lambda_i\lambda_f)\right] = 2\sum_{\lambda_n} k\ M^{J*}(\lambda_f\lambda_n)M^J(\lambda_i\lambda_n)\ , \tag{6.19}$$

In a similar manner as for the partial wave Kadyshevsky integral equation (6.16), also the unitarity relation (6.19) decouples for the combinations (6.17). One gets

$$Im f_{L\pm}(W) = k\ f^*_{L\pm}(W) f_{L\pm}(W)\ , \tag{6.20}$$

which allows for the introduction of the elastic phase-shifts

$$f_{L\pm}(W) = \frac{1}{k}\ e^{i\delta_{L\pm}(W)} \sin\delta_{L\pm}(W)\ . \tag{6.21}$$

From (6.21) we see that once we have found the invariants $f_{L\pm}(W)$ by solving the partial wave Kadyshevsky integral equation (6.16) we can determine the phase-shifts. Now, we must find a relation between the invariants $f_{L\pm}(W)$ and the invariants $f_{1,2}$. This is done by considering (6.12) and (6.14) again for the helicities $\lambda_f, \lambda_i = 1/2\ , \pm 1/2$. Using the formulas

$$\begin{aligned}(J+1/2)d^J_{1/2\ 1/2}(\theta) &= \cos\theta/2\left(P'_{J+1/2}(cos\,\theta) - P'_{J-1/2}(cos\,\theta)\right)\ ,\\ (J+1/2)d^J_{-1/2\ 1/2}(\theta) &= \sin\theta/2\left(P'_{J+1/2}(cos\,\theta) + P'_{J-1/2}(cos\,\theta)\right)\ ,\end{aligned} \tag{6.22}$$

where $P_L(cos\,\theta)$ are Legendre polynomials, and the relations (6.15) one derives

$$f_1 \pm f_2 = 2\sum_J \left(P'_{J+1/2} \mp P'_{J-1/2}\right)\ M^J(\pm 1/2, 1/2)\ . \tag{6.23}$$

Solving for $f_{1,2}$ we get

$$\begin{aligned} f_1 &= \sum_J \Bigg[\Big(M^J(1/2,1/2) + M^J(-1/2,1/2)\Big) P'_{J+1/2} \\ &\qquad - \Big(M^J(1/2,1/2) - M^J(-1/2,1/2)\Big) P'_{J-1/2}\Bigg]\ ,\\ f_2 &= \sum_J \Bigg[\Big(M^J(1/2,1/2) - M^J(-1/2,1/2)\Big) P'_{J+1/2} \\ &\qquad - \Big(M^J(1/2,1/2) + M^J(-1/2,1/2)\Big) P'_{J-1/2}\Bigg]\ . \end{aligned} \tag{6.24}$$

From the combinations in (6.24) we recognize the partial wave amplitudes $f_{L\pm}$ from (6.17) and writing again $J = L + 1/2$ one gets the following expansion in terms of derivatives of the Legendre polynomials

$$
\begin{aligned}
f_1 &= \sum_{L=0} \left[f_{L+} P'_{L+1}(x) - f_{(L+1)-}(x) P'_L \right] \\
&= f_{0+} + \sum_{L=1} \left[f_{L+} P'_{L+1} - f_{L-} P'_{L-1} \right] \ , \\
f_2 &= \sum_{L=1} \left[f_{L-} - f_{L+} \right] P'_L \ . \qquad (6.25)
\end{aligned}
$$

Using the orthogonality relations of the Legendre polynomials (6.25) can be inverted to find that

$$
\begin{aligned}
f_{L\pm} &= \frac{1}{2} \int_{-1}^{+1} dx \ \left[P_L(x) f_1 + P_{L\pm1}(x) f_2 \right] \\
&= f_{1,L} + f_{2,L\pm1} \ , \qquad (6.26)
\end{aligned}
$$

where $x = cos\,\theta$. With (6.26) the (partial wave projections of the) invariants $f_{1,2}$ are related to the invariants $f_{L\pm}$, from which the phase-shifts can be deduced.

6.3 Partial Wave Projection

Via the equations (6.26), (6.11) and (6.8), the partial waves $f_{L\pm}$ can be traced back to the partial wave projection of the invariant amplitudes A'' and B'', which means that we are looking for the partial wave projections of the invariants A, B, A', B'.

Before doing so we include form factors in the same way as in [11]. As mentioned there, they are needed to regulate the high energy behavior and to take into account the extended size of the mesons and baryons. We take them to be

$$
\begin{aligned}
F(\Lambda) &= e^{\frac{-\left(\mathbf{k}_f - \mathbf{k}_i\right)^2}{\Lambda^2}} \qquad \text{for } t\text{-channel} \ , \\
F(\Lambda) &= e^{\frac{-\left(\mathbf{k}_f^2 + \mathbf{k}_i^2\right)}{\Lambda^2}} \qquad \text{for } u, s\text{-channel} \ . \qquad (6.27)
\end{aligned}
$$

The partial wave projection includes an integration over $cos\,\theta = x$. We, therefore, investigate the x-dependence of the invariants. Main concern is the propagators. We want to write them in the form $1/(z \pm x)$, which is especially difficult for the propagators in the t-channel, because of the square root in A_t. We therefore use the identity

$$
\frac{1}{\omega(\omega + a)} = \frac{1}{\omega^2 - a^2} + \frac{2a}{\pi} \int_0^\infty \frac{d\lambda}{\lambda^2 + a^2} \left[\frac{1}{\omega^2 + \lambda^2} - \frac{1}{\omega^2 - a^2} \right] \ , \qquad (6.28)
$$

which holds for $\omega, a \in \mathbb{R}$. With this identity we write the propagators as

$$\begin{aligned}
\frac{1}{2A_t}\,\frac{1}{\Delta_t\cdot n+\bar\kappa-A_t+i\varepsilon} &= -\frac{1}{2p'p}\left[\frac{1}{2}+\frac{\Delta_t\cdot n+\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(\bar\kappa)}\right]\frac{1}{z_t(\bar\kappa)-x}\\
&\quad +\frac{1}{2p'p}\,\frac{\Delta_t\cdot n+\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(\bar\kappa)}\,\frac{1}{z_{t,\lambda}-x}\ ,\\
\frac{1}{2A_t}\,\frac{1}{-\Delta_t\cdot n+\bar\kappa-A_t+i\varepsilon} &= -\frac{1}{2p'p}\left[\frac{1}{2}-\frac{\Delta_t\cdot n-\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(-\bar\kappa)}\right]\\
&\quad\times\frac{1}{z_t(-\bar\kappa)-x}\\
&\quad -\frac{1}{2p'p}\,\frac{\Delta_t\cdot n-\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(-\bar\kappa)}\,\frac{1}{z_{t,\lambda}-x}\ ,\\
\frac{1}{(\bar\kappa+\Delta_u\cdot n)^2-A_u^2} &= -\frac{1}{2p'p}\,\frac{1}{z_u(\bar\kappa)+x}\ , \qquad (6.29)
\end{aligned}$$

where $p'p=|\mathbf{p}'||\mathbf{p}|$ and

$$\begin{aligned}
f_\lambda(\bar\kappa) &= \lambda^2+(\Delta_t\cdot n)^2+\bar\kappa^2+2\bar\kappa\Delta_t\cdot n\ ,\\
z_i(\bar\kappa) &= \frac{1}{2p'p}\left[p'+p+M^2-\bar\kappa^2-2\bar\kappa\Delta_i^0-(\Delta_i^0)^2\right]\ ,\\
z_{t,\lambda} &= \frac{1}{2p'p}\left[p'+p+M^2+\lambda^2\right]\ . \qquad (6.30)
\end{aligned}$$

The invariants are expanded in polynomials of x, like

$$\begin{aligned}
j^{\pm}(t) &= \left[X^j(\pm)+xY^j(\pm)\right]D^{(1)}(\pm\Delta_t,n,\bar\kappa)\\
&= \frac{1}{2p'p}\left[\left(X_1^j(\pm)+xY_1^j(\pm)\right)\frac{F(\Lambda_t)}{z_t(\pm\bar\kappa)-x}\right.\\
&\qquad\left.+\left(X_2^j(\pm)+xY_2^j(\pm)\right)\frac{F(\Lambda_t)}{z_{t,\lambda}-x}\right]\ ,\\
j(u) &= \frac{1}{2p'p}\left(X^j+xY^j+x^2Z^j\right)\frac{F(\Lambda_u)}{z_u(\bar\kappa)+x}\ ,\\
j(s) &= \left(X^j+xY^j+x^2Z^j\right)\frac{F(\Lambda_s)}{\frac{1}{4}(W'+W+\kappa'+\kappa)^2-M_B^2}\ , \qquad (6.31)
\end{aligned}$$

where j is an element of the set (A,B,A',B'). Furthermore, there are the relations in the t-channel

$$\begin{aligned}
X_1^j(\pm) &= -\left[\frac{1}{2}+\frac{\pm\Delta_t^0+\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(\pm\bar\kappa)}\right]X^j(\pm)\ ,\\
X_2^j(\pm) &= \frac{\pm\Delta_t^0+\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(\pm\bar\kappa)}X^j(\pm)\ . \qquad (6.32)
\end{aligned}$$

The coefficients X^j, Y^j and Z^j can easily be extracted from the invariants and they are given for the various exchange processes in appendix C.

With the partial wave projection

$$j_L(i) = \frac{1}{2}\int_{-1}^{1} dx\, P_L(x)\, j(i)\ , \tag{6.33}$$

where $i = t, u, s$, we find the partial wave projections of the invariants

$$\begin{aligned}
j_L^{\pm}(t) &= \frac{1}{2p'p}\Big[\Big(X_1^j(\pm) + z_t(\pm\bar{\kappa})Y_1^j(\pm)\Big)\, U_L(\Lambda_t, z_t(\pm\bar{\kappa})) \\
&\qquad + \Big(X_2^j(\pm) + z_{t,\lambda}Y_2^j(\pm)\Big)\, U_L(\Lambda_t, z_{t,\lambda}) \\
&\qquad - Y_1^j(\pm)R_L(\Lambda_t, z_t(\pm\bar{\kappa})) - Y_2^j(\pm)R_L(\Lambda_t, z_{t,\lambda})\Big] \\
j_L(u) &= \frac{(-1)^L}{2p'p}\Big[\Big(X^j - z_u(\bar{\kappa})Y^j + z_u^2(\bar{\kappa})Z^j\Big)\, U_L(\Lambda_u, z_u(\bar{\kappa})) \\
&\qquad - \Big(-Y^j + z_u(\bar{\kappa})Z^j\Big)\, R_L(\Lambda_u, z_u(\bar{\kappa})) \\
&\qquad - Z^j S_L(\Lambda_u, z_u(\bar{\kappa}))\Big] \\
j_L(s) &= \Big[X^j\, \delta_{L,0} + \frac{1}{3}Y^j\, \delta_{L,1} + \frac{1}{3}\Big(\frac{2}{5}\,\delta_{L,2} + \delta_{L,0}\Big)\, Z^j\Big] \\
&\qquad \times \frac{F(\Lambda_s)}{\frac{1}{4}\left(W' + W + \kappa' + \kappa\right)^2 - M_B^2}\ ,
\end{aligned} \tag{6.34}$$

where

$$\begin{aligned}
U_L(\Lambda, z) &= \frac{1}{2}\int_{-1}^{1} dx\, \frac{P_L(x)F(\Lambda)}{z - x}\ , \\
R_L(\Lambda, z) &= \frac{1}{2}\int_{-1}^{1} dx\, P_L(x)F(\Lambda)\ , \\
S_L(\Lambda, z) &= \frac{1}{2}\int_{-1}^{1} dx\, xP_L(x)F(\Lambda)\ .
\end{aligned} \tag{6.35}$$

Appendices

Appendix A

Proof of the form of $\Phi_\alpha(x,\sigma)$

Here, we prove that

$$\Phi_\alpha(x,\sigma) = \Phi_\alpha(x) + \int_\infty^\sigma d^4y\, D_a(y) R_{\alpha\beta} \Delta(x-y) \boldsymbol{j}_{a,\beta}(y) \ . \tag{A.1}$$

The proof is divided in several steps. We start in section A.1 and A.2 with scalar fields and no derivatives in the interaction Lagrangian. Section A.1 gives a proof up to second order and section A.2 the proof up to all orders.

We extend the proof by including multiple derivatives in the interaction Lagrangian in section A.3 and make a generalization to other types of fields in A.4.

A.1 2^{nd} Order

As mentioned before we consider scalar fields and no derivatives in the interaction Lagrangian. Therefore we proof that

$$\phi(x,\sigma) = \phi(x) + \int_{-\infty}^\sigma d^4y\, \Delta(x-y)\boldsymbol{j}(y) \ , \tag{A.2}$$

is valid up to second order in the coupling constant.

Imagine we have a scalar self interaction of a general form

$$\mathcal{L}_I(x) \ = \ -\mathcal{H}_I(x) = g\phi^n(x) \ , \tag{A.3}$$

We define a quantity $j(x)$, which is the derivative of the interaction Hamiltonian with respect to the scalar field.

$$j(x) \equiv \frac{\partial \mathcal{H}_I}{\partial \phi} = -ng\phi^{n-1}(x) \ , \tag{A.4}$$

Although we have a form of the interaction Hamiltonian in (A.3) it is merely meant to demonstrate the following essential equation

$$[\phi(x), \mathcal{H}_I(y)] = -ng\phi^{n-1}(y) i\Delta(x-y) = i\Delta(x-y)j(y) \ . \tag{A.5}$$

The last important ingredient is the expansion of the evolution operator up to order g^2

$$\begin{aligned} U[\sigma] &= 1 - i\int_{-\infty}^{\sigma} d^4x'\, \mathcal{H}_I(x') \\ &\quad - \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \theta(\sigma_{x'} - \sigma_{y'})\, \mathcal{H}_I(x')\mathcal{H}_I(y') \ , \\ U^{-1}[\sigma] &= 1 + i\int_{-\infty}^{\sigma} d^4x'\, \mathcal{H}_I(x') \\ &\quad - \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \theta(\sigma_{y'} - \sigma_{x'})\, \mathcal{H}_I(x')\mathcal{H}_I(y') \ . \end{aligned} \tag{A.6}$$

To proof (A.2) we start with

$$\begin{aligned} &\phi(x,\sigma) = U^{-1}[\sigma]\phi(x)U[\sigma] \\ =\ & \phi(x) - i\int_{-\infty}^{\sigma} d^4x'\, \phi(x)\mathcal{H}_I(x') + i\int_{-\infty}^{\sigma} d^4x'\, \mathcal{H}_I(x')\phi(x) \\ & - \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \theta[\sigma_{x'} - \sigma_{y'}]\, \phi(x)\mathcal{H}_I(x')\mathcal{H}_I(y') \\ & - \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \theta[\sigma_{y'} - \sigma_{x'}]\, \mathcal{H}_I(x')\mathcal{H}_I(y')\phi(x) \\ & + \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \mathcal{H}_I(x')\phi(x)\mathcal{H}_I(y') \\ =\ & \phi(x) - i\int_{-\infty}^{\sigma} d^4x' \left[\phi(x), \mathcal{H}_I(x')\right] \\ & - \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \theta[\sigma_{x'} - \sigma_{y'}] \left[\phi(x), \mathcal{H}_I(x')\right] \mathcal{H}_I(y') \\ & - \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\, \theta[\sigma_{y'} - \sigma_{x'}]\, \mathcal{H}_I(x') \left[\mathcal{H}_I(y'), \phi(x)\right] \ . \end{aligned} \tag{A.7}$$

Here, we have brought $\phi(x)$ between the interaction Hamiltonians, such that several contributions cancel. What is left are the commutators

$$\begin{aligned}
\phi(x,\sigma) &= \phi(x) + \int_{-\infty}^{\sigma} d^4x' \Delta(x-x') j(x') \\
&\quad -i \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma_{x'}} d^4y' \Delta(x-x') j(x') \mathcal{H}_I(y') \\
&\quad +i \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma_{x'}} d^4y' \Delta(x-x') \mathcal{H}_I(y') j(x') \ , \\
&= \phi(x) + \int_{-\infty}^{\sigma} d^4x' \Delta(x-x') U^{-1}[\sigma_{x'}] j(x') U[\sigma_{x'}] \\
&\quad = \phi(x) + \int_{-\infty}^{\sigma} d^4x' \Delta(x-x') \boldsymbol{j}(x') \ ,
\end{aligned} \tag{A.8}$$

which is what we wanted to proof up to second order in the coupling constant.

Although in principle we have defined $j(x)$ in (A.4) in a different way then in section 3.2, we see by (A.3) that they are equivalent in this example.

A.2 All Orders

In this section we proof (A.2) to all orders. In order to do so we will need the expansion of the U operator and its inverse

$$\begin{aligned}
U[\sigma] &= 1 + \sum_{n=1}^{\infty} (-i)^n \int_{-\infty}^{\sigma} d^4x_1 \dots d^4x_n \, \theta(\sigma_1 - \sigma_2) \dots \theta(\sigma_{n-1} - \sigma_n) \\
&\qquad \times \mathcal{H}_I(x_1) \dots \mathcal{H}_I(x_n) \ , \\
&= \sum_{n=0}^{\infty} U_n[\sigma] \quad , \quad U_0[\sigma] = 1 \ , \\
U^{-1}[\sigma] &= 1 + \sum_{n=1}^{\infty} i^n \int_{-\infty}^{\sigma} d^4x_1 \dots d^4x_n \, \theta(\sigma_n - \sigma_{n-1}) \dots \theta(\sigma_2 - \sigma_1) \\
&\qquad \times \mathcal{H}_I(x_1) \dots \mathcal{H}_I(x_n) \ , \\
&= \sum_{n=0}^{\infty} U_n^{-1}[\sigma] \quad , \quad U_0^{-1}[\sigma] = 1 \ .
\end{aligned} \tag{A.9}$$

Again we start from $\phi(x,\sigma) = U^{-1}[\sigma]\,\phi(x)\,U[\sigma]$. Consider now the $2m^{th}$ order. In the end we need to sum over all m

$$\begin{aligned} U^{-1}[\sigma]\,\phi(x)\,U[\sigma] &= U_0^{-1}\,\phi(x)\,U_{2m} + U_1^{-1}\,\phi(x)\,U_{2m-1} + \\ &\quad \ldots + U_m^{-1}\,\phi(x)\,U_m + \ldots \\ &\quad + U_{2m-1}^{-1}\,\phi(x)\,U_1 + U_{2m}^{-1}\,\phi(x)\,U_0 \;. \end{aligned} \tag{A.10}$$

[1] Bring every $\phi(x)$ in the middle of the interaction Hamiltonians at the cost of commutators

$$\begin{aligned} * \quad & \phi(x)\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{2m}) = \\ = \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})\phi(x)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\ & + \Big[\phi(x),\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)\Big]\,\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\ * \quad & \mathcal{H}_I(x_1)\phi(x)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_{2m}) = \\ = \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})\phi(x)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\ & + \mathcal{H}_I(x_1)\Big[\phi(x),\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_m)\Big]\,\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\ & \ldots \\ * \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{2m-1})\phi(x)\mathcal{H}_I(x_{2m}) = \\ = \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})\phi(x)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\ & + \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)\Big[\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m-1}),\phi(x)\Big]\,\mathcal{H}_I(x_{2m}) \\ * \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{2m})\phi(x) = \\ = \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})\phi(x)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\ & + \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)\Big[\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}),\phi(x)\Big] \;. \end{aligned} \tag{A.11}$$

Next, we concentrate on the $\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})\phi(x)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m})$ part (, which we call in the following formula $\Box$), since it is present in every term. The factors of i and the θ-functions will cause these factors to cancel

$$\begin{aligned} & (-i)^{2m} \quad \theta(\sigma_1-\sigma_2)\theta(\sigma_2-\sigma_3)\theta(\sigma_3-\sigma_4)\ldots\Box \\ & +(-i)^{2m-2} \qquad \theta(\sigma_2-\sigma_3)\theta(\sigma_3-\sigma_4)\ldots\Box \\ & +(-i)^{2m-4}\theta(\sigma_2-\sigma_1) \qquad \theta(\sigma_3-\sigma_4)\ldots\Box \\ & \ldots \\ & +(i)^{2m-4}\ldots\theta(\sigma_{2m-2}-\sigma_{2m-3}) \qquad \theta(\sigma_{2m-1}-\sigma_{2m})\Box \\ & +(i)^{2m-2}\ldots\theta(\sigma_{2m-2}-\sigma_{2m-3})\theta(\sigma_{2m-1}-\sigma_{2m-2}) \qquad \Box \end{aligned}$$

[1] Since m is an integer, $2m$ is even. This is chosen for convenience.

$$+(i)^{2m} \quad \ldots\theta(\sigma_{2m-2}-\sigma_{2m-3})\theta(\sigma_{2m-1}-\sigma_{2m-2})\theta(\sigma_{2m}-\sigma_{2m-1})\Box\ . \tag{A.12}$$

To see this cancellation explicitly we use the rule $\theta(1-2)=1-\theta(2-1)$. Applying this to the first θ-function of the first line of (A.12) we see that the "1" cancels the second line. In the remaining term we apply the mentioned formula to the θ-function containing σ_2 and σ_3. The "1" will cancel the third line etc. In the end all terms will cancel as mentioned before.

Now, we will focus on the commutator part

$$\begin{aligned}
* \quad & \Big[\phi(x), \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)\Big]\,\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\
= \quad & i\Delta(x-x_1)j(x_1)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_{2m}) \\
& +i\Delta(x-x_2)\mathcal{H}_I(x_1)j(x_2)\mathcal{H}_I(x_3)\ldots\mathcal{H}_I(x_{2m}) \\
& +\ldots+i\Delta(x-x_m)\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})j(x_m)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\
\\
* \quad & \mathcal{H}_I(x_1)\Big[\phi(x), \mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_m)\Big]\,\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\
= \quad & i\Delta(x-x_2)\mathcal{H}_I(x_1)j(x_2)\mathcal{H}_I(x_3)\ldots\mathcal{H}_I(x_{2m}) \\
& +i\Delta(x-x_3)\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)j(x_3)\mathcal{H}_I(x_4)\ldots\mathcal{H}_I(x_{2m}) \\
& +\ldots+i\Delta(x-x_m)\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m-1})j(x_m)\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}) \\
& \ldots \\
* \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)\Big[\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m-1}), \phi(x)\Big]\,\mathcal{H}_I(x_{2m}) \\
= \quad & -i\Delta(x-x_{m+1})\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)j(x_{m+1})\mathcal{H}_I(x_{m+2})\ldots\mathcal{H}_I(x_{2m}) \\
& -i\Delta(x-x_{m+2})\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m+1})j(x_{m+2})\mathcal{H}_I(x_{m+3})\ldots\mathcal{H}_I(x_{2m}) \\
& +\ldots+i\Delta(x-x_{2m-1})\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{2m-2})j(x_{2m-1})\mathcal{H}_I(x_{2m}) \\
\\
* \quad & \mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)\Big[\mathcal{H}_I(x_{m+1})\ldots\mathcal{H}_I(x_{2m}), \phi(x)\Big] \\
= \quad & -i\Delta(x-x_{m+1})\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_m)j(x_{m+1})\mathcal{H}_I(x_{m+2})\ldots\mathcal{H}_I(x_{2m}) \\
& -i\Delta(x-x_{m+2})\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{m+1})j(x_{m+2})\mathcal{H}_I(x_{m+3})\ldots\mathcal{H}_I(x_{2m}) \\
& +\ldots+i\Delta(x-x_{2m})\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{2m-1})j(x_{2m})\ .
\end{aligned} \tag{A.13}$$

From this equation we can see that certain terms will combine. But to demonstrate how, we rewrite these combinations. First, we take a term that stands on its own (and include factors of i and the θ-functions)

$$(-i)^{2m}\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_1-\sigma_2)\ldots\theta(\sigma_{2m-1}-\sigma_{2m})$$

$$\times\, i\Delta(x-x_1)j(x_1)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_{2m})$$

$$
\begin{aligned}
&= (-i)^{2m-1}\int_{-\infty}^{\sigma} d^4y \int_{-\infty}^{\sigma_y} d^4x_2 \ldots d^4x_{2m}\,\theta(\sigma_2-\sigma_3)\ldots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
&\qquad\qquad \times \Delta(x-y)j(y)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_{2m}) \\
&= \int_{-\infty}^{\sigma} d^4y\; U_0^{-1}[\sigma_y]\;\Delta(x-y)j(y)\;U_{2m-1}[\sigma_y]\;. \qquad \text{(A.14)}
\end{aligned}
$$

Now we combine the following 2 terms

$$
\begin{aligned}
&(-i)^{2m}\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_1-\sigma_2)\theta(\sigma_2-\sigma_3)\ldots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
&\qquad \times i\Delta(x-x_2)\,\mathcal{H}_I(x_1)j(x_2)\mathcal{H}_I(x_3)\ldots\mathcal{H}_I(x_{2m}) \\
&+(-i)^{2m-2}\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_2-\sigma_3)\ldots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
&\qquad \times i\Delta(x-x_2)\,\mathcal{H}_I(x_1)j(x_2)\mathcal{H}_I(x_3)\ldots\mathcal{H}_I(x_{2m}) \\
&= (-i)^{2m-2}i\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_2-\sigma_1)\mathcal{H}_I(x_1)\,\Delta(x-x_2)j(x_2) \\
&\qquad \times \theta(\sigma_2-\sigma_3)\ldots\theta(\sigma_{2m-1}-\sigma_{2m})\,\mathcal{H}_I(x_3)\ldots\mathcal{H}_I(x_{2m}) \\
&= \int_{-\infty}^{\sigma} d^4y\; U_1^{-1}[\sigma_y]\;\Delta(x-y)j(y)\;U_{2m-2}[\sigma_y]\;. \qquad \text{(A.15)}
\end{aligned}
$$

So far, we have picked certain contributions to demonstrate how they combine and/or can be rewritten. Now, we make it general by picking k contributions

$$
\begin{aligned}
&(-i)^{2m}\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_1-\sigma_2)\theta(\sigma_2-\sigma_3)\ldots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
&\quad \times i\Delta(x-x_k)\,\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{k-1})j(x_k)\mathcal{H}_I(x_{k+1})\ldots\mathcal{H}_I(x_{2m}) \\
&+i(-i)^{2m-1}\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_2-\sigma_3)\ldots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
&\quad \times i\Delta(x-x_k)\,\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{k-1})j(x_k)\mathcal{H}_I(x_{k+1})\ldots\mathcal{H}_I(x_{2m}) \\
&+\ldots \\
&+(i)^{k-2}(-i)^{2m-k+2}\int_{-\infty}^{\sigma} d^4x_1\ldots d^4x_{2m}\,\theta(\sigma_{k-2}-\sigma_{k-3})\ldots\theta(\sigma_2-\sigma_1) \\
&\quad \times i\Delta(x-x_k)\,\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_{k-1})j(x_k)\mathcal{H}_I(x_{k+1})\ldots\mathcal{H}_I(x_{2m}) \\
&\quad \times\theta(\sigma_{k-1}-\sigma_k)\ldots\theta(\sigma_{2m-1}-\sigma_{2m})
\end{aligned}
$$

$$
\begin{aligned}
& +(i)^{k-1}(-i)^{2m-k+1}\int_{-\infty}^{\sigma} d^4x_1 \dots d^4x_{2m}\,\theta(\sigma_{k-1}-\sigma_{k-2})\dots\theta(\sigma_2-\sigma_1) \\
& \qquad \times i\Delta(x-x_k)\,\mathcal{H}_I(x_1)\dots\mathcal{H}_I(x_{k-1})j(x_k)\mathcal{H}_I(x_{k+1})\dots\mathcal{H}_I(x_{2m}) \\
& \qquad \times\theta(\sigma_k-\sigma_{k+1})\dots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
= \; & (i)^{k-1}(-i)^{2m-k}\int_{-\infty}^{\sigma} d^4x_1 \dots d^4x_{2m} \\
& \qquad \times\theta(\sigma_2-\sigma_1)\dots\theta(\sigma_k-\sigma_{k-1})\,\theta(\sigma_k-\sigma_{k+1})\dots\theta(\sigma_{2m-1}-\sigma_{2m}) \\
& \qquad \times\Delta(x-x_k)\,\mathcal{H}_I(x_1)\dots\mathcal{H}_I(x_{k-1})j(x_k)\mathcal{H}_I(x_{k+1})\dots\mathcal{H}_I(x_{2m}) \\
= \; & \int_{-\infty}^{\sigma} d^4y\; U_{k-1}^{-1}[\sigma_y]\;\Delta(x-y)j(y)\;U_{2m-k}[\sigma_y]\;. \qquad \text{(A.16)}
\end{aligned}
$$

Summing over all k $(1 \leq k \leq 2m)$, the total $2m^{th}$ order contribution is

$$
\begin{aligned}
& \int_{-\infty}^{\sigma} d^4y\, U_0^{-1}[\sigma_y]\Delta(x-y)j(y)U_{2m-1}[\sigma_y] \\
+ \; & \int_{-\infty}^{\sigma} d^4y\, U_1^{-1}[\sigma_y]\Delta(x-y)j(y)U_{2m-2}[\sigma_y] \\
+ \; & \dots \\
+ \; & \int_{-\infty}^{\sigma} d^4y\, U_{2m-2}^{-1}[\sigma_y]\Delta(x-y)j(y)U_1[\sigma_y] \\
+ \; & \int_{-\infty}^{\sigma} d^4y\, U_{2m-1}^{-1}[\sigma_y]\Delta(x-y)j(y)U_0[\sigma_y]\;. \qquad \text{(A.17)}
\end{aligned}
$$

Summing over all m we see that

$$
\begin{aligned}
\phi(x,\sigma) &= U^{-1}[\sigma]\phi(x)U[\sigma] \\
&= \phi(x) + \int_{-\infty}^{\sigma} d^4y\,\Delta(x-y)\,U^{-1}[\sigma_y]j(y)U[\sigma_y] \\
&= \phi(x) + \int_{-\infty}^{\sigma} d^4y\,\Delta(x-y)\boldsymbol{j}(y)\;, \qquad \text{(A.18)}
\end{aligned}
$$

where the first term on the rhs of the last line of (A.18) comes from combining the first terms in the expansions of U and its inverse.

A.3 Including derivatives

So far, we have not considered interaction Lagrangians including derivatives. To include this situation in the proof is not very difficult. The main step is to adjust (A.5). Because the Δ-propagator appearing in the formula to be proven (A.2) comes from this formula (A.5)

So, let us start with the following interaction Lagrangian

$$\begin{aligned}\mathcal{L}_I &= g\bar{\psi}\gamma_\mu\psi\cdot\partial^\mu\phi\ , \\ \mathcal{H}_I &= -g\bar{\psi}\gamma_\mu\psi\cdot\partial^\mu\phi + \frac{g^2}{2}\left[\bar{\psi}\not{n}\psi\right]^2\ . \end{aligned} \tag{A.19}$$

The commutator of ϕ with this interaction Hamiltonian (A.19) is

$$[\phi,\mathcal{H}_I(y)] = -ig\left[\bar{\psi}\gamma_\mu\psi\right]_y\partial_y^\mu\Delta(x-y)\ . \tag{A.20}$$

Introducing the vectors $j_a(x)$ and D_a very similar to (3.19)

$$\begin{aligned}j_a &\equiv \left(\frac{\partial\mathcal{H}_I}{\partial\phi},\frac{\partial\mathcal{H}_I}{\partial(\partial_\mu\phi),\ldots}\right)\ , \\ D_a &\equiv (1,\partial_\mu,\ldots)\ , \end{aligned} \tag{A.21}$$

then (A.20) can be rewritten as follows

$$[\phi,\mathcal{H}_I(y)] = iD_a(y)\Delta(x-y)\cdot j_a(y)\ . \tag{A.22}$$

Here we have used only one derivative in (A.19) to come to (A.22). But already from the definitions of j_a and D_a it can be seen that (A.22) is generally valid, so also for cases where the interaction Lagrangian includes multiple factors of derivatives. Using (A.22) for interaction Lagrangians including derivatives (A.18) becomes

$$\phi(x,\sigma) = \phi(x) + \int_{-\infty}^{\sigma} d^4y D_a(y)\Delta(x-y)\cdot \boldsymbol{j}_a(y)\ , \tag{A.23}$$

where $\boldsymbol{j}_a(y)$ is the same as in section 3.2.

A.4 Other Types of Fields

In addition to the previous section there are two remarks: First of all we were considering scalar fields only and second we have not considered the g^2 in the interaction Hamiltonian (A.19), since it is not effecting the scalar sector.

Adjusting (A.23) to other types of fields goes again with adjusting the commutator of the field with the interaction Hamiltonian, as we saw already in the previous section. To illustrate this we imagine interaction Lagrangian (A.19) again, where we now focus on the fermion field. The commutator with the interaction Hamiltonian is

$$\begin{aligned}[\psi(x),\mathcal{H}_I(y)] &= i\left(i\not{\partial}+M\right)\Delta(x-y)\left[-g\gamma_\mu\psi\cdot\partial^\mu\phi + g^2\not{n}\psi\bar{\psi}\not{n}\psi\right]_y \\ &= iD_a(y)\left(i\not{\partial}+M\right)\Delta(x-y)\cdot j_a(y)\ , \end{aligned} \tag{A.24}$$

Looking at (A.18) we see that we need to transform $j_a(y)$

$$\begin{aligned} U^{-1}[\sigma] j_a(y) U[\sigma] &= U^{-1}[\sigma]\left[-g\gamma_\mu\psi\cdot\partial^\mu\phi + g^2\not\eta\psi\bar{\psi}\not\eta\psi\right]_y U[\sigma] \\ &= g\gamma_\mu\boldsymbol{\psi}(y)\left[-U^{-1}[\sigma]\partial^\mu\phi U[\sigma] + g^2\not\eta\boldsymbol{\psi}\bar{\boldsymbol{\psi}}\not\eta\boldsymbol{\psi}\right]_y \\ &= -g\gamma_\mu\boldsymbol{\psi}(y)\partial^\mu\boldsymbol{\phi}(y) = \boldsymbol{j}_a(y) \ , \end{aligned} \tag{A.25}$$

and therefore

$$\psi(x,\sigma) = \psi(x) + \int_{-\infty}^{\sigma} d^4y D_a(y)\left(i\not\partial + M\right)\Delta(x-y)\cdot\boldsymbol{j}_a(y) \ . \tag{A.26}$$

It is not difficult to generalize this. Imagine that

$$\left[\Phi_\alpha(x), \Phi_\beta(y)\right]_\pm = iR_{\alpha\beta}\Delta(x-y) \ , \tag{A.27}$$

then

$$\Phi_\alpha(x,\sigma) = \Phi_\alpha(x) + \int_{-\infty}^{\sigma} d^4y D_a(y) R_{\alpha\beta}\Delta(x-y)\cdot\boldsymbol{j}_{a,\beta}(y) \ , \tag{A.28}$$

which is the equation to be proven.

Appendix B

BMP Theory

According to Haag's theorem [24] in general there does not exist an unitary transformation which relates the fields in the I.R. and the fields in the H.R. On the other hand there is no objection against the existence of an unitary $U[\sigma]$ relating the TU-auxiliary fields and the fields in the I.R. (3.23)

$$\Phi_\alpha(x,\sigma) = U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\ . \tag{B.1}$$

In section 3.2 we have made a consistency check to show that (3.23) is indeed valid. Here, we follow the framework of Bogoliubov and collaborators [28, 29, 30], to which we refer to as the BMP theory, to proof (3.23) in a straightforward way (see section B.3).

The BMP theory was originally constructed to bypass the use of an unitary operator U as a mediator between the fields in the H.R. and in the I.R.

B.1 Set-up

In the description of the BMP theory we will only consider scalar fields. By the assumption of asymptotic completeness the S-matrix is taken to be a functional of the asymptotic fields $\phi_{as,\rho}(x)$, where $as = in, out$. In the following we use in-fields, i.e. $\phi_\rho(x) = \phi_{in,\rho}(x)$

$$\begin{aligned} S &= 1 + \sum_{n=1}^{\infty} \int d^4x_1 \dots d^4x_n\ S_n(x_1\alpha_1, \dots, x_n\alpha_n) \cdot \\ &\qquad\qquad \times :\phi_{\alpha_1}(x_1)\dots\phi_{\alpha_n}(x_n):\ . \end{aligned} \tag{B.2}$$

Here, concepts like unitarity and the stability of the vacuum, i.e. $\langle 0|S|0\rangle = 1$, and the 1-particle states, i.e. $\langle 0|S|1\rangle = 0$ are assumed. Unitarity $S^\dagger S = 1$ gives upon functional differentiation

$$\frac{\delta S^\dagger}{\delta\phi_\rho(x)} S = -S^\dagger \frac{\delta S}{\delta\phi_\rho(x)} \ , \tag{B.3}$$

and a similar relation starting from $S\,S^\dagger = 1$. The *Heisenberg current*, i.e. the current in the H.R., is defined as [1]

$$\mathbf{J}_\rho(x) = -iS^\dagger \frac{\delta S}{\delta\phi_\rho(x)} \ . \tag{B.4}$$

We note that for a hermitean field $\phi_\rho(x)$ the current is also hermitean, due to the relation (B.3). *Microcausality* takes the form, see [29], chapter 17 [2],

$$\frac{\delta \mathbf{J}_\rho(x)}{\delta\phi_\lambda(y)} = 0 \quad , \quad \text{for} \quad x \leq y \ . \tag{B.5}$$

Now, if the S-matrix is of the form as we know it

$$S = T\left[\exp\left(-i \int d^4x \ \mathcal{H}_I(x)\right)\right] \ , \tag{B.6}$$

where $\mathcal{H}_I(x)$ is a (local) function of the asymptotic field $\phi_\alpha(x)$, which is defined even when $\phi_\alpha(x)$ does not satisfy the free field equation, then the microcausility condition (B.5) is satisfied

$$\frac{\delta}{\delta\phi_\beta(y)}\left\{ S^\dagger \frac{\delta S}{\delta\phi_\alpha(x)}\right\} = 0 \ \ \text{for} \ \ x \leq y \ . \tag{B.7}$$

This illustrates that the notion of microcausality is reflected in the expression of the S-matrix as the Time-Ordered exponential. See [29] for the details on this point of view. Furthermore, it follows from (B.4) and (B.6) that

$$\mathbf{J}_\rho(x) = -\frac{\partial \mathcal{H}_I(x)}{\partial\phi_\rho(x)} \ . \tag{B.8}$$

It can be shown that with the current (B.4) the asymptotic fields $\phi_{in/out,\rho}(x)$ satisfy (3.26).

[1] Note that in [30] the out-field is used. Then

$$\mathbf{J}_\rho(x) = i\frac{\delta S}{\delta\phi_\rho(x)} S^\dagger \ .$$

Also, we take a minus sign in the definition of the current.

[2] Here $x \leq y$ means either $(x-y)^2 \geq 0$ and $x^0 < y^0$ or $(x-y)^2 < 0$. So, the point x is in the past of or is spacelike separated from the point y.

B.2 Correspondence with LSZ Theory

Lehmann, Symanzik, and Zimmermann (LSZ) [32] formulated an asymptotic condition utilizing the notion of weak convergence in the Hilbert space of state vectors. See e.g. [12] for an detailed exposition of the LSZ-formalism. Here, the field in the H.R. $\boldsymbol{\phi}(x)$ and the asymptotic fields ϕ_{as} satisfy the equations

$$(\Box + m^2)\boldsymbol{\phi}(x) = \mathbf{J}(x) \quad , \quad (\Box + m^2)\phi_{as}(x) = 0 \ . \tag{B.9}$$

and their relation is given by the YF equations (3.17).

The correspondence is obtained by the identification

$$\mathbf{J}_\rho(x) = -iS^\dagger \ \frac{\delta S}{\delta\phi_\rho(x)} \equiv \left(\Box + m^2\right) \ \boldsymbol{\phi}_\rho(x) \ . \tag{B.10}$$

Also, we want to show the locality properties assumed in the LSZ theory.

Functionally differentiating the current (B.4) and using the unitarity condition (B.3) gives the equations

$$\frac{\delta \mathbf{J}_\rho(x)}{\delta\phi_\sigma(y)} \ = \ -iS^\dagger \ \frac{\delta^2 S}{\delta\phi_\sigma(y)\delta\phi_\rho(x)} - i\mathbf{J}_\sigma(y) \ \mathbf{J}_\rho(x) \ , \tag{B.11a}$$

$$\frac{\delta \mathbf{J}_\sigma(y)}{\delta\phi_\rho(x)} \ = \ -iS^\dagger \ \frac{\delta^2 S}{\delta\phi_\rho(x)\delta\phi_\sigma(y)} - i\mathbf{J}_\rho(x) \ \mathbf{J}_\sigma(y) \ , \tag{B.11b}$$

which yield upon subtraction

$$\frac{\delta \mathbf{J}_\rho(x)}{\delta\phi_\sigma(y)} - \frac{\delta \mathbf{J}_\sigma(y)}{\delta\phi_\rho(x)} \ = \ i\left[\mathbf{J}_\rho(x) \ , \mathbf{J}_\sigma(y)\right] \ . \tag{B.12}$$

Note that for space-like separations, i.e. $(x-y)^2 < 0$, the current commutator vanishes, by means of the microcausality condition (B.5). Moreover, the application of this microcausality condition to equation (B.11b) for $x \neq y$ gives the following important relation

$$H_2(x\rho, y\sigma) \ \equiv \ S^\dagger \ \frac{\delta^2 S}{\delta\phi_\rho(x)\delta\phi_\sigma(y)} = -T\left[\mathbf{J}_\rho(x)\mathbf{J}_\sigma(y)\right] \ . \tag{B.13}$$

It follows that for all x and y

$$H_2(x\rho, y\sigma) = -T\left[\mathbf{J}_\rho(x)\mathbf{J}_\sigma(y)\right] + i\Lambda_{\rho\sigma}(x,y) \ , \tag{B.14}$$

where $\Lambda_{\rho\sigma}$ is a *quasi-local operator*

$$\Lambda_{\rho\sigma}(x,y) = \Lambda_{\sigma\rho}(y,x) = 0 \ \text{ if } \ \mathrm{x} \neq \mathrm{y} \ , \tag{B.15}$$

which is hermitean if $\phi_\rho(x)$ is hermitean.

Substitution of (B.13) into equation (B.11a) gives

$$\frac{\delta \mathbf{J}_\rho(x)}{\delta \phi_\sigma(y)} = i\theta(x^0 - y^0) \Big[\mathbf{J}_\rho(x), \mathbf{J}_\sigma(y)\Big] + \Lambda_{\rho\sigma}(x,y) \ . \qquad \text{(B.16)}$$

Above, the local commutivity of the currents has been shown to follow from microcausality. Using the YF equations (3.25) one can show that for space-like separations the fields in the H.R. commute with the currents and among themselves

$$\Big[\boldsymbol{\phi}_\rho(x), \mathbf{J}_\sigma(y)\Big] = 0 \ \text{ for } \ (\mathrm{x}-\mathrm{y})^2 < 0 \ , \qquad \text{(B.17a)}$$

$$\Big[\boldsymbol{\phi}_\rho(x), \boldsymbol{\phi}_\sigma(y)\Big] = 0 \ \text{ for } \ (\mathrm{x}-\mathrm{y})^2 < 0 \ . \qquad \text{(B.17b)}$$

This can be done as follows: Since the S-operator is an expansion in asymptotic fields, so is $\mathbf{J}(x)$ by means of its definition in terms of this S-operator. Now, from the commutation relations of the asymptotic fields one has

$$\Big[\phi_\rho(x), \mathbf{J}_\sigma(y)\Big] = i\int d^4x' \ \Delta(x-x')\frac{\delta \mathbf{J}_\sigma(y)}{\delta\phi_\rho(x')} \ . \qquad \text{(B.18)}$$

Using (B.16) and (B.18) one gets with the YF equation (3.25) that

$$\Big[\boldsymbol{\phi}_\rho(x), \mathbf{J}_\sigma(y)\Big] =$$

$$= -\int d^4x' \Big(\theta(x'^0 - y^0)\theta(x^0 - x'^0) - \theta(y^0 - x'^0)\theta(x'^0 - x^0)\Big)\Delta(x-x')$$

$$\times \Big[\mathbf{J}_\rho(x'), \mathbf{J}_\sigma(y)\Big] + i\int d^4x' \ \Delta(x-x')\Lambda_{\rho\sigma}(x',y) \ . \qquad \text{(B.19)}$$

Since $\Lambda_{\rho\sigma}(x',y)$ in the last term on the rhs of (B.19) is only non-zero for $x' = y$, we see that this last term vanishes for space-like separations, because of the properties of $\Delta(x-y)$. As far as the remaining terms are concerned, they vanish in the special case $x^0 = y^0$. This can be seen as follows: By means of the θ-functions the only possible point of interest is $x'^0 = x^0 = y^0$. Now, $\Delta(x-x')$ vanishes in this point, as well as $[\mathbf{J}_\rho(x'), \mathbf{J}_\sigma(y)]$ (see (B.12) and the text below). Because of Lorentz invariance (B.19) vanishes for all $(x-y)^2 < 0$, as was claimed (B.17a). Similarly one can prove the second commutator (B.17b) to vanish for space-like separations.

With (B.12) (and the text below it), (B.17a) and (B.17b) we have shown the locality properties as assumed in LSZ formalism.

B.3 Application to Takahashi-Umezawa scheme

In appendix A we proved (3.21) from (3.23) provided that the unitary operator is the time evolution operator connected to the S-matrix (see section 3.2). Here, we do exactly the opposite. Introducing the auxiliary field, similar to (3.21), by

$$\phi(x,\sigma) \equiv \phi(x) - \int_{-\infty}^{\sigma} d^4x' \; \Delta(x-x') \; \mathbf{J}(x') \; , \tag{B.20}$$

we prove that

$$\Big[\phi(x,\sigma), \phi(y,\sigma)\Big] = \Big[\phi(x), \phi(y)\Big] = i\Delta(x-y) \; . \tag{B.21}$$

Using (B.20) gives

$$\begin{aligned} & \Big[\phi(x,\sigma), \phi(y,\sigma)\Big] - \Big[\phi(x), \phi(y)\Big] \\ = \;& -\int_{-\infty}^{\sigma} d^4y' \; \Delta(y-y') \Big[\phi(x), \mathbf{J}(y')\Big] + \int_{-\infty}^{\sigma} d^4x' \; \Delta(x-x') \Big[\phi(y), \mathbf{J}(x')\Big] \\ & + \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y' \; \Delta(x-x')\Delta(y-y') \Big[\mathbf{J}(x'), \mathbf{J}(y')\Big] \; . \end{aligned} \tag{B.22}$$

Now, we use (B.12) and (B.18) to rewrite (B.22)

$$\begin{aligned} & \Big[\phi(x,\sigma), \phi(y,\sigma)\Big] - \Big[\phi(x), \phi(y)\Big] \\ = \;& -i \int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\infty} d^4x' \Delta(x-x')\Delta(y-y') \frac{\delta \mathbf{J}(y')}{\delta\phi(x')} \\ & +i \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\infty} d^4y' \; \Delta(x-x')\Delta(y-y') \frac{\delta \mathbf{J}(x')}{\delta\phi(y')} \\ & -i \int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y' \; \Delta(x-x')\Delta(y-y') \left(\frac{\delta \mathbf{J}(x')}{\delta\phi(y')} - \frac{\delta \mathbf{J}(y')}{\delta\phi(x')} \right) \\ = \;& 0 \; . \end{aligned} \tag{B.23}$$

Cancellation takes place in (B.23) when the second integral of the first two term on the rhs in (B.23) is split up: $\int_{-\infty}^{\infty} = \int_{-\infty}^{\sigma} + \int_{\sigma}^{\infty}$. The remaining terms are zero because of the microcausility condition (B.5).

This justifies the existence of a unitary operator $U[\sigma]$ such that

$$\phi(x,\sigma) = U^{-1}[\sigma] \; \phi(x) \; U[\sigma] \; . \tag{B.24}$$

In section 3.2 we have already shown that this U-operator is connected to the S-matrix and satisfies the Tomonaga-Schwinger equation (3.30). Following the steps in section 3.2, the interaction Hamiltonian can be obtained (3.33). Therefore, the interaction Hamiltonian can also be deduced using BMP theory.

Appendix C

Kadyshevsky Amplitudes and Invariants

C.1 Meson Exchange

Scalar Meson Exchange, diagram (a)

$$M^{(a)}_{\kappa',\kappa} = g_{PPS}g_S\,[\bar{u}(p')u(p)]\,D^{(1)}(\Delta_t, n, \bar{\kappa})\ , \tag{C.1}$$

where $D^{(1)}(\Delta_t, n, \bar{\kappa}) = \frac{1}{2A_t} \cdot \frac{1}{\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon}$

$$A_S = g_{PPS}g_S\,D^{(1)}(\Delta_t, n, \bar{\kappa})\ . \tag{C.2}$$

$$X^A_S = g_{PPS}g_S\ . \tag{C.3}$$

Scalar Meson Exchange, diagram (b)

$$M^{(b)}_{\kappa',\kappa} = g_{PPS}g_S\,[\bar{u}(p's')u(p)]\,D^{(1)}(-\Delta_t, n, \bar{\kappa})\ . \tag{C.4}$$

$$A_S = g_{PPS}g_S\,D^{(1)}(-\Delta_t, n, \bar{\kappa})\ . \tag{C.5}$$

$$X^A_S = g_{PPS}g_S\ . \tag{C.6}$$

Pomeron Exchange

$$M_{\kappa'\kappa} = \frac{g_{PPP}g_P}{M}\left[\bar{u}(p's')u(p)\right] . \tag{C.7}$$

$$A_P = \frac{g_{PPP}g_P}{M} . \tag{C.8}$$

The partial wave projection is obtained by applying (6.33) straightforward

Vector Meson Exchange, diagram (a)

$$\begin{aligned} M^{(a)}_{\kappa',\kappa} = & -g_{VPP}\ \bar{u}(p's')\Bigg[2g_V \not{Q} - \frac{g_V}{M_V^2}\left((M_f - M_i) + \kappa'\not{n}\right) \\ & \times\left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) + 2\bar{\kappa}n\cdot Q\right) \\ & + \frac{f_V}{2M_V}\left(2(M_f + M_i)\not{Q} + \frac{1}{2}\left(u_{pq'} + u_{p'q}\right) - \frac{1}{2}\left(s_{p'q'} + s_{pq}\right)\right) \\ & - \frac{f_V}{2M_V^3}\Bigg(\frac{1}{2}\left(M_f^2 + M_i^2\right) + \frac{1}{2}\left(m_f^2 + m_i^2\right) \\ & \quad - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\right) \\ & \quad + (M_f + M_i)\,\kappa'\not{n} + \frac{1}{4}\left(\kappa' - \kappa\right)^2 - (p' + p)\cdot n\bar{\kappa}\Bigg) \\ & \times\left(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) + 2\bar{\kappa}n\cdot Q\right)\Bigg] u(ps) \\ & \times D^{(1)}(\Delta_t, n, \bar{\kappa}) . \end{aligned} \tag{C.9}$$

$$\begin{aligned} A_V = & -g_{VPP}\Bigg[-\frac{g_V}{M_V^2}\left(M_f - M_i\right)\Bigg(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) \\ & + 2\bar{\kappa}n\cdot Q\Bigg) + \frac{f_V}{4M_V}\left(u_{pq'} + u_{p'q} - s_{p'q'} - s_{pq}\right) - \frac{f_V}{2M_V^3}\Bigg(\frac{1}{2}\left(M_f^2 + M_i^2\right) \\ & + \frac{1}{2}\left(m_f^2 + m_i^2\right) - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\right) + \frac{1}{4}\left(\kappa' - \kappa\right)^2 \\ & - (p' + p)\cdot n\bar{\kappa}\Bigg)\Bigg(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) \\ & + 2\bar{\kappa}n\cdot Q\Bigg)\Bigg]\ D^{(1)}(\Delta_t, n, \bar{\kappa}) , \end{aligned}$$

$$
\begin{aligned}
B_V &= -2g_{VPP}\left[g_V + \frac{f_V}{2M_V}\left(M_f + M_i\right)\right] D^{(1)}(\Delta_t, n, \bar{\kappa}) \ , \\
A'_V &= \frac{g_{VPP}\kappa'}{M_V^2}\left[g_V + \frac{f_V}{2M_V}\left(M_f + M_i\right)\right]\left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q})\right. \\
&\qquad \left. - \left(m_f^2 - m_i^2\right) + 2\bar{\kappa} n \cdot Q\right) D^{(1)}(\Delta_t, n, \bar{\kappa}) \ . \qquad \text{(C.10)}
\end{aligned}
$$

$$
\begin{aligned}
X_V^A &= -g_{VPP}\left[-\frac{g_V}{M_V^2}\left(M_f - M_i\right)\left(\frac{1}{4}\left(E' + \mathcal{E}'\right)^2 - \frac{1}{4}\left(E + \mathcal{E}\right)^2 - \frac{1}{4}\left(M_f^2 - M_i^2\right)\right.\right. \\
&\quad \left. -\frac{3}{4}\left(m_f^2 - m_i^2\right) + \frac{1}{2}\left(E'\mathcal{E} - E\mathcal{E}'\right) + \bar{\kappa}(\mathcal{E}' + \mathcal{E})\right) + \frac{f_V}{4M_V}\left(M_f^2 + M_i^2\right. \\
&\quad \left. + m_f^2 + m_i^2 - 2\left(E'\mathcal{E} + E\mathcal{E}'\right) - \left(E' + \mathcal{E}'\right)^2 - \left(E + \mathcal{E}\right)^2\right) - \frac{f_V}{4M_V^3}\left(M_f^2\right. \\
&\quad + M_i^2 + m_f^2 + m_i^2 + \frac{1}{2}\left(\kappa' - \kappa\right)^2 - \frac{1}{2}\left(M_f^2 + 3M_i^2 + 3m_f^2 + m_i^2\right. \\
&\quad \left.\left. - 2E'E - 2\mathcal{E}'\mathcal{E} - 4\mathcal{E}'E + \left(E + \mathcal{E}\right)^2\right) - 2\left(E' + E\right)\bar{\kappa}\right) \\
&\quad \times \left(\frac{1}{4}\left(E' + \mathcal{E}'\right)^2 - \frac{1}{4}\left(E + \mathcal{E}\right)^2 - \frac{1}{4}\left(M_f^2 - M_i^2\right) - \frac{3}{4}\left(m_f^2 - m_i^2\right)\right. \\
&\qquad \left.\left. + \frac{1}{2}\left(E'\mathcal{E} - E\mathcal{E}'\right) + \bar{\kappa}(\mathcal{E}' + \mathcal{E})\right)\right] \ , \\
Y_V^A &= -\frac{g_{VPP}\, f_V\, p'p}{M_V}\left[1 + \frac{1}{4M_V^2}\left(\left(E' + \mathcal{E}'\right)^2 - \left(E + \mathcal{E}\right)^2 - \left(M_f^2 - M_i^2\right)\right.\right. \\
&\qquad \left.\left. - 3\left(m_f^2 - m_i^2\right) + 2\left(E'\mathcal{E} - E\mathcal{E}'\right) + 4\bar{\kappa}(\mathcal{E}' + \mathcal{E})\right)\right] \ , \\
X_V^B &= -2g_{VPP}\left[g_V + \frac{f_V}{2M_V}\left(M_f + M_i\right)\right] \ , \\
X_V^{A'} &= \frac{g_{VPP}\kappa'}{4M_V^2}\left[g_V + \frac{f_V}{2M_V}\left(M_f + M_i\right)\right]\left(\left(E' + \mathcal{E}'\right)^2 - \left(E + \mathcal{E}\right)^2\right. \\
&\quad \left. - \left(M_f^2 - M_i^2\right) - 3\left(m_f^2 - m_i^2\right) + 2\left(E'\mathcal{E} - E\mathcal{E}'\right) + 4\bar{\kappa}(\mathcal{E}' + \mathcal{E})\right) \ . \\
&\qquad \text{(C.11)}
\end{aligned}
$$

Vector Meson Exchange, diagram (b)

$$
\begin{aligned}
M^{(b)}_{\kappa',\kappa} \;=\;& -g_{VPP}\; \bar{u}(p's') \Bigg[2g_V \not{Q} - \frac{g_V}{M_V^2} \left((M_f - M_i) - \kappa \not{n} \right) \\
& \times \left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) - 2\bar{\kappa} n \cdot Q \right) \\
& + \frac{f_V}{2M_V} \left(2(M_f + M_i)\not{Q} + \frac{1}{2}\left(u_{pq'} + u_{p'q}\right) - \frac{1}{2}\left(s_{p'q'} + s_{pq}\right) \right) \\
& - \frac{f_V}{2M_V^3} \left(\frac{1}{2}\left(M_f^2 + M_i^2\right) + \frac{1}{2}\left(m_f^2 + m_i^2\right) \right. \\
& \qquad - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq} \right) \\
& \qquad \left. - \left(M_f + M_i\right) \kappa \not{n} + \frac{1}{4}\left(\kappa' - \kappa\right)^2 + (p' + p) \cdot n \bar{\kappa} \right) \\
& \times \left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) - 2\bar{\kappa} n \cdot Q \right) \Bigg] u(ps) \\
& \times D^{(1)}(-\Delta_t, n, \bar{\kappa}) \; . \qquad \text{(C.12)}
\end{aligned}
$$

$$
\begin{aligned}
A_V \;=\;& -g_{VPP} \Bigg[-\frac{g_V}{M_V^2} \left(M_f - M_i\right) \left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) \right. \\
& \left. - 2\bar{\kappa} n \cdot Q \right) + \frac{f_V}{4M_V}\left(u_{pq'} + u_{p'q} - s_{p'q'} - s_{pq}\right) - \frac{f_V}{2M_V^3} \left(\frac{1}{2}\left(M_f^2 + M_i^2\right) \right. \\
& + \frac{1}{2}\left(m_f^2 + m_i^2\right) - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq} \right) + \frac{1}{4}\left(\kappa' - \kappa\right)^2 \\
& \left. + (p' + p) \cdot n\bar{\kappa} \right) \left(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) \right. \\
& \left. - 2\bar{\kappa} n \cdot Q \right) \Bigg] \; D^{(1)}(-\Delta_t, n, \bar{\kappa}) \; , \\
B_V \;=\;& -2g_{VPP} \left[g_V + \frac{f_V}{2M_V}\left(M_f + M_i\right) \right] D^{(1)}(-\Delta_t, n, \bar{\kappa}) \; , \\
A'_V \;=\;& -\frac{g_{VPP}\kappa}{M_V^2} \left[g_V + \frac{f_V}{2M_V}\left(M_f + M_i\right) \right] \left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) \right. \\
& \qquad \left. - \left(m_f^2 - m_i^2\right) - 2\bar{\kappa} n \cdot Q \right) D^{(1)}(-\Delta_t, n, \bar{\kappa}) \; . \qquad \text{(C.13)}
\end{aligned}
$$

$$
\begin{aligned}
X_V^A &= -g_{VPP}\Bigg[-\frac{g_V}{M_V^2}\left(M_f-M_i\right)\Bigg(\frac{1}{4}\left(E'+\mathcal{E}'\right)^2-\frac{1}{4}\left(E+\mathcal{E}\right)^2-\frac{1}{4}\left(M_f^2-M_i^2\right) \\
&\quad -\frac{3}{4}\left(m_f^2-m_i^2\right)+\frac{1}{2}\left(E'\mathcal{E}-E\mathcal{E}'\right)-\bar{\kappa}(\mathcal{E}'+\mathcal{E})\Bigg)+\frac{f_V}{4M_V}\Big(M_f^2+M_i^2 \\
&\quad +m_f^2+m_i^2-2\left(E'\mathcal{E}+E\mathcal{E}'\right)-\left(E'+\mathcal{E}'\right)^2-\left(E+\mathcal{E}\right)^2\Big)-\frac{f_V}{4M_V^3}\Bigg(M_f^2 \\
&\quad +M_i^2+m_f^2+m_i^2+\frac{1}{2}\left(\kappa'-\kappa\right)^2-\frac{1}{2}\Big(M_f^2+3M_i^2+3m_f^2+m_i^2 \\
&\quad -2E'E-2\mathcal{E}'\mathcal{E}-4\mathcal{E}'E+\left(E+\mathcal{E}\right)^2\Big)+2\left(E'+E\right)\bar{\kappa}\Bigg) \\
&\quad \times\Bigg(\frac{1}{4}\left(E'+\mathcal{E}'\right)^2-\frac{1}{4}\left(E+\mathcal{E}\right)^2-\frac{1}{4}\left(M_f^2-M_i^2\right)-\frac{3}{4}\left(m_f^2-m_i^2\right) \\
&\quad +\frac{1}{2}\left(E'\mathcal{E}-E\mathcal{E}'\right)-\bar{\kappa}(\mathcal{E}'+\mathcal{E})\Bigg)\Bigg]\ , \\
Y_V^A &= -\frac{g_{VPP}\,f_V\,p'p}{M_V}\Bigg[1+\frac{1}{4M_V^2}\Bigg(\left(E'+\mathcal{E}'\right)^2-\left(E+\mathcal{E}\right)^2-\left(M_f^2-M_i^2\right) \\
&\qquad -3\left(m_f^2-m_i^2\right)+2\left(E'\mathcal{E}-E\mathcal{E}'\right)-4\bar{\kappa}(\mathcal{E}'+\mathcal{E})\Bigg)\Bigg]\ , \\
X_V^B &= -2g_{VPP}\left[g_V+\frac{f_V}{2M_V}\left(M_f+M_i\right)\right]\ , \\
X_V^{A'} &= -\frac{g_{VPP}\kappa}{4M_V^2}\left[g_V+\frac{f_V}{2M_V}\left(M_f+M_i\right)\right]\Bigg(\left(E'+\mathcal{E}'\right)^2-\left(E+\mathcal{E}\right)^2 \\
&\quad -\left(M_f^2-M_i^2\right)-3\left(m_f^2-m_i^2\right)+2\left(E'\mathcal{E}-E\mathcal{E}'\right)-4\bar{\kappa}(\mathcal{E}'+\mathcal{E})\Bigg)\ .
\end{aligned}
\tag{C.14}
$$

C.2 Baryon Exchange/Resonance

Baryon Exchange, Scalar coupling

$$
M^S_{\kappa',\kappa} = \frac{g_S^2}{2}\,\bar{u}(p's')\left[\frac{1}{2}\left(M_f+M_i\right)+M_B-\not{Q}+\not{n}\bar{\kappa}\right]u(ps)D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ , \tag{C.15}
$$

where the denominator function is $D^{(2)}\left(\Delta_i,n,\bar{\kappa}\right)=\left[\left(\bar{\kappa}+\Delta_i\cdot n\right)^2-A_i^2\right]^{-1}$,

$i = u, s$.

$$\begin{aligned}
A_S &= \frac{g_S^2}{2}\left[\frac{1}{2}\left(M_f + M_i\right) + M_B\right] D^{(2)}\left(\Delta_u, n, \bar{\kappa}\right) \ , \\
B_S &= -\frac{g_S^2}{2} D^{(2)}\left(\Delta_u, n, \bar{\kappa}\right) \ , \\
A'_S &= \frac{g_S^2}{2}\,\bar{\kappa}\, D^{(2)}\left(\Delta_u, n, \bar{\kappa}\right) \ . \qquad \text{(C.16)}
\end{aligned}$$

$$\begin{aligned}
X_S^A &= -\frac{g_S^2}{2}\left[\frac{1}{2}\left(M_f + M_i\right) + M_B\right] \ , \\
X_S^B &= \frac{g_S^2}{2} \ , \\
X_S^{A'} &= -\frac{g_S^2}{2}\,\bar{\kappa} \ . \qquad \text{(C.17)}
\end{aligned}$$

Baryon Exchange, Pseudo Scalar coupling

The expressions for baryon exchange with pseudo scalar coupling are the same as (C.15)-(C.17) with the substitution $M_B \to -M_B$.

Baryon Resonance, Scalar coupling

$$M^S_{\kappa',\kappa} = \frac{g_S^2}{2}\,\bar{u}(p's')\left[\frac{1}{2}\left(M_f + M_i\right) + M_B + \not{Q} + \not{n}\bar{\kappa}\right] u(ps) D^{(2)}\left(\Delta_s, n, \bar{\kappa}\right) \ . \qquad \text{(C.18)}$$

$$\begin{aligned}
A_S &= \frac{g_S^2}{2}\left[\frac{1}{2}\left(M_f + M_i\right) + M_B\right] D^{(2)}\left(\Delta_s, n, \bar{\kappa}\right) \ , \\
B_S &= \frac{g_S^2}{2} D^{(2)}\left(\Delta_s, n, \bar{\kappa}\right) \ , \\
A'_S &= \frac{g_S^2}{2}\,\bar{\kappa}\, D^{(2)}\left(\Delta_s, n, \bar{\kappa}\right) \ . \qquad \text{(C.19)}
\end{aligned}$$

$$\begin{aligned}
X_S^A &= -\frac{g_S^2}{2}\left[\frac{1}{2}\left(M_f + M_i\right) + M_B\right] \ , \\
X_S^B &= -\frac{g_S^2}{2} \ , \\
X_S^{A'} &= -\frac{g_S^2}{2}\,\bar{\kappa} \ . \qquad \text{(C.20)}
\end{aligned}$$

Baryon Resonance, Pseudo Scalar coupling

The expressions for baryon resonance with pseudo scalar coupling are the same as (C.18)-(C.20) with the substitution $M_B \to -M_B$.

Baryon Exchange Vector coupling

$$
\begin{aligned}
M^V_{\kappa',\kappa} \;=\; & \frac{f_V^2}{2m_\pi^2}\bar{u}(p's')\Bigg[-\left(\frac{1}{2}(M_f+M_i)-M_B\right) \\
& \times\Bigg(-\frac{1}{2}\left(M_f^2+M_i^2\right)+\frac{1}{2}\left(u_{p'q}+u_{pq'}\right)+\left(M_f+M_i\right)\not{Q} \\
& \quad -\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n+\frac{1}{2}(\kappa'-\kappa)[\not{n},\not{Q}]-\frac{1}{2}(\kappa'-\kappa)^2\Bigg) \\
& -\frac{1}{2}\left(u_{pq'}-M_i^2\right)\left(\frac{1}{2}(M_f-M_i)+\not{Q}+\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
& -\frac{1}{2}\left(u_{p'q}-M_f^2\right)\left(-\frac{1}{2}(M_f-M_i)+\not{Q}-\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\
& +\bar{\kappa}\Bigg(-\frac{1}{2}(M_f-M_i)(p'-p)\cdot n-(p'-p)\cdot n\,\not{Q}+2Q\cdot n\,\not{Q} \\
& \quad -\frac{1}{2}(M_f-M_i)(\kappa'-\kappa)+\frac{1}{2}(M_f-M_i)[\not{n},\not{Q}] \\
& \quad +\frac{1}{2}\left(M_f^2+M_i^2\right)\not{n}-\frac{1}{2}\left(u_{p'q}+u_{pq'}\right)\not{n}\Bigg)\Bigg]u_i(p)D^{(2)}(\Delta_u,n,\bar{\kappa}) \ .
\end{aligned}
\tag{C.21}
$$

$$
\begin{aligned}
A_V \;=\; & \frac{f_V^2}{2m_\pi^2}\Bigg[-\left(\frac{1}{2}(M_f+M_i)-M_B\right)\Bigg(-\frac{1}{2}\left(M_f^2+M_i^2\right)+\frac{1}{2}(u_{p'q}+u_{pq'}) \\
& -\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\Bigg)-\frac{\bar{\kappa}}{2}(M_f-M_i) \\
& \times(p'-p)\cdot n+\frac{1}{4}\left(u_{p'q}-u_{pq'}-M_f^2+M_i^2\right)(M_f-M_i) \\
& -\frac{\bar{\kappa}}{2}(M_f-M_i)(\kappa'-\kappa)\Bigg]D^{(2)}(\Delta_u,n,\bar{\kappa}) \ , \\
B_V \;=\; & \frac{f_V^2}{2m_\pi^2}\Bigg[-\left(\frac{1}{2}(M_f+M_i)-M_B\right)(M_f+M_i)+\frac{1}{2}\Bigg(M_f^2+M_i^2 \\
& -u_{p'q}-u_{pq'}\Bigg)-\bar{\kappa}(p'-p)\cdot n+2\bar{\kappa}n\cdot Q\Bigg]D^{(2)}(\Delta_u,n,\bar{\kappa}) \ ,
\end{aligned}
$$

$$\begin{aligned}
A'_V &= \frac{f_V^2}{4m_\pi^2}\left[\left(M_i^2-u_{pq'}\right)\kappa'+\left(M_f^2-u_{p'q}\right)\kappa\right]D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ ,\\
B'_V &= -\frac{f_V^2}{4m_\pi^2}\left[\kappa' M_i-\kappa M_f-(\kappa'-\kappa)M_B\right]D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ . \qquad \text{(C.22)}
\end{aligned}$$

$$\begin{aligned}
X_V^A &= -\frac{f_V^2}{2m_\pi^2}\left[-\left(\frac{1}{2}\left(M_f+M_i\right)-M_B\right)\left(\frac{1}{2}\left(m_f^2+m_i^2\right)-E'\mathcal{E}-E\mathcal{E}'\right.\right.\\
&\qquad \left.-\frac{1}{2}(\kappa'-\kappa)\left(E'-E\right)-\frac{1}{2}(\kappa'-\kappa)^2\right)-\frac{\bar{\kappa}}{2}\left(M_f-M_i\right)\\
&\qquad \times\left(E'-E\right)-\frac{1}{4}\left(m_f^2-m_i^2+2E'\mathcal{E}-2E\mathcal{E}'\right)\left(M_f-M_i\right)\\
&\qquad \left.-\frac{\bar{\kappa}}{2}\left(M_f-M_i\right)\left(\kappa'-\kappa\right)\right]\ ,\\
Y_V^A &= \frac{f_V^2\,p'p}{m_\pi^2}\left[\frac{1}{2}\left(M_f+M_i\right)-M_B\right]\ ,\\
X_V^B &= \frac{f_V^2}{2m_\pi^2}\left[\left(\frac{1}{2}\left(M_f+M_i\right)-M_B\right)\left(M_f+M_i\right)+\frac{1}{2}\left(m_f^2+m_i^2\right.\right.\\
&\qquad \left.\left.-2E'\mathcal{E}-2E\mathcal{E}'\right)+\bar{\kappa}\left(E'-E\right)\cdot n-\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\right]\ ,\\
Y_V^B &= \frac{f_V^2\,p'p}{m_\pi^2}\ ,\\
X_V^{A'} &= \frac{f_V^2}{4m_\pi^2}\left[\kappa'\left(m_f^2-2E\mathcal{E}'\right)+\kappa\left(m_i^2-2E'\mathcal{E}\right)\right]\ ,\\
Y_V^{A'} &= \frac{f_V^2\bar{\kappa}\,p'p}{m_\pi^2}\ ,\\
X_V^{B'} &= \frac{f_V^2}{4m_\pi^2}\left[\kappa' M_i-\kappa M_f-\left(\kappa'-\kappa\right)M_B\right]\ . \qquad \text{(C.23)}
\end{aligned}$$

Baryon Exchange, Pseudo Vector coupling

The expressions for baryon exchange with pseudo vector coupling are the same as (C.21)-(C.23) with the substitution $M_B \to -M_B$.

Baryon Resonance, Vector coupling

$$
\begin{aligned}
M^V_{\kappa',\kappa} &= \frac{f_V^2}{m_\pi^2}\bar{u}(p's')\Bigg[-\left(\frac{1}{2}\left(M_f+M_i\right)-M_B\right)\\
&\times\Bigg(-\frac{1}{2}\left(M_f^2+M_i^2\right)+\frac{1}{2}\left(s_{p'q'}+s_{pq}\right)-\left(M_f+M_i\right)\not{Q}\\
&\quad -\frac{1}{2}\left(\kappa'-\kappa\right)\left(p'-p\right)\cdot n-\frac{1}{2}\left(\kappa'-\kappa\right)\left[\not{n},\not{Q}\right]-\frac{1}{2}\left(\kappa'-\kappa\right)^2\Bigg)\\
&+\frac{1}{2}\left(s_{p'q'}-M_f^2\right)\left(\frac{1}{2}\left(M_f-M_i\right)+\not{Q}+\frac{1}{2}\left(\kappa'-\kappa\right)\not{n}\right)\\
&+\frac{1}{2}\left(s_{pq}-M_i^2\right)\left(-\frac{1}{2}\left(M_f-M_i\right)+\not{Q}-\frac{1}{2}\left(\kappa'-\kappa\right)\not{n}\right)\\
&+\bar{\kappa}\Bigg(-\frac{1}{2}\left(M_f-M_i\right)\left(p'-p\right)\cdot n+\left(p'-p\right)\cdot n\;\not{Q}+2Q\cdot n\;\not{Q}\\
&\quad -\frac{1}{2}\left(M_f-M_i\right)\left(\kappa'-\kappa\right)-\frac{1}{2}\left(M_f-M_i\right)\left[\not{n},\not{Q}\right]\\
&\quad +\frac{1}{2}\left(M_f^2+M_i^2\right)\not{n}-\frac{1}{2}\left(s_{p'q'}+s_{pq}\right)\not{n}\Bigg)\Bigg]u_i(p)D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\;.
\end{aligned}
\tag{C.24}
$$

$$
\begin{aligned}
A_V &= \frac{f_V^2}{2m_\pi^2}\Bigg[-\left(\frac{1}{2}\left(M_f+M_i\right)-M_B\right)\Bigg(\frac{1}{2}(s_{p'q'}+s_{pq})-\frac{1}{2}\left(M_f^2+M_i^2\right)\\
&\quad -\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\Bigg)-\frac{\bar{\kappa}}{2}\left(M_f-M_i\right)\\
&\quad \times\left(p'-p\right)\cdot n+\frac{1}{4}\left(s_{p'q'}-s_{pq}-M_f^2+M_i^2\right)\left(M_f-M_i\right)\\
&\quad -\frac{\bar{\kappa}}{2}\left(M_f-M_i\right)\left(\kappa'-\kappa\right)\Bigg]D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\;,\\
B_V &= \frac{f_V^2}{2m_\pi^2}\Bigg[\left(\frac{1}{2}\left(M_f+M_i\right)-M_B\right)\left(M_f+M_i\right)+\frac{1}{2}\Bigg(s_{p'q'}+s_{pq}\\
&\quad -M_f^2-M_i^2\Bigg)+\bar{\kappa}(p'-p)\cdot n+2\bar{\kappa}n\cdot Q\Bigg]D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\;,\\
A_V' &= \frac{f_V^2}{4m_\pi^2}\left[\left(M_i^2-s_{pq}\right)\kappa'+\left(M_f^2-s_{p'q'}\right)\kappa\right]D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\;,\\
B_V' &= \frac{f_V^2}{4m_\pi^2}\left[\kappa'M_i-\kappa M_f-(\kappa'-\kappa)M_B\right]D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\;.
\end{aligned}
\tag{C.25}
$$

$$
\begin{aligned}
X_V^A &= -\frac{f_V^2}{2m_\pi^2}\left[-\frac{1}{2}\left(\frac{1}{2}(M_f+M_i)-M_B\right)\left((E'+\mathcal{E}')^2+(E+\mathcal{E})^2\right.\right.\\
&\quad\left.-\left(M_f^2+M_i^2\right)-\frac{1}{2}(\kappa'-\kappa)(E'-E)-\frac{1}{2}(\kappa'-\kappa)^2\right)\\
&\quad-\frac{\bar\kappa}{2}(M_f-M_i)(E'-E)+\frac{1}{4}\left((E'+\mathcal{E}')^2-(E+\mathcal{E})^2\right.\\
&\quad\left.\left.-M_f^2+M_i^2\right)(M_f-M_i)-\frac{\bar\kappa}{2}(M_f-M_i)(\kappa'-\kappa)\right]\ ,\\
X_V^B &= -\frac{f_V^2}{2m_\pi^2}\left[\left(\frac{1}{2}(M_f+M_i)-M_B\right)(M_f+M_i)+\frac{1}{2}(E'+\mathcal{E}')^2\right.\\
&\quad\left.+\frac{1}{2}(E+\mathcal{E})^2-\frac{1}{2}\left(M_f^2+M_i^2\right)+\bar\kappa(E'-E)+\bar\kappa(\mathcal{E}'+\mathcal{E})\right]\ ,\\
X_V^{A'} &= -\frac{f_V^2}{4m_\pi^2}\left[\left(M_i^2-(E+\mathcal{E})^2\right)\kappa'+\left(M_f^2-(E'+\mathcal{E}')^2\right)\kappa\right]\ ,\\
X_V^{B'} &= -\frac{f_V^2}{4m_\pi^2}\left[\kappa' M_i-\kappa M_f-(\kappa'-\kappa)M_B\right]\ . \qquad (C.26)
\end{aligned}
$$

Baryon Resonance, Pseudo Vector coupling

The expressions for baryon resonance with pseudo vector coupling are the same as (C.24)-(C.26) with the substitution $M_B \to -M_B$.

$\frac{3}{2}^+$ Baryon Exchange, Gauge invariant coupling

$$
\begin{aligned}
&M_{\kappa',\kappa} = -\frac{g_{gi}^2}{2}\,\bar u(p's')\Bigg[\\
&\frac{1}{2}\bar P_u^2\left(\frac{1}{2}(M_f+M_i)+M_\Delta-\not Q+\bar\kappa\not n\right)\left(m_f^2+m_i^2-t_{q'q}\right)\\
&-\frac{1}{3}\bar P_u^2\left(\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)\not q\not q'+\frac{1}{2}\left(u_{pq'}-M_i^2\right)\not q\right.\\
&\qquad\left.+\frac{1}{2}\left(s_{pq}+t_{q'q}-M_i^2-m_f^2-3m_i^2\right)\not q'+\bar\kappa\not n\not q\not q'\right)\\
&-\frac{1}{12}\left(\left(\bar P_u^2+\frac{M_\Delta}{2}(M_f-M_i)\right)\not q+\frac{M_\Delta}{2}\left(s_{pq}-M_i^2-2m_i^2\right)\right.\\
&\qquad\left.-\frac{M_\Delta}{2}\not q'\not q+M_\Delta\bar\kappa\not n\not q\right)\left(\bar P_u\cdot q'\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{12}\left(\left(\bar{P}_u^2+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\not{q}'+\frac{M_\Delta}{2}\left(M_i^2-u_{pq'}\right)\right.\\
&\qquad\left.-\frac{M_\Delta}{2}\not{q}\not{q}'+M_\Delta\bar{\kappa}\not{n}\not{q}'\right)\left(\bar{P}_u\cdot q\right)\\
&-\frac{1}{24}\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta-\not{Q}+\bar{\kappa}\not{n}\right)\left(\bar{P}_u\cdot q'\right)\left(\bar{P}_u\cdot q\right)\Big]\,u(ps)\\
&\times D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ . \qquad\qquad (C.27)
\end{aligned}
$$

Here, $\bar{P}_u^2$ is defined in (5.47). All the expressions for the *slashed* terms (i.e. $\not{q}$, $\not{q}'$, etc.), can be found in (C.68). Furthermore

$$
\begin{aligned}
\bar{P}_u\cdot q' &= \Big(-M_f^2+M_i^2-3m_f^2-m_i^2+s_{p'q'}-u_{pq'}+t_{q'q}-2\bar{\kappa}(p'-p)\cdot n\\
&\qquad +4\bar{\kappa}n\cdot Q-\left(\kappa'^2-\kappa^2\right)\Big)\ ,\\
\bar{P}_u\cdot q' &= \Big(M_f^2-M_i^2-m_f^2-3m_i^2+s_{pq}-u_{p'q}+t_{q'q}+2\bar{\kappa}(p'-p)\cdot n\\
&\qquad +4\bar{\kappa}n\cdot Q+\left(\kappa'^2-\kappa^2\right)\Big)\ . \qquad\qquad (C.28)
\end{aligned}
$$

$$
\begin{aligned}
A_\Delta &= -\frac{g_{gi}^2}{2}\left\{\frac{1}{2}\bar{P}_u^2\left[\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right]\left(m_f^2+m_i^2-t_{q'q}\right)\right.\\
&-\frac{1}{3}\bar{P}_u^2\left[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\left(\frac{1}{2}(u_{p'q}+u_{pq'})-\frac{1}{2}\left(M_f^2+M_i^2\right)\right.\right.\\
&\qquad\left.-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right)+\frac{1}{4}\left(u_{pq'}-M_i^2\right)\\
&\qquad\times\left(M_f-M_i\right)-\frac{1}{4}\left(s_{pq}+t_{q'q}-M_i^2-m_f^2-3m_i^2\right)\left(M_f-M_i\right)\\
&\qquad\left.-\bar{\kappa}\left(M_f-M_i\right)n\cdot Q\right]\\
&-\frac{1}{12}\left[\frac{1}{2}\left(\bar{P}_u^2+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\left(M_f-M_i\right)+\frac{1}{2}M_\Delta\right.\\
&\qquad\times\left(s_{pq}-M_i^2-2m_i^2\right)-\frac{1}{2}M_\Delta\left(\frac{1}{2}(s_{p'q'}+s_{pq})-\frac{1}{2}\left(M_f^2+M_i^2\right)\right.\\
&\qquad\left.-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right)+\bar{\kappa}M_\Delta\Big(n\cdot p'+n\cdot Q\\
&\qquad\left.+\frac{1}{2}(\kappa'-\kappa)\Big)\right]\left(\bar{P}_u\cdot q'\right)
\end{aligned}
$$

$$-\frac{1}{12}\left[\frac{1}{2}\left(\bar{P}_u^2+\frac{M_\Delta}{2}(M_f-M_i)\right)(M_f-M_i)-\frac{1}{2}M_\Delta\left(M_i^2-u_{pq'}\right)\right.$$
$$+\frac{1}{2}M_\Delta\left(\frac{1}{2}(u_{p'q}+u_{pq'})-\frac{1}{2}\left(M_f^2+M_i^2\right)\right.$$
$$\left.-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right)-\bar{\kappa}M_\Delta\left(-n\cdot p'+n\cdot Q\right.$$
$$\left.\left.-\frac{1}{2}(\kappa'-\kappa)\right)\right]\left(\bar{P}_u\cdot q\right)$$
$$\left.-\frac{1}{24}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]\left(\bar{P}_u\cdot q'\right)\left(\bar{P}_u\cdot q\right)\right\}D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ . \tag{C.29}$$

$$B_\Delta = -\frac{g_{gi}^2}{2}\left\{-\frac{1}{2}\bar{P}_u^2\left(m_f^2+m_i^2-t_{q'q}\right)\right.$$
$$-\frac{1}{3}\bar{P}_u^2\left[\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)(M_f+M_i)+\frac{1}{2}\left(u_{pq'}-M_i^2\right)\right.$$
$$+\frac{1}{2}\left(s_{pq}+t_{q'q}-M_i^2-m_f^2-3m_i^2\right)+2\bar{\kappa}(p'-p)\cdot n$$
$$\left.+\frac{1}{2}\left(\kappa'^2-\kappa^2\right)\right]$$
$$-\frac{1}{12}\left(\bar{P}_u^2+M_\Delta M_f\right)\left(\bar{P}_u\cdot q'\right)+\frac{1}{12}\left(\bar{P}_u^2-M_\Delta M_i\right)\left(\bar{P}_u\cdot q\right)$$
$$\left.+\frac{1}{24}\left(\bar{P}_u\cdot q'\right)\left(\bar{P}_u\cdot q\right)\right\}D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ . \tag{C.30}$$

$$A'_\Delta = -\frac{g_{gi}^2}{2}\left\{\frac{\bar{\kappa}}{2}\bar{P}_u^2\left(m_f^2+m_i^2-t_{q'q}\right)\right.$$
$$-\frac{1}{3}\bar{P}_u^2\left[\frac{1}{4}(\kappa'-\kappa)\left(u_{pq'}-M_i^2\right)-\frac{1}{4}(\kappa'-\kappa)\left(s_{pq}+t_{q'q}-M_i^2\right.\right.$$
$$\left.-m_f^2-3m_i^2\right)+\bar{\kappa}\left(-\frac{1}{2}\left(M_f^2+M_i^2\right)+\frac{1}{2}(u_{p'q}+u_{pq'})\right.$$
$$\left.\left.-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-(\kappa'-\kappa)Q\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right)\right]$$
$$-\frac{1}{24}\left[(\kappa'-\kappa)\bar{P}_u^2-M_\Delta\left(\kappa M_f+\kappa' M_i\right)\right]\left[s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right.$$
$$\left.+2t_{q'q}-4m_f^2-4m_i^2+8\bar{\kappa}n\cdot Q\right]$$
$$\left.+\frac{\bar{\kappa}}{24}\left(\bar{P}_u\cdot q'\right)\left(\bar{P}_u\cdot q\right)\right\}D^{(2)}\left(\Delta_u,n,\bar{\kappa}\right)\ . \tag{C.31}$$

$$B'_\Delta = \frac{g_{gi}^2}{12}\left\{\bar{P}_u^2\left[M_i\kappa' - M_f\kappa + M_\Delta(\kappa'-\kappa)\right] + \frac{M_\Delta\kappa'}{4}\left(\bar{P}_u\cdot q'\right) - \frac{M_\Delta\kappa}{4}\left(\bar{P}_u\cdot q\right)\right\} D^{(2)}(\Delta_u, n, \bar{\kappa}) \ . \tag{C.32}$$

$$\begin{aligned}
X_\Delta^A = \frac{g_{gi}^2}{2}\Bigg\{ & \left(\bar{P}_u^2\right)_{CM}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]\mathcal{E}'\mathcal{E} \\
& -\frac{1}{3}\left(\bar{P}_u^2\right)_{CM}\left[\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)\left(\frac{1}{2}\left(M_f^2+M_i^2+m_f^2+m_i^2\right.\right.\right. \\
& \quad -2E'\mathcal{E}-2\mathcal{E}'E) - \frac{1}{2}\left(M_f^2+M_i^2\right) - \frac{1}{2}(\kappa'-\kappa)(E'-E) \\
& \quad \left.-\frac{1}{2}(\kappa'-\kappa)^2\right) + \frac{1}{4}\left(m_f^2-2E\mathcal{E}'\right)(M_f-M_i) - \frac{1}{4}\left((E+\mathcal{E})^2\right. \\
& \quad \left.\left.-2\mathcal{E}'\mathcal{E}-M_i^2-2m_i^2\right)(M_f-M_i) - \frac{1}{2}\bar{\kappa}(M_f-M_i)(\mathcal{E}'+\mathcal{E})\right] \\
& -\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar{P}_u^2\right)_{CM}+\frac{M_\Delta}{2}(M_f-M_i)\right)(M_f-M_i)+\frac{1}{2}M_\Delta\left((E+\mathcal{E})^2\right.\right. \\
& \quad \left.-M_i^2-2m_i^2\right) - \frac{1}{2}M_\Delta\left(\frac{1}{2}(E'+\mathcal{E}')^2+\frac{1}{2}(E+\mathcal{E})^2-\frac{1}{2}\left(M_f^2+M_i^2\right)\right. \\
& \quad \left.-\frac{1}{2}(\kappa'-\kappa)(E'-E)-\frac{1}{2}(\kappa'-\kappa)^2\right)+\bar{\kappa}M_\Delta\left(E'+\frac{1}{2}(\mathcal{E}'+\mathcal{E})\right. \\
& \quad \left.\left.+\frac{1}{2}(\kappa'-\kappa)\right)\right]\left(\bar{P}_u\cdot q'\right)_{CM} \\
& -\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar{P}_u^2\right)_{CM}+\frac{M_\Delta}{2}(M_f-M_i)\right)(M_f-M_i)-\frac{1}{2}M_\Delta\left(m_f^2-2E\mathcal{E}'\right)\right. \\
& \quad +\frac{1}{2}M_\Delta\left(\frac{1}{2}\left(m_f^2+m_i^2-2E'\mathcal{E}-2\mathcal{E}'E\right)-\frac{1}{2}(\kappa'-\kappa)(E'-E)\right. \\
& \quad \left.\left.-\frac{1}{2}(\kappa'-\kappa)^2\right)-\bar{\kappa}M_\Delta\left(-E'+\frac{1}{2}(\mathcal{E}'+\mathcal{E})-\frac{1}{2}(\kappa'-\kappa)\right)\right]\left(\bar{P}_u\cdot q\right)_{CM} \\
& -\frac{1}{24}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]\left(\bar{P}_u\cdot q'\right)_{CM}\left(\bar{P}_u\cdot q\right)_{CM}\Bigg\} \ ,
\end{aligned} \tag{C.33}$$

where

$$\left(\bar{P}_u^2\right)_{CM} = \left[\frac{1}{2}\left(M_f^2+M_i^2+m_f^2+m_i^2-2E'\mathcal{E}-2\mathcal{E}'E\right)+\kappa'\kappa + \bar{\kappa}\left(E'+E-\mathcal{E}'-\mathcal{E}\right)\right]$$

$$\begin{aligned}
\left(\bar{P}_u\cdot q'\right)_{CM} &= \Big[-M_f^2-3m_f^2+\left(E'+\mathcal{E}'\right)^2+2E\mathcal{E}'-2\mathcal{E}'\mathcal{E}-2\bar{\kappa}(E'-E)\\
&\quad +2\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)-\left(\kappa'^2-\kappa^2\right)\Big]\ ,\\
\left(\bar{P}_u\cdot q\right)_{CM} &= \Big[-M_i^2-3m_i^2+\left(E+\mathcal{E}\right)^2+2E'\mathcal{E}-2\mathcal{E}'\mathcal{E}+2\bar{\kappa}\left(E'-E\right)\\
&\quad +2\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)+\left(\kappa'^2-\kappa^2\right)\Big]\ . \qquad \text{(C.34)}
\end{aligned}$$

$$\begin{aligned}
Y_\Delta^A &= \frac{g_{gi}^2\,p'p}{2}\Bigg\{\left[\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right]2\mathcal{E}'\mathcal{E}\\
&-\frac{5}{3}\left(\bar{P}_u^2\right)_{CM}\left[\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right]\\
&-\frac{2}{3}\Bigg[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\Bigg(\frac{1}{2}\left(M_f^2+M_i^2+m_f^2+m_i^2-2E'\mathcal{E}\right.\\
&\quad\left.-2\mathcal{E}'E\right)-\frac{1}{2}\left(M_f^2+M_i^2\right)-\frac{1}{2}(\kappa'-\kappa)\left(E'-E\right)-\frac{1}{2}(\kappa'-\kappa)^2\Bigg)\\
&\quad-\frac{1}{4}\left(\left(E+\mathcal{E}\right)^2-2\mathcal{E}'\mathcal{E}-M_i^2-2m_i^2\right)\left(M_f-M_i\right)\\
&\quad+\frac{1}{4}\left(m_f^2-2E\mathcal{E}'\right)\left(M_f-M_i\right)-\frac{1}{2}\,\bar{\kappa}\left(M_f-M_i\right)\left(\mathcal{E}'+\mathcal{E}\right)\Bigg]\\
&-\frac{1}{12}\Big[-M_f^2-M_i^2-3m_f^2-3m_i^2+\left(E'+\mathcal{E}'\right)^2+\left(E+\mathcal{E}\right)^2\\
&\quad+2E'\mathcal{E}+2E\mathcal{E}'-4\mathcal{E}'\mathcal{E}+4\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\Big]\left[M_f-M_i\right]\\
&-\frac{M_\Delta}{6}\left(\bar{P}_u\cdot q\right)_{CM}\Bigg\}\ . \qquad \text{(C.35)}
\end{aligned}$$

$$Z_\Delta^A = -\frac{5g_{gi}^2(p'p)^2}{3}\left[\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right]\ . \qquad \text{(C.36)}$$

$$\begin{aligned}
X_\Delta^B &= \frac{g_{gi}^2}{2}\Bigg\{-\left(\bar{P}_u^2\right)_{CM}\mathcal{E}'\mathcal{E}-\frac{1}{3}\left(\bar{P}_u^2\right)_{CM}\Bigg[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\left(M_f+M_i\right)\\
&\quad+\frac{1}{2}\left(\left(E+\mathcal{E}\right)^2-2\mathcal{E}'\mathcal{E}-M_i^2-2m_i^2\right)\\
&\quad+\frac{1}{2}\left(m_f^2-2E\mathcal{E}'\right)+2\bar{\kappa}\left(E'-E\right)+\frac{1}{2}\left(\kappa'^2-\kappa^2\right)\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{12}\left[\left(\bar{P}_u^2\right)_{CM} + M_\Delta M_f\right]\left(\bar{P}_u\cdot q'\right)_{CM} \\
&+\frac{1}{12}\left[\left(\bar{P}_u^2\right)_{CM} - M_\Delta M_i\right]\left(\bar{P}_u\cdot q\right)_{CM} \\
&+\frac{1}{24}\left(\bar{P}_u\cdot q'\right)_{CM}\left(\bar{P}_u\cdot q\right)_{CM}\Bigg\} \ . \qquad \text{(C.37)}
\end{aligned}
$$

$$
\begin{aligned}
Y_\Delta^B &= \frac{g_{gi}^2\, p'p}{2}\Bigg\{-2\mathcal{E}'\mathcal{E} + \frac{1}{3}\left(\bar{P}_u^2\right)_{CM} \\
&-\frac{2}{3}\Bigg[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\left(M_f+M_i\right)+\frac{1}{2}\Bigg(\left(E+\mathcal{E}\right)^2-2\mathcal{E}'\mathcal{E} \\
&\qquad -M_i^2-2m_i^2\Bigg)+\frac{1}{2}\left(m_f^2-2E\mathcal{E}'\right)+\bar{\kappa}\left(\left(\kappa'-\kappa\right)+2(E'-E)\right)\Bigg] \\
&+\frac{1}{6}\Bigg[M_f^2-M_i^2+3m_f^2-3m_i^2-\left(E'+\mathcal{E}'\right)^2+\left(E+\mathcal{E}\right)^2 \\
&\qquad -2E\mathcal{E}'+2E'\mathcal{E}+4\bar{\kappa}\left(E'-E\right)+2\left(\kappa'^2-\kappa^2\right)\Bigg]\Bigg\} \ . \qquad \text{(C.38)}
\end{aligned}
$$

$$
Z_\Delta^B = \frac{g_{gi}^2(p'p)^2}{3} \ . \qquad \text{(C.39)}
$$

$$
\begin{aligned}
X_\Delta^{A'} &= \frac{g_{gi}^2}{2}\Bigg\{\bar{\kappa}\left(\bar{P}_u^2\right)_{CM}\mathcal{E}'\mathcal{E} - \frac{1}{3}\left(\bar{P}_u^2\right)_{CM}\Bigg[\frac{1}{4}\left(\kappa'-\kappa\right)\Big(m_f^2-2E\mathcal{E}'+2\mathcal{E}'\mathcal{E} \\
&\qquad -\left(E+\mathcal{E}\right)^2+M_i^2+2m_i^2\Big)+\bar{\kappa}\Bigg(\frac{1}{2}\left(m_f^2+m_i^2\right)-E'\mathcal{E}-\mathcal{E}'E \\
&\qquad -\frac{1}{2}(\kappa'-\kappa)\left(E'-E\right)-\frac{1}{2}\left(\kappa'-\kappa\right)^2-\frac{1}{2}(\kappa'-\kappa)\left(\mathcal{E}'+\mathcal{E}\right)\Bigg)\Bigg] \\
&-\frac{1}{12}\left[(\kappa'-\kappa)\left(\bar{P}_u^2\right)_{CM}-M_\Delta\left(\kappa M_f+\kappa' M_i\right)\right]\Big[\left(E'+\mathcal{E}'\right)^2 \\
&\qquad +\left(E+\mathcal{E}\right)^2+2E'\mathcal{E}+2E\mathcal{E}'-2\mathcal{E}'\mathcal{E}-M_f^2-M_i^2 \\
&\qquad -3m_f^2-3m_f^2+4\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\Big] \\
&+\frac{\bar{\kappa}}{24}\left(\bar{P}_u\cdot q'\right)_{CM}\left(\bar{P}_u\cdot q\right)_{CM}\Bigg\} \ . \qquad \text{(C.40)}
\end{aligned}
$$

$$
\begin{aligned}
Y_{\Delta}^{A'} &= \frac{g_{gi}^2\, p'p}{2}\Bigg\{2\bar{\kappa}\mathcal{E}'\mathcal{E} - \frac{5\bar{\kappa}}{3}\left(\bar{P}_u^2\right)_{CM} \\
&\quad -\frac{2}{3}\left[\frac{1}{4}\left(\kappa'-\kappa\right)\left(m_f^2 - 2E\mathcal{E}' - \left(E+\mathcal{E}\right)^2 + 2\mathcal{E}'\mathcal{E} + M_i^2 + 2m_i^2\right)\right. \\
&\qquad + \bar{\kappa}\left(\frac{1}{2}\left(m_f^2+m_i^2\right) - E'\mathcal{E} - \mathcal{E}'E - \frac{1}{2}(\kappa'-\kappa)\left(E'-E\right)\right. \\
&\qquad\quad \left.\left. -\frac{1}{2}\left(\kappa'-\kappa\right)^2 - \frac{1}{2}(\kappa'-\kappa)\left(\mathcal{E}'+\mathcal{E}\right)\right)\right] \\
&\quad -\frac{(\kappa'-\kappa)}{12}\Big[-M_f^2 - M_i^2 - 3m_f^2 - 3m_i^2 + \left(E'+\mathcal{E}'\right)^2 + \left(E+\mathcal{E}\right)^2 \\
&\qquad +2E'\mathcal{E} + 2E\mathcal{E}' - 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\Big]\Bigg\}\ . \qquad \text{(C.41)}
\end{aligned}
$$

$$
Z_{\Delta}^{A'} = -\frac{5g_{gi}^2(p'p)^2\bar{\kappa}}{3}\ . \qquad \text{(C.42)}
$$

$$
\begin{aligned}
X_{\Delta}^{B'} &= -\frac{g_{gi}^2}{12}\Bigg\{\left(\bar{P}_u^2\right)_{CM}\left[M_i\kappa' - M_f\kappa + M_\Delta\left(\kappa'-\kappa\right)\right] \\
&\qquad +\frac{M_\Delta\,\kappa'}{4}\left(\bar{P}_u\cdot q'\right)_{CM} - \frac{M_\Delta\,\kappa}{4}\left(\bar{P}_u\cdot q\right)_{CM}\Bigg\}\ . \qquad \text{(C.43)}
\end{aligned}
$$

$$
Y_{\Delta}^{B'} = -\frac{g_{gi}^2 p'p}{6}\left[M_i\kappa' - M_f\kappa + M_\Delta\left(\kappa'-\kappa\right)\right]\ . \qquad \text{(C.44)}
$$

$\frac{3}{2}^+$ Baryon Resonance, Gauge invariant coupling

$$
\begin{aligned}
&M_{\kappa',\kappa} = -\frac{g_{gi}^2}{2}\,\bar{u}(p's')\Bigg[\\
&\frac{1}{2}\,\bar{P}_s^2\left(\frac{1}{2}(M_f+M_i) + M_\Delta + \not{Q} + \bar{\kappa}\not{n}\right)\left(m_f^2 + m_i^2 - t_{q'q}\right) \\
&-\frac{1}{3}\,\bar{P}_s^2\left(\left(\frac{1}{2}\,\left(M_f+M_i\right) + M_\Delta\right)\not{q}'\not{q} - \frac{1}{2}\,\left(s_{pq} - M_i^2\right)\not{q}'\right. \\
&\qquad \left. -\frac{1}{2}\,\left(u_{pq'} + t_{q'q} - M_i^2 - 3m_f^2 - m_i^2\right)\not{q} + \bar{\kappa}\not{n}\not{q}'\not{q}\right)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{12}\left(\left(\bar{P}_s^2+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\not{q}'+\frac{M_\Delta}{2}\left(M_i^2+2m_f^2-u_{pq'}\right)\right.\\
&\qquad\left.+\frac{M_\Delta}{2}\not{q}\not{q}'+M_\Delta\bar{\kappa}\not{n}\not{q}'\right)\left(\bar{P}_s\cdot q\right)\\
&+\frac{1}{12}\left(\left(\bar{P}_s^2+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\not{q}+\frac{M_\Delta}{2}\left(s_{pq}-M_i^2\right)\right.\\
&\qquad\left.+\frac{M_\Delta}{2}\not{q}'\not{q}+M_\Delta\bar{\kappa}\not{n}\not{q}\right)\left(\bar{P}_s\cdot q'\right)\\
&\left.-\frac{1}{24}\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta+\not{Q}+\bar{\kappa}\not{n}\right)\left(\bar{P}_s\cdot q'\right)\left(\bar{P}_s\cdot q\right)\right]u(ps)\\
&\times D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\ , \qquad \text{(C.45)}
\end{aligned}
$$

where $\bar{P}_s^2$ is defined in (5.47) and the slashed terms are as, before, defined in (C.68). The inner products in (C.45) are

$$
\begin{aligned}
\bar{P}_s\cdot q' &= \Big(-M_f^2+M_i^2+3m_f^2+m_i^2+s_{p'q'}-u_{pq'}-t_{q'q}-2\bar{\kappa}(p'-p)\cdot n\\
&\qquad +4\bar{\kappa}n\cdot Q-\left(\kappa'^2-\kappa^2\right)\Big)\\
\bar{P}_s\cdot q &= \Big(M_f^2-M_i^2+m_f^2+3m_i^2+s_{pq}-u_{p'q}-t_{q'q}+2\bar{\kappa}(p'-p)\cdot n\\
&\qquad +4\bar{\kappa}n\cdot Q+\left(\kappa'^2-\kappa^2\right)\Big) \qquad \text{(C.46)}
\end{aligned}
$$

$$
\begin{aligned}
A_\Delta &= -\frac{g_{gi}^2}{2}\left\{\frac{1}{2}\bar{P}_s^2\left[\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right]\left(m_f^2+m_i^2-t_{q'q}\right)\right.\\
&-\frac{1}{3}\bar{P}_s^2\left[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\left(\frac{1}{2}(s_{p'q'}+s_{pq})-\frac{1}{2}\left(M_f^2+M_i^2\right)\right.\right.\\
&\qquad\left.-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right)+\frac{1}{4}\left(s_{pq}-M_i^2\right)\\
&\qquad\times\left(M_f-M_i\right)+\frac{1}{4}\left(M_i^2+3m_f^2+m_i^2-u_{pq'}-t_{q'q}\right)\left(M_f-M_i\right)\\
&\qquad\left.+\bar{\kappa}\left(M_f-M_i\right)n\cdot Q\right]\\
&+\frac{1}{12}\left[\frac{1}{2}\left(\bar{P}_s^2+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\left(M_f-M_i\right)-\frac{1}{2}M_\Delta\right.\\
&\qquad\times\left(M_i^2+2m_f^2-u_{pq'}\right)-\frac{1}{2}M_\Delta\left(\frac{1}{2}(u_{p'q}+u_{pq'})-\frac{1}{2}\left(M_f^2+M_i^2\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n - \frac{1}{2}(\kappa'-\kappa)^2\Big) - \bar{\kappa} M_\Delta \Big(- n\cdot p' + n\cdot Q \\
& - \frac{1}{2}(\kappa'-\kappa)\Big)\Big] \left(\bar{P}_s\cdot q\right) \\
+\frac{1}{12}&\left[\frac{1}{2}\left(\bar{P}_s^2 + \frac{M_\Delta}{2}\left(M_f - M_i\right)\right)\left(M_f - M_i\right) + \frac{1}{2} M_\Delta \left(s_{pq} - M_i^2\right)\right. \\
& + \frac{1}{2} M_\Delta \Big(\frac{1}{2}(s_{p'q'} + s_{pq}) - \frac{1}{2}\left(M_f^2 + M_i^2\right) \\
& - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n - \frac{1}{2}(\kappa'-\kappa)^2\Big) + \bar{\kappa} M_\Delta \Big(n\cdot p' + n\cdot Q \\
& \left. + \frac{1}{2}(\kappa'-\kappa)\Big)\right] \left(\bar{P}_s\cdot q'\right) \\
-\frac{1}{24}&\left[\frac{1}{2}\left(M_f + M_i\right) + M_\Delta\right]\left(\bar{P}_s\cdot q'\right)\left(\bar{P}_s\cdot q\right)\Big\} D^{(2)}\left(\Delta_u, n, \bar{\kappa}\right) \ . \qquad \text{(C.47)}
\end{aligned}
$$

$$
\begin{aligned}
B_\Delta \ = \ & -\frac{g_{gi}^2}{2}\Big\{\frac{1}{2}\bar{P}_s^2\left(m_f^2 + m_i^2 - t_{q'q}\right) \\
& -\frac{1}{3}\bar{P}_s^2\Big[-\left(\frac{1}{2}\left(M_f + M_i\right) + M_\Delta\right)\left(M_f + M_i\right) - \frac{1}{2}\left(s_{pq} - M_i^2\right) \\
& \qquad + \frac{1}{2}\left(M_i^2 + 3m_f^2 + m_i^2 - u_{pq'} - t_{q'q}\right) - 2\bar{\kappa}(p'-p)\cdot n \\
& \qquad - \frac{1}{2}\left(\kappa'^2 - \kappa^2\right)\Big] \\
& -\frac{1}{12}\left(\bar{P}_s^2 + M_\Delta M_f\right)\left(\bar{P}_s\cdot q\right) + \frac{1}{12}\left(\bar{P}_s^2 - M_\Delta M_i\right)\left(\bar{P}_s\cdot q'\right) \\
& +\frac{1}{24}\left(\bar{P}_s\cdot q'\right)\left(\bar{P}_s\cdot q\right)\Big\} D^{(2)}\left(\Delta_s, n, \bar{\kappa}\right) \ . \qquad \text{(C.48)}
\end{aligned}
$$

$$
\begin{aligned}
A'_\Delta \ = \ & -\frac{g_{gi}^2}{2}\Big\{\frac{\bar{\kappa}}{2}\bar{P}_s^2\left(m_f^2 + m_i^2 - t_{q'q}\right) \\
& -\frac{1}{3}\bar{P}_s^2\Big[\frac{1}{4}\left(\kappa'-\kappa\right)\left(s_{pq} - M_i^2\right) + \frac{1}{4}\left(\kappa'-\kappa\right)\big(M_i^2 + 3m_f^2 + m_i^2 \\
& \qquad -u_{pq'} - t_{q'q}\big) + \bar{\kappa}\Big(-\frac{1}{2}\left(M_f^2 + M_i^2\right) + \frac{1}{2}(s_{p'q'} + s_{pq}) \\
& \qquad - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n + (\kappa'-\kappa)Q\cdot n - \frac{1}{2}\left(\kappa'-\kappa\right)^2\Big)\Big]
\end{aligned}
$$

$$+\frac{1}{24}\left[(\kappa'-\kappa)\bar{P}_s^2 - M_\Delta\left(\kappa M_f+\kappa' M_i\right)\right]\left[s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}\right.$$
$$\left.-2t_{q'q}+4m_f^2+4m_i^2+8\bar{\kappa}n\cdot Q\right]$$
$$\left.-\frac{\bar{\kappa}}{24}\left(\bar{P}_s\cdot q'\right)\left(\bar{P}_s\cdot q'\right)\right\} D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\ . \tag{C.49}$$

$$B'_\Delta = -\frac{g_{gi}^2}{12}\left\{\bar{P}_s^2\left[M_i\kappa' - M_f\kappa + M_\Delta\left(\kappa'-\kappa\right)\right]\right.$$
$$\left.-\frac{\kappa' M_\Delta}{4}\left(\bar{P}_s\cdot q\right)+\frac{\kappa M_\Delta}{4}\left(\bar{P}_s\cdot q'\right)\right\} D^{(2)}\left(\Delta_s,n,\bar{\kappa}\right)\ . \tag{C.50}$$

$$X_\Delta^A = \frac{g_{gi}^2}{2}\left\{\left(\bar{P}_s^2\right)_{CM}\left[\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right]\mathcal{E}'\mathcal{E}\right.$$
$$-\frac{1}{3}\left(\bar{P}_s^2\right)_{CM}\left[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\left(\frac{1}{2}\left(E'+\mathcal{E}'\right)^2+\frac{1}{2}\left(E+\mathcal{E}\right)^2\right.\right.$$
$$\left.-\frac{1}{2}\left(M_f^2+M_i^2\right)-\frac{1}{2}(\kappa'-\kappa)\left(E'-E\right)-\frac{1}{2}(\kappa'-\kappa)^2\right)$$
$$+\frac{1}{4}\left(m_f^2+2E\mathcal{E}'+2\mathcal{E}'\mathcal{E}\right)\left(M_f-M_i\right)+\frac{1}{4}\left(\left(E+\mathcal{E}\right)^2-M_i^2\right)$$
$$\left.\times\left(M_f-M_i\right)+\frac{\bar{\kappa}}{2}\left(M_f-M_i\right)\left(\mathcal{E}'+\mathcal{E}\right)\right]$$
$$+\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar{P}_s^2\right)_{CM}+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\left(M_f-M_i\right)\right.$$
$$-\frac{1}{2}M_\Delta\left(m_f^2+2E\mathcal{E}'\right)-\frac{1}{2}M_\Delta\left(\frac{1}{2}\left(m_f^2+m_i^2\right)\right.$$
$$\left.-2\left(E'\mathcal{E}+E\mathcal{E}'\right)-\frac{1}{2}(\kappa'-\kappa)\left(E'-E\right)-\frac{1}{2}(\kappa'-\kappa)^2\right)$$
$$\left.-\bar{\kappa}M_\Delta\left(-E'+\frac{1}{2}\left(\mathcal{E}'+\mathcal{E}\right)-\frac{1}{2}(\kappa'-\kappa)\right)\right]\left(\bar{P}_s\cdot q\right)_{CM}$$
$$+\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar{P}_s^2\right)_{CM}+\frac{M_\Delta}{2}\left(M_f-M_i\right)\right)\left(M_f-M_i\right)\right.$$
$$+\frac{1}{2}M_\Delta\left(\left(E+\mathcal{E}\right)^2-M_i^2\right)+\frac{1}{2}M_\Delta\left(\frac{1}{2}\left(E'+\mathcal{E}'\right)^2+\frac{1}{2}\left(E+\mathcal{E}\right)^2\right.$$

$$
\begin{aligned}
& - \frac{1}{2}\left(M_f^2 + M_i^2\right) - \frac{1}{2}(\kappa' - \kappa)\left(E' - E\right) - \frac{1}{2}(\kappa' - \kappa)^2\Bigg) \\
& + \bar{\kappa} M_\Delta \left(E' + \frac{1}{2}\left(\mathcal{E}' + \mathcal{E}\right) + \frac{1}{2}(\kappa' - \kappa)\right)\Bigg] \left(\bar{P}_s \cdot q'\right)_{CM} \\
-\frac{1}{24} & \left[\frac{1}{2}\left(M_f + M_i\right) + M_\Delta\right] \left(\bar{P}_s \cdot q'\right)_{CM} \left(\bar{P}_s \cdot q\right)_{CM}\Bigg\} \ , \qquad \text{(C.51)}
\end{aligned}
$$

where

$$
\begin{aligned}
\left(\bar{P}_s^2\right)_{CM} &= \left[\frac{1}{2}\left(E' + \mathcal{E}'\right)^2 + \frac{1}{2}\left(E + \mathcal{E}\right)^2 + \kappa'\kappa + \bar{\kappa}\left(E' + E + \mathcal{E}' + \mathcal{E}\right)\right] \ , \\
\left(\bar{P}_s \cdot q'\right)_{CM} &= \Bigg[-M_f^2 + m_f^2 + \left(E' + \mathcal{E}'\right)^2 + 2E\mathcal{E}' + 2\mathcal{E}'\mathcal{E} - 2\bar{\kappa}\left(E' - E\right) \\
&\quad + 2\bar{\kappa}\left(\mathcal{E}' + \mathcal{E}\right) - \left(\kappa'^2 - \kappa^2\right)\Bigg] \ , \\
\left(\bar{P}_s \cdot q\right)_{CM} &= \Bigg[-M_i^2 + m_i^2 + \left(E + \mathcal{E}\right)^2 + 2E'\mathcal{E} + 2\mathcal{E}'\mathcal{E} + 2\bar{\kappa}\left(E' - E\right) \\
&\quad + 2\bar{\kappa}\left(\mathcal{E}' + \mathcal{E}\right) + \left(\kappa'^2 - \kappa^2\right)\Bigg] \qquad \text{(C.52)}
\end{aligned}
$$

$$
\begin{aligned}
Y_\Delta^A &= \frac{g_{gi}^2\, p'p}{2}\Bigg\{-\left(\bar{P}_s^2\right)_{CM}\left[\frac{1}{2}\left(M_f + M_i\right) + M_\Delta\right] \\
&+ \frac{M_\Delta}{6}\Bigg[-\frac{1}{2}M_f^2 + \frac{1}{2}M_i^2 + 2M_f M_i + \frac{1}{2}\left(3m_f^2 + m_i^2\right) - \left(E'\mathcal{E} - E\mathcal{E}'\right) \\
&\quad - \frac{1}{2}\left(E' + \mathcal{E}'\right)^2 - \frac{3}{2}\left(E + \mathcal{E}\right)^2 - 4\bar{\kappa}E' - \left(\kappa'^2 - \kappa^2\right)\Bigg] \\
&+ \frac{1}{6}\left[\frac{1}{2}\left(M_f + M_i\right) + M_\Delta\right]\Bigg[-M_f^2 - M_i^2 + m_f^2 + m_i^2 + \left(E' + \mathcal{E}'\right)^2 \\
&\quad + \left(E + \mathcal{E}\right)^2 + 2E'\mathcal{E} + 2E\mathcal{E}' + 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}\left(\mathcal{E}' + \mathcal{E}\right)\Bigg]\Bigg\} \ . \qquad \text{(C.53)}
\end{aligned}
$$

$$
Z_\Delta^A = -\frac{g_{gi}^2 (p'p)^2}{3}\left[\frac{1}{2}\left(M_f + M_i\right) + M_\Delta\right] \ . \qquad \text{(C.54)}
$$

$$
\begin{aligned}
X_\Delta^B &= \frac{g_{gi}^2}{2}\Bigg\{\left(\bar{P}_s^2\right)_{CM}\mathcal{E}'\mathcal{E} + \frac{1}{3}\left(\bar{P}_s^2\right)_{CM}\Bigg[\left(\frac{1}{2}\left(M_f+M_i\right)+M_\Delta\right)\left(M_f+M_i\right) \\
&\qquad - \frac{1}{2}\left(m_f^2 + 2E\mathcal{E}' + 2\mathcal{E}'\mathcal{E}\right) + \frac{1}{2}\left(\left(E+\mathcal{E}\right)^2 - M_i^2\right) + 2\bar{\kappa}\left(E'-E\right) \\
&\qquad + \frac{1}{2}\left(\kappa'^2 - \kappa^2\right)\Bigg] \\
&\quad - \frac{1}{12}\left[\left(\bar{P}_s^2\right)_{CM} + M_\Delta M_f\right]\left(\bar{P}_s\cdot q\right)_{CM} \\
&\quad + \frac{1}{12}\left[\left(\bar{P}_s^2\right)_{CM} - M_\Delta M_i\right]\left(\bar{P}_s\cdot q'\right)_{CM} \\
&\quad + \frac{1}{24}\left(\bar{P}_s\cdot q'\right)_{CM}\left(\bar{P}_s\cdot q\right)_{CM}\Bigg\} \ . \qquad \text{(C.55)}
\end{aligned}
$$

$$
\begin{aligned}
Y_\Delta^B &= -\frac{g_{gi}^2\, p'p}{6}\Bigg\{\left(\bar{P}_s^2\right)_{CM} - M_\Delta\left(M_f+M_i\right) \\
&\qquad + \frac{1}{2}\Big[-M_f^2 - M_i^2 + m_f^2 + m_i^2 + \left(E'+\mathcal{E}'\right)^2 + \left(E+\mathcal{E}\right)^2 \\
&\qquad + 2E\mathcal{E}' + 2E'\mathcal{E} + 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\Big]\Bigg\} \ . \qquad \text{(C.56)}
\end{aligned}
$$

$$
Z_\Delta^B = \frac{g_{gi}^2(p'p)^2}{3} \ . \qquad \text{(C.57)}
$$

$$
\begin{aligned}
X_\Delta^{A'} &= \frac{g_{gi}^2}{2}\Bigg\{\bar{\kappa}\left(\bar{P}_s^2\right)_{CM}\mathcal{E}'\mathcal{E} - \frac{1}{3}\left(\bar{P}_s^2\right)_{CM}\Bigg[\frac{1}{4}\left(\kappa'-\kappa\right)\left(\left(E+\mathcal{E}\right)^2 - M_i^2\right) \\
&\qquad + \frac{1}{4}\left(\kappa'-\kappa\right)\left(m_f^2 + 2E\mathcal{E}' + 2\mathcal{E}'\mathcal{E}\right) + \bar{\kappa}\Bigg(-\frac{1}{2}\left(M_f^2 + M_i^2\right) \\
&\qquad + \frac{1}{2}\left(E'+\mathcal{E}'\right)^2 + \frac{1}{2}\left(E+\mathcal{E}\right)^2 - \frac{1}{2}(\kappa'-\kappa)\left(E'-E\right) \\
&\qquad + \frac{1}{2}(\kappa'-\kappa)\left(\mathcal{E}'+\mathcal{E}\right) - \frac{1}{2}\left(\kappa'-\kappa\right)^2\Bigg)\Bigg] \\
&\quad + \frac{1}{24}\Big[(\kappa'-\kappa)\left(\bar{P}_s^2\right)_{CM} - M_\Delta\left(\kappa' M_i + \kappa M_f\right)\Big]\Big[-M_f^2 - M_i^2 \\
&\qquad + m_f^2 + m_i^2 + \left(E'+\mathcal{E}'\right)^2 + \left(E+\mathcal{E}\right)^2 + 2E'\mathcal{E} + 2E\mathcal{E}' \\
&\qquad + 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\Big]
\end{aligned}
$$

$$-\frac{\bar{\kappa}}{24}\left(\bar{P}_s\cdot q'\right)_{CM}\left(\bar{P}_s\cdot q\right)_{CM}\Big\} \ . \tag{C.58}$$

$$\begin{aligned} Y_{\Delta}^{A'} &= -\frac{g_{gi}^2\,p'p}{2}\Big\{\bar{\kappa}\left(\bar{P}_s^2\right)_{CM} - \frac{M_\Delta}{3}\left[\kappa' M_i + \kappa M_f\right] \\ &\quad -\frac{\bar{\kappa}}{6}\Big[-M_f^2 - M_i^2 + m_f^2 + m_i^2 + \left(E'+\mathcal{E}'\right)^2 + \left(E+\mathcal{E}\right)^2 \\ &\quad + 2E'\mathcal{E} + 2E\mathcal{E}' + 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}\left(\mathcal{E}'+\mathcal{E}\right)\Big]\Big\} \ . \end{aligned} \tag{C.59}$$

$$Z_{\Delta}^{A'} = -\frac{g_{gi}^2(p'p)^2}{3} \ . \tag{C.60}$$

$$\begin{aligned} X_{\Delta}^{B'} &= \frac{g_{gi}^2}{12}\Big\{\left(\bar{P}_s^2\right)_{CM}\Big[\kappa' M_i - \kappa M_f + \left(\kappa'-\kappa\right)M_\Delta\Big] \\ &\quad -\frac{\kappa' M_\Delta}{4}\left(\bar{P}_s\cdot q\right)_{CM} + \frac{\kappa M_\Delta}{4}\left(\bar{P}_s\cdot q'\right)_{CM}\Big\} \ . \end{aligned} \tag{C.61}$$

$$Y_{\Delta}^{B'} = \frac{g_{gi}^2 M_\Delta\,p'p}{12}\left(\kappa'-\kappa\right) \ . \tag{C.62}$$

C.3 Useful relations

C.3.1 Feynman

In Feynman formalism the following relations are quit useful

$$\begin{aligned} 2(q'\cdot q) &= m_f^2 + m_i^2 - t \ , \\ 2(p'\cdot p) &= M_f^2 + M_i^2 - t \ , \\ 2(p'\cdot q') &= s - M_f^2 - m_f^2 \ , \\ 2(p\cdot q) &= s - M_i^2 - m_i^2 \ , \\ 2(p\cdot q') &= M_i^2 + m_f^2 - u \ , \\ 2(p'\cdot q) &= M_f^2 + m_i^2 - u \ . \end{aligned} \tag{C.63}$$

$$s + u + t = M_f^2 + M_i^2 + m_f^2 + m_i^2 \ . \tag{C.64}$$

$$\begin{aligned}
\not{q} &= \frac{1}{2}(M_f - M_i) + \not{Q} , \\
\not{q}' &= -\frac{1}{2}(M_f - M_i) + \not{Q} , \\
\not{q}\not{q}' &= (M_f + M_i)\not{Q} - \frac{1}{2}\left(M_f^2 + M_i^2\right) + u , \\
\not{q}'\not{q} &= -(M_f + M_i)\not{Q} - \frac{1}{2}\left(M_f^2 + M_i^2\right) + s . \qquad \text{(C.65)}
\end{aligned}$$

C.3.2 Kadyshevsky

In Kadyshevsky formalism there are similar relations

$$\begin{aligned}
2(q' \cdot q) &= m_f^2 + m_i^2 - t_{q'q} , \\
2(p' \cdot p) &= M_f^2 + M_i^2 - t_{p'p} , \\
2(p' \cdot q') &= s_{p'q'} - M_f^2 - m_f^2 , \\
2(p \cdot q) &= s_{pq} - M_i^2 - m_i^2 , \\
2(p \cdot q') &= M_i^2 + m_f^2 - u_{pq'} , \\
2(p' \cdot q) &= M_f^2 + m_i^2 - u_{p'q} . \qquad \text{(C.66)}
\end{aligned}$$

$$\begin{aligned}
s_{p'q'} + s_{pq} + u_{p'q} + u_{pq'} + t_{p'p} + t_{q'q} &= 2\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right) \\
&\quad + (\kappa' - \kappa)^2 , \\
2\sqrt{s_{p'q'}s_{pq}} + u_{p'q} + u_{pq'} + t_{p'p} + t_{q'q} &= 2\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right) . \qquad \text{(C.67)}
\end{aligned}$$

$$\begin{aligned}
\not{q}' &= -\frac{1}{2}(M_f - M_i) + \not{Q} - \frac{1}{2}\not{n}(\kappa' - \kappa) , \\
\not{q} &= \frac{1}{2}(M_f - M_i) + \not{Q} + \frac{1}{2}\not{n}(\kappa' - \kappa) , \\
\not{q}'\not{q} &= -(M_f + M_i)\not{Q} + \frac{1}{2}\left(s_{p'q'} + s_{pq}\right) - \frac{1}{2}\left(M_f^2 + M_i^2\right) \\
&\quad -\frac{1}{2}(\kappa' - \kappa)(p' - p) \cdot n - \frac{1}{2}(\kappa' - \kappa)[\not{n}, \not{Q}] - \frac{1}{2}(\kappa' - \kappa)^2 , \\
\not{q}\not{q}' &= (M_f + M_i)\not{Q} + \frac{1}{2}\left(u_{p'q} + u_{pq'}\right) - \frac{1}{2}\left(M_f^2 + M_i^2\right) \\
&\quad -\frac{1}{2}(\kappa' - \kappa)(p' - p) \cdot n + \frac{1}{2}(\kappa' - \kappa)[\not{n}, \not{Q}] - \frac{1}{2}(\kappa' - \kappa)^2 , \\
\not{n}\not{q}' &= \frac{1}{2}(M_f + M_i)\not{n} - (n \cdot p') + \frac{1}{2}[\not{n}, \not{Q}] + n \cdot Q - \frac{1}{2}(\kappa' - \kappa) ,
\end{aligned}$$

$$\begin{aligned}
\not{n}\not{q} &= -\frac{1}{2}\,(M_f+M_i)\,\not{n}+(n\cdot p')+\frac{1}{2}\,[\not{n},\not{Q}]+n\cdot Q+\frac{1}{2}\,(\kappa'-\kappa)\ , \\
\not{n}\not{q}'\not{q} &= -\frac{1}{2}\,\left(M_f^2+M_i^2\right)\not{n}+\frac{1}{2}\,(s_{p'q'}+s_{pq})\,\not{n}+\frac{1}{2}\,(M_f-M_i)\,[\not{n},\not{Q}] \\
&\quad +(M_f-M_i)\,n\cdot Q-\frac{1}{2}\,(\kappa'-\kappa)n\cdot(p'-p)\not{n}+(\kappa'-\kappa)\,(n\cdot Q)\,\not{n} \\
&\quad -(\kappa'-\kappa)\not{Q}-2n\cdot(p'-p)\not{Q}-\frac{1}{2}\,(\kappa'-\kappa)^2\not{n}\ , \\
\not{n}\not{q}\not{q}' &= -\frac{1}{2}\,\left(M_f^2+M_i^2\right)\not{n}+\frac{1}{2}\,(u_{p'q}+u_{pq'})\,\not{n}-\frac{1}{2}\,(M_f-M_i)\,[\not{n},\not{Q}] \\
&\quad -(M_f-M_i)\,n\cdot Q-\frac{1}{2}\,(\kappa'-\kappa)n\cdot(p'-p)\not{n}-(\kappa'-\kappa)\,(n\cdot Q)\,\not{n} \\
&\quad +(\kappa'-\kappa)\not{Q}+2n\cdot(p'-p)\not{Q}-\frac{1}{2}\,(\kappa'-\kappa)^2\not{n}\ . \qquad \text{(C.68)}
\end{aligned}$$

Part II

Dirac Quantization of Higher Spin Fields

Chapter 7

Introduction

This part of the thesis is about the quantization of higher spin ($1 \leq j \leq 2$) fields and their propagators. Besides the interest in their own, the physical interest in these various fields comes from very different areas in (high energy) physics. The massive spin-1 field is extremely important in the electro-weak part of the Standard Model and in phenomenological One-Boson-Exchange (OBE) models. Needless to mention the physical interest in the photon.

As far as the spin-3/2 field is concerned, ever since the pioneering work of [45] and [46] it has been considered by many authors for several reasons. The spin-3/2 field plays a significant role in low energy hadron scattering, where it appears as a resonance. Also in supergravity (for a review see [47]) and superstring theory the spin-3/2 field plays an important role, since it appears in these theories as a massless gravitino. Besides the role it plays in the tensor-force in OBE-models the spin-2 field mainly appears in (super-) gravity and string theories as the massless graviton.

The quantization of such fields can roughly be divided in three areas: free field quantization, the quantization of the system where it is coupled to (an) auxiliary field(s) and the quantization of an interacting field. The latter area in the spin-3/2 case is known to have problems and inconsistencies (see for instance [48], [49] and [41]). Although very interesting, in this part we will focus our attention on the first two areas.

In chapter 8 we start with the quantization of the massive, free fields. We do this for all spin cases ($j = 1$, $3/2$, 2) at the same time using Dirac's prescription [50]. The inclusion of the spin-1 field case is merely meant to demonstrate Dirac's procedure in a simple case and to have a complete description of higher spin field quantization.

The free spin-3/2 field quantization is in the same line as in references [51, 52, 40, 53]. In [51] the massless free spin-3/2 field was quantized in the transverse gauge. The authors of [52, 40] quantize the massive free theory, which is also what we do. We will follow Dirac's prescription straightforwardly by first determining all Lagrange multipliers and constraints. Afterwards the Dirac bracket (Db) is introduced and we calculate the equal time anti commutation (ETAC) relations among all components of the field. In both [52] and [40, 53] the step to the Dirac bracket is made earlier, without determining all Lagrange multipliers and constraints. In [52] it is mentioned that this involves "technical difficulties and much labor" and in [40, 53] the focus is on the number of constraints and therefore not so much on their specific forms. As a result [52] and [40, 53] both calculate only the ETAC relations between the spatial components of the spin-3/2 field, whereas we obtain them all.

A Dirac constraint analysis of the free spin-2 field can be found for instance in [54, 55, 56]. In these references the massless ([54, 55]) case and massive ([56]) case is considered. We stress, however, that our description of the quantization not only differs from [56] in the sense that the nature of one of the obtained constraints is different, which we will discuss below, also we obtain all constraints and Lagrange multipliers by applying Dirac's procedure straightforward. We present a full analysis of the constrained system. After introducing the Dirac bracket (Db) we give all equal time commutation (ETC) relations between the various components of the spin-2 field.

Having quantized the free theories properly we make use of a free field expansion identity and with these ingredients we obtain the propagators. We notice that they are not explicitly covariant, as is mentioned for instance in [57] for general cases $j \geq 1$.

To cure this problem we are inspired by [58] and allow for auxiliary fields in the free Lagrangian in chapter 9. To be more specific we couple the gauge conditions of the massless cases to auxiliary fields and also allow for mass terms of these auxiliary fields, with which free (gauge) parameters are introduced. As in for instance [58], we obtain a covariant vector field propagator, independently of the choice of the parameter.

In the spin-3/2 case several systems of a spin-3/2 field coupled to auxiliary fields are considered in [59, 60, 61]. In [60, 61] are for several of such systems four dimensional commutation relations obtained. In the only massive case which the authors of [61] consider, two auxiliary fields are introduced to couple (indirectly) to the constraint equations [1] of a spin-3/2 field. The authors of [59] use the Lagrange multiplier [2] method, where this multiplier

[1] $i\partial\!\!\!/\psi = 0$ is a constraint in the sense that it reduces the number of degrees of freedom of a general ψ_μ field. It is not a constraint in the sense of Dirac, since it is a dynamical equation.

[2] These Lagrange multipliers are the ones used in the original sense and are therefore

is coupled to the covariant gauge condition of the massless spin-3/2 field in the Rarita-Schwinger (RS) framework (to be defined below). They notice that the Lagrange multiplier has to be a spinor and in this sense it can also be viewed as an auxiliary field. We follow the same line by coupling our auxiliary field to the above mentioned gauge condition. In [59] the quantization is performed outside the RS framework in order to circumvent the appearance of singularities. We remain within the RS framework and deal with these singularities relying on Dirac's method. Therefore we stay in line with the considerations of chapter 8. A covariant propagator is obtained for one specific choice of the parameter ($b = 0$). This propagator is the same as the one obtained in [59]. We notice that also in [62] a covariant propagator is obtained, but these authors make use of two spin-1/2 fields.

Coupled systems of spin-2 and auxiliary fields were for various reasons considered in for instance [63, 64, 65, 66, 67]. In [64] an auxiliary boson field is coupled to the "De Donder" gauge condition in the Lagrangian which also contains Faddeev-Popov ghosts. In [65] an auxiliary field is coupled to the divergence of the tensor field in such a way that the auxiliary field can be viewed as a Lagrange multiplier. These authors mention that if an other auxiliary field is introduced, coupled to the trace of the tensor field in order to get the other spin-2 condition, four dimensional commutation relations for the tensor field can not be written down. We present a description in which this is possible relying on Dirac's procedure. Also in the tensor field case we obtain a covariant propagator, independently of the choice of the parameter. Most probably a similar procedure of coupling gauge conditions to auxiliary fields in order to obtain a covariant propagator is also applicable for even higher spin ($j > 2$) fields.

Having obtained all the various covariant propagators we discuss several choices of the parameters (if possible) and the massless limits of these propagators. We show that the propagators do not only have a smooth massless limit but that they also connect to the ones obtained in the massless case (including (an) auxiliary field(s)).

When coupled to conserved currents we see that it is possible to obtain the correct massless spin-j propagators carrying only the helicities $\lambda = \pm j_z$. This does not require a choice of the parameter in the spin-1 case, but in the spin-3/2 and in the spin-2 case we have to make the choices $b = 0$ [3] and $c = \pm\infty$. As far as these last two cases is concerned, it is a different situation then taking the massive propagator, couple it to conserved currents and putting the mass to zero as noticed in [68] and [69], respectively. A discussion on the

different then the ones used in Dirac's formalism.

[3]This choice we already made in order to obtain a covariant propagator.

latter matter in (anti)-de Sitter spaces can be found in [70, 71, 72]. We stress however, that in the spin-3/2 and the spin-2 case this limit is only smooth if the massive propagator contains ghosts.

Chapter 8

Free Fields

As mentioned in the introduction we deal with the free theories in this chapter. We start in section 8.1 with the Lagrangians and the equations of motion that can be deduced from them. We explicitly quantize the theories in section 8.2 and calculate the propagators in section 8.3.

8.1 Equations of Motion

As a starting point we take the Lagrangian for free, massive fields ($j = 1, 3/2, 2$). In case of the spin-3/2 there is, according to [53, 73, 74, 75, 76], a class of Lagrangians describing the particularities of a spin-3/2 field. Also in the spin-2 case several authors ([65, 77, 78, 79]) describe a class of Lagrangians (with one or more free parameters) which give the correct Euler-Lagrange equations for a spin-2 field. By taking this spin-2 field to be real and symmetric from the outset only one parameter remains

$$\mathcal{L}_1 = -\frac{1}{2}\left(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu\right) + \frac{1}{2} M_1^2 A^\mu A_\mu \ , \tag{8.1a}$$

$$\begin{aligned}\mathcal{L}_{3/2,A} = \bar{\psi}^\mu \Big[& (i\partial\!\!\!/ - M_{3/2}) g_{\mu\nu} + A(\gamma_\mu i\partial_\nu + \gamma_\nu i \partial_\mu) + B\gamma_\mu i\partial\!\!\!/ \gamma_\nu \\ & + C M_{3/2} \gamma_\mu \gamma_\nu \Big] \psi^\nu \ ,\end{aligned} \tag{8.1b}$$

$$\begin{aligned}\mathcal{L}_{2,A} = & \frac{1}{4}\partial^\alpha h^{\mu\nu} \partial_\alpha h_{\mu\nu} - \frac{1}{2}\partial_\mu h^{\mu\nu}\partial^\alpha h_{\alpha\nu} - \frac{1}{4} B\, \partial_\nu h^\beta_\beta \partial^\nu h^\alpha_\alpha \\ & - \frac{1}{2} A\, \partial_\alpha h^{\alpha\beta} \partial_\beta h^\nu_\nu - \frac{1}{4} M_2^2 h^{\mu\nu} h_{\mu\nu} + \frac{1}{4} C M_2^2 h^\mu_\mu h^\nu_\nu \ ,\end{aligned} \tag{8.1c}$$

where $B = \frac{1}{2}(3A^2 + 2A + 1)$, $C = 3A^2 + 3A + 1$ and $A \neq -\frac{1}{2}$, but arbitrary otherwise. We improperly[1] refer to (8.1b) as the RS case.

Since we do not need to be so general we choose $A = -1$ and end-up with a particular spin-3/2 Lagrangian also used in [47, 51, 52, 40, 53, 61] and in case of the spin-2 field we get the well-know Fierz-Pauli Lagrangian [45] also used in for instance [80, 81, 82]

$$\begin{aligned}
\mathcal{L}_{3/2} &= -\frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\rho\left(\partial_\sigma\psi_\nu\right) + \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\left(\partial_\sigma\bar{\psi}_\mu\right)\gamma_5\gamma_\rho\psi_\nu \\
&\quad - M_{3/2}\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu \ , \qquad (8.2a) \\
\mathcal{L}_2 &= \frac{1}{4}\,\partial^\alpha h^{\mu\nu}\partial_\alpha h_{\mu\nu} - \frac{1}{2}\,\partial_\mu h^{\mu\nu}\partial^\alpha h_{\alpha\nu} - \frac{1}{4}\,\partial_\nu h^\beta_\beta\partial^\nu h^\alpha_\alpha + \frac{1}{2}\,\partial_\alpha h^{\alpha\beta}\partial_\beta h^\nu_\nu \\
&\quad - \frac{1}{4}\,M_2^2 h^{\mu\nu}h_{\mu\nu} + \frac{1}{4}\,M_2^2 h^\mu_\mu h^\nu_\nu \ . \qquad (8.2b)
\end{aligned}$$

Although we have picked particular Lagrangians we can always go back to the general case by redefining the fields in the following sense

$$\begin{array}{ll}
\psi'_\mu = O_\mu^\alpha(A)\psi_\alpha \ , & O_\mu^\alpha(A) = g_\mu^\alpha - \frac{A+1}{2}\,\gamma_\mu\gamma^\alpha \ , \\
h'_{\mu\nu} = O_{\mu\nu}^{\alpha\beta}(A)h_{\alpha\beta} \ , & O_{\mu\nu}^{\alpha\beta}(A) = \frac{1}{2}\left(g_\mu^\alpha g_\nu^\beta + g_\mu^\beta g_\nu^\alpha - (A+1)g_{\mu\nu}g^{\alpha\beta}\right) \ .
\end{array} \qquad (8.3)$$

The transformation in the first line of (8.3) was also mentioned in [53]. Requiring that the transformation matrices in (8.3) are non-singular ($detO \neq 0$) gives again the constraint $A \neq -\frac{1}{2}$.

The Euler-Lagrange equations following from the free field Lagrangians lead to the correct equations of motion (EoM)

$$\begin{array}{llll}
(\Box + M_1^2)A^\mu = 0 \ , & \partial\cdot A = 0 \ , & & \\
(i\partial\!\!\!/ - M_{3/2})\psi_\mu = 0 \ , & \gamma\cdot\psi = 0 \ , & i\partial\cdot\psi = 0 \ , & \\
(\Box + M_2^2)h^{\mu\nu} = 0 \ , & \partial_\mu h^{\mu\nu} = 0 \ , & h^\mu_\mu = 0 \ . & (8.4)
\end{array}$$

The massless versions of the Lagrangians $\mathcal{L}_1$, $\mathcal{L}_{3/2}$ and $\mathcal{L}_2$ [2] exhibit a gauge freedom: they are invariant under the transformations $A^\mu \to A^{\mu\prime} = A^\mu + \partial^\mu\Lambda$, $\psi_\mu \to \psi'_\mu = \psi_\mu + \partial_\mu\epsilon$ and $h^{\mu\nu} \to h^{\mu\nu\prime} = h^{\mu\nu} + \partial^\mu\eta^\nu + \partial^\nu\eta^\mu$ as well as $h^{\mu\nu} \to h^{\mu\nu\prime} = h^{\mu\nu} + \partial^\mu\partial^\nu\Lambda$, respectively. Here, Λ, ϵ and η^μ are scalar, spinor and vector fields, respectively.

In the spin-1 case a popular gauge is the Lorentz gauge $\partial\cdot A = 0$. Imposing this gauge conditions automatically ensures the EoM $\Box A^\mu = 0$ and puts the

[1] Although the authors of [46] mention a general class, they expose one specific Lagrangian which would correspond to the choice $A = -\frac{1}{3}$

[2] The massless version of (8.2b) is the linearized Einstein-Hilbert Lagrangian discussed in many textbooks as for instance [83]

constraint $\Box\Lambda = 0$. This last constraint is used to eliminate the residual helicity state $\lambda = 0$.

A popular gauge in the spin-3/2 case is the covariant gauge $\gamma \cdot \psi = 0$, which causes similar effects, namely the correct EoM $i\partial\!\!\!/\psi = 0$ and $i\partial \cdot \psi = 0$ and the constraint $i\partial\!\!\!/\epsilon = 0$. Since the ϵ-field is a free spinor, it is used to transform away the helicity states $\lambda = \pm 1/2$ of the free ψ_μ field.

Since the spin-2 Lagrangian has two symmetries, two gauge conditions need to be imposed. The gauge conditions $h^\alpha_\alpha = 0$ and $\partial_\alpha h^{\alpha\beta} = 0$ give the correct EoM. From the effects these gauge conditions have on the auxiliary fields ($\Box\eta^\mu = 0$, $\partial \cdot \eta = 0$ and $\Box\Lambda = 0$) we see that these equations describe a massless spin-1 field and a massless spin-0 field. Therefore these fields can be used to ensure that the tensor field $h^{\mu\nu}$ only has $\lambda = \pm 2$ helicity states.

In our case the mass terms in the Lagrangian break the gauge symmetry. Although, the correct EoM (8.4) are obtained the freedom in the choice of the field can not be exploited to transform away helicity states. Therefore, the massive fields contain all helicity states.

8.2 Quantization

For the quantization of our systems we use Dirac's Hamilton method for constrained systems [50]. In case of the (real) vector and tensor fields the accompanying canonical momenta are defined in the usual way. Since we use complex fields in case of the spin-3/2 field we consider ψ_μ and $\psi^\dagger_\mu$ as independent fields being elements of a Grassmann algebra. For the definition of the accompanying canonical momenta we rely on [84]. Although, the authors of [84] use spin-1/2 fields, the prescription for the canonical momenta does not change. The canonical momenta are defined as

$$\pi^\nu_a = \frac{\partial^r \mathcal{L}}{\partial \dot{\psi}_{a,\nu}} \quad , \quad \pi^{\nu\ddagger}_a = \frac{\partial^r \mathcal{L}}{\partial \dot{\psi}^*_{a,\nu}} \ , \tag{8.5}$$

where r means that the differentiation is performed from right to left. We use the ‡-notation to distinguish the canonical momentum coming from the complex conjugate field from the one coming form the original field, since they need not (and in fact will not) be the same.

Using this prescription (8.5) we obtain the canonical momenta from our

Lagrangians (8.1a), (8.2a) and (8.2b)

$$\pi_1^0 = 0 \ , \qquad \pi_1^n = -\dot{A}^n + \partial^n A^0 \ ,$$

$$\pi_{3/2}^0 = 0 \ , \qquad \pi_{3/2}^{0\ \ddagger} = 0 \ ,$$
$$\pi_{3/2}^n = \tfrac{i}{2}\,\psi_k^\dagger \sigma^{kn} \ , \qquad \pi_{3/2}^{n\ \ddagger} = \tfrac{i}{2}\,\sigma^{nk}\psi_k \ ,$$

$$\pi_2^{00} = -\tfrac{1}{2}\,\partial_n h^{n0} \ , \qquad \pi_2^{0m} = -\partial_n h^{nm} + \tfrac{1}{2}\,\partial^m h^{00}$$
$$\pi_2^{nm} = \tfrac{1}{2}\,\dot{h}^{nm} - \tfrac{1}{2}\,g^{nm}\dot{h}_k^k + \tfrac{1}{2}\,g^{nm}\partial_k h^{k0} \ , \qquad + \tfrac{1}{2}\,\partial^m h_n^n \ , \tag{8.6}$$

from which the velocities can be deduced

$$\begin{aligned}
\dot{A}^n &= -\pi_1^n + \partial^n A^0 \ , \\
\dot{h}^{nm} &= 2\pi_2^{nm} - g^{nm}\pi_{2k}^{\ k} + \frac{1}{2}\,g^{nm}\partial_k h^{k0} \ , \\
\dot{h}_k^k &= -\pi_{2k}^{\ k} + \frac{3}{2}\,\partial_k h^{k0} \ ,
\end{aligned} \tag{8.7}$$

and the constraint equations. These constraints are called *primary* constraints

$$\theta_1^0 = \pi_1^0 \ ,$$

$$\theta_{3/2}^0 = \pi_{3/2}^0 \ , \qquad \theta_{3/2}^{0\ \ddagger} = \pi_{3/2}^{0\ \ddagger} \ ,$$
$$\theta_{3/2}^n = \pi_{3/2}^n - \tfrac{i}{2}\,\psi_k^\dagger\sigma^{kn} \ , \qquad \theta_{3/2}^{n\ \ddagger} = \pi_{3/2}^{n\ \ddagger} - \tfrac{i}{2}\,\sigma^{nk}\psi_k \ ,$$

$$\theta_2^{00} = \pi_2^{00} + \tfrac{1}{2}\,\partial_n h^{n0} \ , \qquad \theta_2^{0m} = \pi_2^{0m} + \partial_n h^{nm} - \tfrac{1}{2}\,\partial^m h^{00} - \tfrac{1}{2}\,\partial^m h_n^n \ , \tag{8.8}$$

and they vanish in the weak sense, to which we will come back below.

If we want these constraints to remain zero we impose the time derivative of these constraints to be zero. We find it most easily to define the time derivative via the Poisson bracket (Pb) $\dot{\theta} = \{\theta, H\}_P + \partial\theta/\partial t$ [3]. We, therefore, need the Hamiltonians.

In constructing the Hamiltonians we need to explain the concept of strong and weak equations: a strong equation is, as opposed to a weak equation, an equation that remains to be valid when the relevant quantities (p,q,$\dot{q}$) are varied independently by a small quantity of order ϵ (see [50]). Dirac has shown [50] that the Hamiltonian obtained in the usual way is a weak equation

[3] In practice it will turn out that the constraints are not explicitly dependent on time t

[4] and does not give the correct EoM. This can be repaired by adding the primary constraints (8.8) to the Hamiltonian by means of Lagrange multipliers in order to make it a so-called strong equation. What we get is

$$\begin{aligned}
H_w &= \int d^3x\ \mathcal{H}_w(x) = \int d^3x \left(\sum_i \pi_i \dot{q}_i - \mathcal{L} \right) \ , \\
\mathcal{H}_{1,S} &= -\frac{1}{2}\,\pi_1^n \pi_{1,n} + \pi_1^n \partial_n A_0 + \frac{1}{2}\,\partial_m A_n \partial^m A^n - \frac{1}{2}\,\partial_m A_n \partial^n A^m \\
&\quad -\frac{1}{2}\,M_1^2 A^0 A_0 - \frac{1}{2}\,M_1^2 A^n A_n + \lambda_{1,0}\theta_1^0 \ , \\
\mathcal{H}_{3/2,S} &= \frac{1}{2}\,\epsilon^{\mu\nu\rho k}\bar{\psi}_\mu \gamma_5 \gamma_\rho \left(\partial_k \psi_\nu\right) - \frac{1}{2}\,\epsilon^{\mu\nu\rho k}\left(\partial_k \bar{\psi}_\mu\right)\gamma_5\gamma_\rho\psi_\nu + M_{3/2}\bar{\psi}_\mu \sigma^{\mu\nu}\psi_\nu \\
&\quad +\lambda_{3/2,0}\theta_{3/2}^0 + \lambda_{3/2,n}\theta_{3/2}^n + \lambda_{3/2,0}^\ddagger {\theta_{3/2}^0}^\ddagger + \lambda_{3/2,n}^\ddagger {\theta_{3/2}^n}^\ddagger \ , \\
\mathcal{H}_{2,S} &= \pi_2^{nm}\pi_{2,nm} - \frac{1}{2}\,{\pi_2}_n^n {\pi_2}_m^m + \frac{1}{2}\,{\pi_2}_n^n \partial^m h_{m0} - \frac{1}{2}\,\partial^k h^{n0}\partial_k h_{n0} \\
&\quad -\frac{1}{4}\,\partial^k h^{nm}\partial_k h_{nm} + \frac{1}{8}\,\partial_n h^{n0}\partial^m h_{m0} + \frac{1}{2}\,\partial_n h^{nm}\partial^k h_{km} \\
&\quad +\frac{1}{2}\,\partial_m h^{00}\partial^m h_n^n + \frac{1}{4}\,\partial_m h_n^n \partial^m h_k^k - \frac{1}{2}\,\partial_n h^{nm}\partial_m h_{00} \\
&\quad -\frac{1}{2}\,\partial_n h^{nm}\partial_m h_k^k + \frac{1}{2}\,M_2^2 h^{n0}h_{n0} + \frac{1}{4}\,M_2^2 h^{nm}h_{nm} \\
&\quad -\frac{1}{2}\,M_2^2 h^{00}h_m^m - \frac{1}{4}\,M_2^2 h_n^n h_m^m + \lambda_{2,00}\theta_2^{00} + \lambda_{2,0m}\theta_2^{0m} \ . \qquad (8.9)
\end{aligned}$$

For the definition of the Pb we rely on [51] and [84]. There, it is defined as

$$\{E(x), F(y)\}_P = \left[\frac{\partial^r E(x)}{\partial q_a(x)}\,\frac{\partial^l F(y)}{\partial p^a(y)} - (-1)^{n_E n_F}\frac{\partial^r F(y)}{\partial q_a(y)}\,\frac{\partial^l E(x)}{\partial p^a(x)} \right] \delta^3(x-y) \ , \qquad (8.10)$$

where n_E, n_F is 0 (1) in case $E(x), F(x)$ is even (odd). With this form of the Pb (8.10) we already anticipate that bosons satisfy commutation relations and fermions anti-commutation relations in a quantum theory.

Now, we can impose the time derivatives of the constraints (8.8) to be zero using (8.9) and (8.10)

$$\left\{\theta_1^0(x), H_{1,S}\right\}_P = \partial_n \pi_1^n + M_1^2 A^0 = 0 \equiv \Phi_1^0(x) \ , \qquad (8.11a)$$

[4] In constructing the usual Hamiltonian explicit use can be made of the constraints, since these are also weak equations

$$\begin{aligned}
\left\{\theta^0_{3/2}(x), H_{3/2,S}\right\}_P &= \epsilon^{\mu 0\rho k}\left(\partial_k\bar{\psi}_\mu\right)\gamma_5\gamma_\rho - M_{3/2}\bar{\psi}_\mu\sigma^{\mu 0} = 0 \\
&\equiv -\Phi^0_{3/2}{}^\ddagger(x)\ , && (8.11b) \\
\left\{\theta^0_{3/2}{}^\ddagger(x), H_{3/2,S}\right\}_P &= -\epsilon^{\mu 0\rho k}\gamma^0\gamma_5\gamma_\rho\left(\partial_k\psi_\mu\right) + M_{3/2}\gamma^0\sigma^{0\mu}\psi_\mu = 0 \\
&\equiv -\Phi^0_{3/2}(x)\ , && (8.11c) \\
\left\{\theta^n_{3/2}(x), H_{3/2,S}\right\}_P &= \epsilon^{\mu n\rho k}\left(\partial_k\bar{\psi}_\mu\right)\gamma_5\gamma_\rho - M_{3/2}\bar{\psi}_\mu\sigma^{\mu n} \\
&\quad + i\lambda^\ddagger_{3/2,k}\sigma^{kn} = 0\ , && (8.11d) \\
\left\{\theta^n_{3/2}{}^\ddagger(x), H_{3/2,S}\right\}_P &= -\epsilon^{\mu n\rho k}\gamma^0\gamma_5\gamma_\rho\left(\partial_k\psi_\mu\right) + M_{3/2}\gamma^0\sigma^{n\mu}\psi_\mu \\
&\quad + i\sigma^{nk}\lambda_{3/2,k} = 0\ , && (8.11e)
\end{aligned}$$

$$\begin{aligned}
\left\{\theta^{00}_2(x), H_{2,S}\right\}_P &= \frac{1}{2}\left[\left(\partial^k\partial_k + M_2^2\right)h^m_m - \partial_n\partial_m h^{nm}\right] = 0 \\
&\equiv \frac{1}{2}\,\Phi^0_2(x)\ , && (8.11f) \\
\left\{\theta^{0m}_2(x), H_{2,Tot}\right\}_P &= 2\partial_k\pi_2^{km} - \left(\partial^k\partial_k + M_2^2\right)h^{0m} = 0 \\
&\equiv \Phi^m_2(x)\ . && (8.11g)
\end{aligned}$$

[5] In two cases ((8.11d) and (8.11e)) Lagrange multipliers are determined. In all other cases new constraints are obtained. These are called *secondary* constraints. We also impose the time derivatives of these secondary constraints to be zero

$$\left\{\Phi^0_1(x), H_{1,S}\right\}_P = M_1^2(\partial_n A^n + \lambda^0_1) = 0\ , \qquad (8.12a)$$

$$\begin{aligned}
\left\{\Phi^0_{3/2}(x), H_{3/2,S}\right\}_P &= \sigma^{nk} i\partial_n\lambda_{3/2,k} + M_{3/2}\gamma^k\lambda_{3/2,k} = 0\ , && (8.12b) \\
\left\{\Phi^0_{3/2}{}^\ddagger(x), H_{3/2,S}\right\}_P &= i\partial_k\lambda^\ddagger_{3/2,n}\sigma^{nk} + M_{3/2}\lambda^\ddagger_{3/2,k}\gamma^k = 0\ , && (8.12c)
\end{aligned}$$

$$\begin{aligned}
\left\{\Phi^0_2(x), H_{2,S}\right\}_P &= -2\partial_n\partial_m\pi_2^{nm} - M^2\pi_2{}^n_n + \left(\partial^k\partial_k + \frac{3}{2}\,M_2^2\right)\partial^n h_{n0} \\
&= 0 \equiv -\Phi^{(1)}_2(x)\ , && (8.12d) \\
\left\{\Phi^m_2(x), H_{2,S}\right\}_P &= -M_2^2\left[\lambda^{0m}_2 + \partial_k h^{km} - \partial^m h^{00} - \partial^m h^n_n\right] \\
&= 0\ . && (8.12e)
\end{aligned}$$

The first line (8.12a) determines the Lagrange multiplier λ^0_1. Since this was the only Lagrange multiplier in the spin-1 case all Lagrange multipliers of

[5] If Φ is a constraint, then so is $a\Phi$. The constants in front of the constraints in (8.11) are chosen for convenience and have no physical meaning.

this case are determined and therefore all constraints are said to be *second class* constraints.

In a situation where all constraints are second class every constraint has at least one non-vanishing Pb with another constraint. If there is a constraint that has non-vanishing Pb's with all other constraints, this constraint is said to be *first class*. In such a situation there is also an undetermined Lagrange multiplier.

Equation (8.12e) determines the Lagrange multiplier λ_2^{0m} and equation (8.12d) brings about yet another (tertiary) constraint. Its vanishing time derivative yields

$$\left\{\Phi_2^{(1)}(x), H_{2,S}\right\}_P = M_2^2\left[\left(2\partial^k\partial_k + \frac{3}{2}M_2^2\right)h^{00} + \left(\frac{3}{2}\partial^k\partial_k + M_2^2\right)h_n^n - \frac{3}{2}\partial_n\partial_m h^{nm} - 2\partial_n\lambda_2^{n0}\right] = 0\ . \tag{8.13}$$

We see that we have in the spin-3/2 case as well as in the spin-2 case two equations involving the same Lagrange multipliers. In the spin-3/2 case these are (8.11e) and (8.12b) for $\lambda_{3/2,k}$ and (8.11d) and (8.12c) for $\lambda^\ddagger_{3/2,k}$. In the spin-2 case these are (8.12e) and (8.13) for λ_2^{n0}. Combining these equations for consistency, and using $\Phi^0_{3/2}$, ${\Phi^0_{3/2}}^\ddagger$ as well as Φ_2^0 as weakly vanishing constraints, yields the last constraints

$$\Phi^{(1)}_{3/2} = \gamma^0\psi_0 + \gamma^k\psi_k\ , \tag{8.14a}$$

$$\Phi^{(1)\,\ddagger}_{3/2} = -\psi_0^\dagger\gamma^0 + \psi_k^\dagger\gamma^k\ , \tag{8.14b}$$

$$\Phi_2^{(2)} = h_0^0 + h_n^n\ , \tag{8.14c}$$

It is important to note that these constraints are only obtained when combining other results, as describes above. This is not done in [56]. Therefore these authors do not find $\Phi_2^{(2)}$, leaving θ_2^{00} as a first class constraint. Imposing vanishing time derivatives of these constraints ((8.14a)-(8.14c))

$$\begin{aligned}
\left\{\Phi^{(1)}_{3/2}(x), H_{3/2,S}\right\}_P &= -\gamma^0\lambda_{3/2,0} - \gamma^k\lambda_{3/2,k} = 0\ ,\\
\left\{\Phi^{(1)\,\ddagger}_{3/2}(x), H_{3/2,S}\right\}_P &= \lambda^\ddagger_{3/2,0}\gamma^0 - \lambda^\ddagger_{3/2,k}\gamma^k = 0\ ,\\
\left\{\Phi_2^{(2)}(x), H_{2,S}\right\}_P &= \lambda_2^{00} - \pi_{2k}^{\ k} + \frac{3}{2}\partial_k h^{k0} = 0\ ,
\end{aligned} \tag{8.15}$$

determines the last Lagrange multipliers $\lambda_{3/2,0}$, $\lambda^{\ddagger}_{3/2,0}$ and λ_2^{00}.

In the massless spin-1 case the vanishing of the time-derivative of $\Phi_1^0(x)$ would automatically be satisfied as can be seen from (8.12a). In this case λ_1^0 would not be determined which means that both constraints are first class.

We notice that in combining the equations that involve $\lambda_{3/2,k}$ ((8.11e), (8.12b)) and $\lambda^{\ddagger}_{3/2,k}$ ((8.11d), (8.12c)) we obtain the constraints $\Phi^{(1)}_{3/2}$ and $\Phi^{(1)\,\ddagger}_{3/2}$ being proportional to $M^2_{3/2}$. This means that in the massless case these equations are already consistent with each other and that $\lambda_{3/2,0}$ and $\lambda^{\ddagger}_{3/2,0}$ can not be determined leaving $\theta^0_{3/2}$ and $\theta^0_{3/2}{}^{\ddagger}$ to be a first class constraint ([51])[6].

The situation in the massless spin-2 case is even more clear. From (8.12e) and (8.13) it is evident that the time derivatives of Φ_2^m and $\Phi_2^{(1)}$ will already be zero and that λ_2^{0k} can not be determined. Therefore $\Phi_2^{(2)}$ will not be obtained from which λ_2^{00} also can not be determined, leaving θ_2^{00} and θ_2^{0n} to be first class constraints ([54, 55]) [7].

The fact that there are first class constraints (or undetermined Lagrange multipliers) in the massless cases is a reflection of the gauge symmetry. In the spin-1 and the spin-3/2 case only one Lagrange multiplier is undetermined meaning there is only one gauge symmetry (of course the massless spin-3/2 action is also invariant under the hermitian gauge transformation, that is why $\lambda^{\ddagger}_{3/2,k}$ is also undetermined). In the massless spin-2 case, however, there are two Lagrange multipliers undetermined, meaning that there are two gauge symmetries as we have mentioned before.

In the massive cases all Lagrange multipliers can be determined, which means that all constraints are second class. Therefore every constraint has at least one non-vanishing Pb with another constraint. The complete set of con-

[6] In this case also $\partial_n\theta^n_{3/2}$ and $\partial_n\theta^n_{3/2}{}^{\ddagger}$ become first class.

[7] Actually all constraints become first class.

straints (primary, secondary, ...) is

$$\begin{array}{ll}
\theta_1^0 = \pi_1^0 \ , & \Phi_1^0 = \partial_n \pi_1^n + M_1^2 A^0 \ , \\
 & \\
\theta_{3/2}^0 = \pi_{3/2}^0 \ , & \theta_{3/2}^{0\ \ddagger} = \pi^{0\ddagger} \ , \\
\Phi_{3/2}^{(1)} = \gamma \cdot \psi \ , & \Phi_{3/2}^{(1)\ \ddagger} = -\psi_0^\dagger \gamma^0 + \psi_k^\dagger \gamma^k \ , \\
\theta_{3/2}^n = \pi_{3/2}^n - \frac{i}{2}\, \psi_k^\dagger \sigma^{kn} \ , & \theta_{3/2}^{n\ \ddagger} = \pi^{n\ddagger} - \frac{i}{2}\, \sigma^{nk} \psi_k \ , \\
\Phi_{3/2}^0 = -i \partial_k \sigma^{kl} \psi_l - M_{3/2} \gamma^k \psi_k \ , & \Phi_{3/2}^{0\ \ddagger} = -\psi_n^\dagger \sigma^{nk} i \overleftarrow{\partial_k} - M_{3/2} \psi_k^\dagger \gamma^k \ , \\
 & \\
\theta_2^{00} = \pi_2^{00} + \frac{1}{2}\, \partial_n h^{n0} \ , & \Phi_2^0 = \left(\partial^k \partial_k + M_2^2\right) h_m^m - \partial_n \partial_m h^{nm} \ , \\
\theta_2^{0m} = \pi_2^{0m} + \partial_n h^{nm} - \frac{1}{2}\, \partial^m h^{00} & \Phi_2^m = 2\partial_k \pi^{km} - (\partial^k \partial_k + M_2^2) h^{0m} \ , \\
\qquad - \frac{1}{2}\, \partial^m h_n^n \ , & \Phi_2^{(2)} = h_0^0 + h_n^n \ , \\
\Phi_2^{(1)} = 2\partial_n \partial_m \pi_2^{nm} + M_2^2 \pi_{2n}^{\ n} & \\
\qquad - \left(\partial^k \partial_k + \frac{3}{2}\, M_2^2\right) \partial^n h_{n0} \ , &
\end{array} \tag{8.16}$$

We want to make linear combinations of constraints in order to reduce the number of non-vanishing Pb among these constraints. In the end we will arrive at a situation where every constraint has only one non-vanishing Pb with another constraint. Therefore, we make the following linear combinations

$$\begin{aligned}
\tilde{\theta}_{3/2}^n &= \theta_{3/2}^n - \theta_{3/2}^0 \gamma_0 \gamma^n = \pi_{3/2}^n - \pi_{3/2}^0 \gamma_0 \gamma^n - \frac{i}{2}\, \psi_k^\dagger \sigma^{kn} \ , \\
\tilde{\Phi}_{3/2}^0 &= \Phi_{3/2}^0 + \left(-\partial_m + \frac{i}{2}\, M_{3/2} \gamma_m\right) \tilde{\theta}_{3/2}^m \\
&= -\partial_m \pi_{3/2}^{m\ \ddagger} + \frac{i}{2}\, M_{3/2} \gamma_m \pi_{3/2}^{m\ \ddagger} - \partial_m \gamma^m \gamma^0 \pi^{0\ddagger} + \frac{3i}{2}\, M_{3/2} \gamma_0 \pi_{3/2}^{0\ \ddagger} \\
&\quad - \frac{i}{2} \partial_k \sigma^{km} \psi_m - \frac{1}{2}\, M_{3/2} \gamma^k \psi_k \ , \\
\tilde{\theta}^{n\ddagger} &= \theta_{3/2}^{n\ \ddagger} + \gamma^n \gamma^0 \theta_{3/2}^{0\ \ddagger} = \pi_{3/2}^{n\ \ddagger} + \gamma^n \gamma^0 \pi_{3/2,0}^\ddagger - \frac{i}{2}\, \sigma^{nk} \psi_k \ , \\
\tilde{\Phi}_{3/2}^{0\ddagger} &= \Phi_{3/2}^{0\ \ddagger} + \tilde{\theta}_{3/2}^{m\ddagger} \left(-\overleftarrow{\partial}_m + \frac{i}{2}\, M_{3/2} \gamma_m\right) \\
&= -\partial_m \pi_{3/2}^m + \frac{i}{2}\, M_{3/2} \pi_{3/2}^m \gamma_m + \partial_m \pi_{3/2}^0 \gamma_0 \gamma^m - \frac{3i}{2}\, M_{3/2} \pi_{3/2}^0 \gamma_0 \\
&\quad - \frac{i}{2}\, \partial_k \psi_m^\dagger \sigma^{mk} - \frac{1}{2}\, M_{3/2} \psi_k^\dagger \gamma^k \ ,
\end{aligned}$$

$$\begin{aligned}
\tilde{\Phi}_2^n &= \Phi_2^n - 2\partial^n \theta_2^{00} = 2\partial_k \pi_2^{kn} - 2\partial^n \pi_2^{00} - \left(\partial^k \partial_k + M_2^2\right) h^{0n} - \partial^n \partial_k h^{0k} \ , \\
\tilde{\Phi}_2^0 &= \Phi_2^0 + 2\partial_n \theta_2^{n0} = 2\partial_n \pi_2^{0n} + \partial_n \partial_m h^{nm} - \partial^k \partial_k h^{00} + M_2^2 h_k^k \ ,
\end{aligned}$$

$$\begin{aligned}\tilde{\Phi}_2^{(1)} &= \Phi_2^{(1)} - (2\partial^k\partial_k + 3M_2^2)\theta_2^{00} - 2\partial_n\tilde{\Phi}_2^n \\ &= -2\partial_n\partial_m\pi_2^{nm} + M_2^2\pi_{2k}^{\ k} + 2\partial^k\partial_k\pi_2^{00} - 3M_2^2\pi_2^{00} + \left(2\partial^k\partial_k - M_2^2\right)\partial_n h^{0n} \ .\end{aligned} \tag{8.17}$$

The remaining non-vanishing Pb's are

$$\begin{aligned}\left\{\theta_1^0(x), \Phi_1^0(y)\right\}_P &= -M_1^2\delta^3(x-y) \ , \\ \left\{\tilde{\theta}_{3/2}^n(x), \tilde{\theta}_{3/2}^{m\ddagger}(y)\right\}_P &= -i\sigma^{mn}\delta^3(x-y) \ , \\ \left\{\tilde{\Phi}_{3/2}^0(x), \tilde{\Phi}_{3/2}^{0\ddagger}(y)\right\}_P &= -\frac{3i}{2}M_{3/2}^2\delta^3(x-y) \ , \\ \left\{\theta_{3/2}^0(x), \Phi_{3/2}^{(1)\ \ddagger}(y)\right\}_P &= \gamma^0\delta^3(x-y) \ , \\ \left\{\theta_2^{00}(x), \Phi_2^{(2)}(y)\right\}_P &= -\delta^3(x-y) \ , \\ \left\{\tilde{\Phi}_2^0(x), \tilde{\Phi}_2^{(1)}(y)\right\}_P &= 3M_2^4\,\delta^3(x-y) \ , \\ \left\{\theta_2^{0n}(x), \tilde{\Phi}_2^m(y)\right\}_P &= M_2^2 g^{nm}\,\delta^3(x-y) \ .\end{aligned} \tag{8.18}$$

In a proper (quantum) theory we want the constraint to vanish. Although, here, they vanish in the weak sense there still exist non-vanishing Pb relations among them. This means in a quantum theory that ETC and ETAC relations exist among the constraints. We, therefore, introduce the new Pb à la Dirac [50]: The Dirac bracket (Db), such that the Db among the constraints vanishes

$$\begin{aligned}\{E(x), F(y)\}_D &= \{E(x), F(y)\}_P - \int d^3z_z d^3z_2\,\{E(x), \theta_a(z_1)\}_P \\ &\quad \times C_{ab}(z_1 - z_2)\,\{\theta_b(z_2), F(y)\}_P \ ,\end{aligned} \tag{8.19}$$

where the inverse functions $C_{ab}(z_1 - z_2)$ are defined as follows

$$\int d^3z\,\{\theta_a(x), \theta_c(z)\}_P\,C_{cb}(z-y) = \delta_{ab}\delta^3(x-y) \ , \tag{8.20}$$

and can be deduced from (8.18).

The ETC and ETAC relations are obtained by multiplying the Db by a factor of i [8]. What we get is

$$\begin{aligned}
\left[A^0(x), A^n(y)\right]_0 &= \frac{i\partial^n}{M_1^2}\,\delta^3(x-y)\ , \\
\left[\dot{A}^0(x), A^0(y)\right]_0 &= -\frac{i}{M_1^2}\,\partial^n\partial_n\,\delta^3(x-y)\ , \\
\left[\dot{A}^n(x), A^m(y)\right]_0 &= i\left(g^{nm}+\frac{\partial^n\partial^m}{M_1^2}\right)\delta^3(x-y)\ ,
\end{aligned}$$

$$\begin{aligned}
\left\{\psi^0(x), \psi^{0\dagger}(y)\right\}_0 &= -\frac{2}{3M_{3/2}^2}\,\nabla^2\,\delta^3(x-y)\ , \\
\left\{\psi^0(x), \psi^{m\dagger}(y)\right\}_0 &= \frac{1}{M_{3/2}}\left[\frac{2}{3M_{3/2}}\left(i\gamma^k\partial_k\right)\gamma^0 i\partial^m+\frac{1}{3}\left(i\gamma^k\partial_k\right)\gamma^0\gamma^m\right. \\
&\qquad \left.+\,\gamma^0 i\partial^m\right]\delta^3(x-y)\ , \\
\left\{\psi^n(x), \psi^{0\dagger}(y)\right\}_0 &= \frac{1}{M_{3/2}}\left[\frac{2}{3M_{3/2}}\left(i\gamma^k\partial_k\right)i\partial^n\gamma^0+\frac{1}{3}\gamma^n\gamma^0\left(i\gamma^k\partial_k\right)+i\partial^n\gamma^0\right] \\
&\qquad \times\,\delta^3(x-y)\ , \\
\left\{\psi^n(x), \psi^{m\dagger}(y)\right\}_0 &= -\left[g^{nm}-\frac{1}{3}\,\gamma^n\gamma^m+\frac{2}{3M_{3/2}^2}\,\partial^n\partial^m+\frac{1}{3M_{3/2}}\left(\gamma^n i\partial^m\right.\right. \\
&\qquad \left.\left.-\,i\partial^n\gamma^m\right)\right]\delta^3(x-y)\ ,
\end{aligned}$$

$$\begin{aligned}
\left[h^{00}(x), h^{0l}(y)\right]_0 &= \frac{4i}{3M_2^4}\,\partial^j\partial_j\partial^l\delta^3(x-y)\ , \\
\left[h^{0m}(x), h^{kl}(y)\right]_0 &= \frac{-i}{M_2^2}\left[\frac{4}{3M^2}\,\partial^m\partial^k\partial^l-\frac{2}{3}\,\partial^m g^{kl}+\partial^k g^{ml}+\partial^l g^{mk}\right] \\
&\qquad \times\,\delta^3(x-y)\ , \\
\left[\dot{h}^{00}(x), h^{00}(y)\right]_0 &= -\frac{4i}{3M_2^4}\,\partial^i\partial_i\partial^j\partial_j\delta^3(x-y)\ , \\
\left[\dot{h}^{0m}(x), h^{0l}(y)\right]_0 &= \frac{i}{M_2^2}\left[\frac{4}{3M_2^2}\,\partial^m\partial^l\,\partial^j\partial_j+\frac{1}{3}\,\partial^m\partial^l+\partial^j\partial_j g^{ml}\right]\delta^3(x-y)\ ,
\end{aligned}$$

[8] Of course, this is not the only step to be made when passing to a quantum theory. Also the fields should be regarded as state operators, etc.

$$
\begin{aligned}
\left[\dot{h}^{00}(x), h^{kl}(y)\right]_0 &= \frac{i}{M_2^2}\left[\frac{4}{3M_2^2}\,\partial^k\partial^l\,\partial^j\partial_j + 2\partial^k\partial^l - \frac{2}{3}\,\partial^j\partial_j g^{kl}\right]\delta^3(x-y)\ , \\
\left[\dot{h}^{nm}(x), h^{kl}(y)\right]_0 &= i\left[-g^{nk}g^{ml} - g^{nl}g^{mk} + \frac{2}{3}\,g^{nm}g^{kl}\right. \\
&\quad - \frac{1}{M_2^2}\left(\partial^n\partial^k g^{ml} + \partial^m\partial^k g^{nl} + \partial^n\partial^l g^{mk} + \partial^m\partial^l g^{nk}\right) \\
&\quad \left. + \frac{2}{3M_2^2}\left(\partial^n\partial^m g^{kl} + g^{nm}\partial^k\partial^l\right) - \frac{4}{3M_2^2}\,\partial^n\partial^m\partial^k\partial^l\right] \\
&\quad \times \delta^3(x-y)\ .
\end{aligned}
\tag{8.21}
$$

This concludes the quantization of free, massive higher spin ($j = 1,\ 3/2,\ 2$) fields. As a final remark we notice that the ET(A)C relations in (8.21) amongst the various components of the spin-3/2, spin-2 field and their velocities are independent of the choice of the parameter A in (8.1).

8.3 Propagators

Having quantized the free fields in the previous section (section 8.2) we now want to obtain the propagators. In order to do so we need to calculate the commutation relations for non-equal times, which is done using the following identities as solutions to the field equations (first column of (8.4))

$$
\begin{aligned}
A^\mu(x) &= \int d^3z\,\left[\partial_0^z\Delta(x-z;M_1^2)A^\mu(z) - \Delta(x-z;M_1^2)\partial_0^z A^\mu(z)\right]\ , \\
\psi^\mu(x) &= i\int d^3z(i\not{\partial}_x + M_{3/2})\gamma_0\Delta(x-z;M_{3/2}^2)\psi^\mu(z)\ , \\
h^{\mu\nu}(x) &= \int d^3z\,\left[\partial_0^z\Delta(x-z;M_2^2)h^{\mu\nu}(z) - \Delta(x-z;M_2^2)\partial_0^z h^{\mu\nu}(z)\right]\ .
\end{aligned}
\tag{8.22}
$$

Using these equations (8.22) and the ETC and ETAC relations we obtained before (8.21) we calculate the commutation relations for unequal times

$$
\begin{aligned}
[A^\mu(x), A^\nu(y)] &= -i\left(g^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{M_1^2}\right)\Delta(x-y;M_1^2) \\
&= P_1^{\mu\nu}(\partial)\,i\Delta(x-y;M_1^2)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\left\{\psi^\mu(x), \bar\psi^\nu(y)\right\} &= -i\left(i\partial\!\!\!/ + M_{3/2}\right)\Bigg[g^{\mu\nu} - \frac{1}{3}\,\gamma^\mu\gamma^\nu + \frac{2\partial^\mu\partial^\nu}{3M_{3/2}^2} \\
&\quad -\frac{1}{3M_{3/2}}\left(\gamma^\mu i\partial^\nu - \gamma^\nu i\partial^\mu\right)\Bigg]\,\Delta(x-y;M_{3/2}^2) \\
&= \left(i\partial\!\!\!/ + M_{3/2}\right)P_{3/2}^{\mu\nu}(\partial)\,i\Delta(x-y;M_{3/2}^2)\ , \\
\left[h^{\mu\nu}(x), h^{\alpha\beta}(y)\right] &= i\Bigg[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - \frac{2}{3}\,g^{\mu\nu}g^{\alpha\beta} \\
&\quad + \frac{1}{M_2^2}\left(\partial^\mu\partial^\alpha g^{\nu\beta} + \partial^\nu\partial^\alpha g^{\mu\beta} + \partial^\mu\partial^\beta g^{\nu\alpha} + \partial^\nu\partial^\beta g^{\mu\alpha}\right) \\
&\quad - \frac{2}{3M_2^2}\left(\partial^\mu\partial^\nu g^{\alpha\beta} + g^{\mu\nu}\partial^\alpha\partial^\beta\right) + \frac{4}{3M_2^2}\,\partial^\mu\partial^\nu\partial^\alpha\partial^\beta\Bigg] \\
&\quad \times\Delta(x-y;M_2^2) = 2P_2^{\mu\nu\alpha\beta}(\partial)\,i\Delta(x-y;M_2^2)\ , \qquad (8.23)
\end{aligned}
$$

where the $P_j(\partial)$, $j = 1,\, 3/2,\, 2$ are the (on mass shell) spin projection operators. The factor 2 in the last line of (8.23) can be transformed away by redefining the spin-2 field. (8.23) yields for the propagators

$$
\begin{aligned}
D_F^{\mu\nu}(x-y) &= -i < 0|T\left[A^\mu(x)A^\nu(y)\right]|0> \\
&= -i\theta(x^0-y^0)P_1^{\mu\nu}(\partial)\Delta^{(+)}(x-y;M_1^2) \\
&\quad -i\theta(y^0-x^0)P_1^{\mu\nu}(\partial)\Delta^{(-)}(x-y;M_1^2) \\
&= P_1^{\mu\nu}(\partial)\Delta_F(x-y;M_1^2) - i\delta_0^\mu\delta_0^\nu\,\delta^4(x-y)\ . \qquad (8.24)
\end{aligned}
$$

$$
\begin{aligned}
S_F^{\mu\nu}(x-y) &= -i < 0|T\left(\psi^\mu(x)\bar\psi^\nu(y)\right)|0> \\
&= -i\theta(x^0-y^0)\left(i\partial\!\!\!/ + M_{3/2}\right)P_{3/2}^{\mu\nu}(\partial)\Delta^{(+)}(x-y;M_{3/2}^2) \\
&\quad -i\theta(y^0-x^0)\left(i\partial\!\!\!/ + M_{3/2}\right)P_{3/2}^{\mu\nu}(\partial)\Delta^{(-)}(x-y;M_{3/2}^2) \\
&= \left(i\partial\!\!\!/ + M_{3/2}\right)P_{3/2}^{\mu\nu}(\partial)\Delta_F(x-y;M_{3/2}^2) \\
&\quad -\gamma_0\left[\frac{2}{3M_{3/2}^2}\left(\delta_0^\mu\delta_m^\nu + \delta_0^\nu\delta_m^\mu\right)i\partial^m + \frac{1}{3M_{3/2}}\left(\delta_m^\mu\delta_0^\nu - \delta_m^\nu\delta_0^\mu\right)\gamma^m\right] \\
&\qquad \times\,\delta^4(x-y) \\
&\quad -\frac{2}{3M_{3/2}^2}\left(i\partial\!\!\!/ + M_{3/2}\right)\delta_0^\mu\delta_0^\nu\delta^4(x-y)\ . \qquad (8.25)
\end{aligned}
$$

$$
\begin{aligned}
&D_F^{\mu\nu\alpha\beta}(x-y) = -i < 0|T\left[h^{\mu\nu}(x)h^{\alpha\beta}(y)\right]|0> \\
= &\ -i\theta(x^0-y^0)2P_2^{\mu\nu\alpha\beta}(\partial)\Delta^{(+)}(x-y;M_2^2) \\
&\ -i\theta(y^0-x^0)2P_2^{\mu\nu\alpha\beta}(\partial)\Delta^{(-)}(x-y;M_2^2)
\end{aligned}
$$

$$
\begin{aligned}
&= 2P_2^{\mu\nu\alpha\beta}(\partial)\Delta_F(x-y;M_2^2) \\
&+\frac{1}{M_2^2}\Big[\delta_0^\mu\delta_0^\alpha g^{\nu\beta}+\delta_0^\nu\delta_0^\alpha g^{\mu\beta}+\delta_0^\mu\delta_0^\beta g^{\nu\alpha}+\delta_0^\nu\delta_0^\beta g^{\mu\alpha} \\
&-\frac{2}{3}\left(\delta_0^\mu\delta_0^\nu g^{\alpha\beta}+g^{\mu\nu}\delta_0^\alpha\delta_0^\beta\right)+\frac{4}{3}\Big(\delta_0^\mu\delta_0^\nu\delta_0^\alpha\delta_0^\beta(\partial^0\partial_0-\partial^k\partial_k-M_2^2) \\
&+\delta_0^\mu\delta_0^\nu\delta_0^\alpha\delta_b^\beta\partial^0\partial^b+\delta_0^\mu\delta_0^\nu\delta_a^\alpha\delta_0^\beta\partial^0\partial^a+\delta_0^\mu\delta_n^\nu\delta_0^\alpha\delta_0^\beta\partial^0\partial^n+\delta_m^\mu\delta_0^\nu\delta_0^\alpha\delta_0^\beta\partial^0\partial^m \\
&+\delta_0^\mu\delta_0^\nu\delta_a^\alpha\delta_b^\beta\partial^a\partial^b+\delta_0^\mu\delta_n^\nu\delta_0^\alpha\delta_b^\beta\partial^n\partial^b+\delta_m^\mu\delta_0^\nu\delta_0^\alpha\delta_b^\beta\partial^m\partial^b+\delta_0^\mu\delta_n^\nu\delta_a^\alpha\delta_0^\beta\partial^n\partial^a \\
&+\delta_m^\mu\delta_0^\nu\delta_a^\alpha\delta_0^\beta\partial^m\partial^a+\delta_m^\mu\delta_n^\nu\delta_0^\alpha\delta_0^\beta\partial^m\partial^n\Big)\Big]\,\delta^4(x-y)\ . \qquad (8.26)
\end{aligned}
$$

The use of $\Delta^{(+)}(x-y)$ and $\Delta^{(-)}(x-y)$ is similar to what is written in [12] in case of scalar fields

$$
\begin{aligned}
< 0|\phi(x)\phi(y)|0 > &= \Delta^{(+)}(x-y)\ , \\
< 0|\phi(y)\phi(x)|0 > &= \Delta^{(-)}(x-y)\ . \qquad (8.27)
\end{aligned}
$$

As can be seen from ((8.24)-(8.26)) the propagators are not covariant; they contain non-covariant, local terms, as is mentioned in for instance [57].

Chapter 9

Auxiliary Fields

The goal of this chapter is to come to covariant propagators. The way we do this is to introduce auxiliary fields. Since we also allow for mass terms we have extra parameters which can be seen as gauge parameters. We discuss certain choices of these parameters. Also we discuss the massless limits of the propagators in section 9.4 and give momentum representations of the fields in section 9.5. Apart from that, the organization of this chapter is exactly the same as the previous one (chapter 8).

9.1 Equations of Motion

As a starting point we take the Lagrangians (8.1a), (8.2a) and (8.2b). To these Lagrangians we add auxiliary fields coupled to the gauge conditions of the massless theory, as discussed in the text below (8.4). We also allow for mass terms of these auxiliary fields, which introduces parameters to be seen as gauge parameters

$$\mathcal{L}_B = \mathcal{L}_1 + M_1 B\partial^\mu A_\mu + \frac{1}{2}\, a M_1^2 B^2 \ , \tag{9.1a}$$

$$\mathcal{L}_\chi = \mathcal{L}_{3/2} + M_{3/2}\bar{\chi}\gamma^\mu\psi_\mu + M_{3/2}\bar{\psi}_\mu\gamma^\mu\chi + bM_{3/2}\bar{\chi}\chi \ , \tag{9.1b}$$

$$\mathcal{L}_{\eta\epsilon} = \mathcal{L}_2 + M_2\partial_\mu h^{\mu\nu}\eta_\nu + M_2^2 h^\mu_\mu\epsilon + \frac{1}{2}\, cM_2^2\eta^\mu\eta_\mu \ . \tag{9.1c}$$

In (9.1c) we did not allow for a mass term for the ϵ field. We will come back to this point below.

These Lagrangians ((9.1a)-(9.1c)) lead to the following EoM's.

$$\begin{aligned} \left(\Box + M_1^2\right) A^\mu &= (1-a)M_1\partial^\mu B \ , \\ \left(\Box + M_B^2\right)\left(\Box + M_1^2\right) A^\mu &= 0 \ , \\ \left(\Box + M_B^2\right) B &= 0 \ , \end{aligned} \tag{9.2}$$

where $M_B = aM_1^2$. Furthermore we have the constraint relation $\partial^\mu A_\mu = -aM_1 B$.

$$\begin{aligned}
\left(i\partial\!\!\!/ - M_{3/2}\right)\psi_\mu &= -\frac{b+2}{2}\,M_{3/2}\gamma_\mu\chi - bi\partial_\mu\chi\ ,\\
\left(i\partial\!\!\!/ + M_\chi\right)\left(i\partial\!\!\!/ - M_{3/2}\right)\psi_\mu &= -(3b^2+5b+2)M_{3/2}i\partial_\mu\chi\ ,\\
\left(\Box + M_\chi^2\right)\left(i\partial\!\!\!/ - M_{3/2}\right)\psi_\mu &= 0\ ,\\
\left(i\partial\!\!\!/ - M_\chi\right)\chi &= 0\ ,
\end{aligned} \tag{9.3}$$

where $M_\chi = (3b/2+2)M_{3/2}$. The auxiliary field is related to the original spin-3/2 field via the equations $\gamma\cdot\psi = -b\chi$ and $i\partial\cdot\psi = -\frac{1}{2}\,(1+b)(3b+4)M_{3/2}\chi$.

$$\begin{aligned}
\left(\Box + M_2^2\right)h^{\mu\nu} &= -\,(1+c)\,M_2\,(\partial^\mu\eta^\nu + \partial^\nu\eta^\mu)\\
&\quad + \frac{2\,(1+c)}{1-c}M_2^2 g^{\mu\nu}\epsilon\ ,\\
\left(\Box + M_\eta^2\right)\left(\Box + M_2^2\right)h^{\mu\nu} &= \frac{2\,(1+c)^2}{1-c}\,M_2^2\\
&\quad \times\left(2\partial^\mu\partial^\nu - \frac{c}{3+c}M_2^2 g^{\mu\nu}\right)\epsilon\ ,\\
\left(\Box + M_\epsilon^2\right)\left(\Box + M_\eta^2\right)\left(\Box + M_2^2\right)h^{\mu\nu} &= 0\ ,\\
\left(\Box + M_\eta^2\right)\eta^\mu &= -\frac{2\,(1+c)}{1-c}\,M_2\partial^\mu\epsilon\ ,\\
\left(\Box + M_\epsilon^2\right)\left(\Box + M_\eta^2\right)\eta^\mu &= 0\ ,\\
\left(\Box + M_\epsilon^2\right)\epsilon &= 0\ ,
\end{aligned} \tag{9.4}$$

where $M_\eta^2 = -cM_2^2$ and $M_\epsilon^2 = -\frac{2c}{3+c}M_2^2$. The constraint relations are $h^\mu_\mu = 0$, $\partial_\mu h^{\mu\nu} = -cM_2\eta^\nu$ and $\partial\cdot\eta = \frac{4M_2}{1-c}\,\epsilon$

From the last line of (9.4) we see that the ϵ-field is a free Klein-Gordon field. This equation comes about quite natural from the Euler-Lagrange equations. This would not be so if we allowed for a mass term of this ϵ-field in the Lagrangian (9.1c). Then it must be imposed that ϵ is a free Klein-Gordon field which makes the calculations unnatural and unnecessary difficult.

9.2 Quantization

As mentioned before the quantization procedure runs exactly the same as in the previous chapter (section 8.2). We, therefore, determine the canonical momenta to be

$$
\begin{array}{ll}
\pi_1^0 = M_1 B \ , & \pi_B = 0 \ , \\
\pi_1^n = -\dot{A}^n + \partial^n A^0 \ , & \\
 & \\
\pi_{3/2}^0 = 0 \ , & \pi_{3/2}^{0\ \ddagger} = 0 \ , \\
\pi_{3/2}^n = \frac{i}{2}\, \psi_k^\dagger \sigma^{kn} \ , & \pi_{3/2}^{n\ \ddagger} = \frac{i}{2}\, \sigma^{nk}\psi_k \ , \\
\pi_\chi = 0 \ , & \pi_\chi^\ddagger = 0 \ , \\
 & \\
\pi_2^{00} = -\frac{1}{2}\, \partial_n h^{n0} + M_2 \eta^0 \ , & \pi_\eta^0 = 0 \ , \\
\pi_2^{0m} = -\partial_n h^{nm} + \frac{1}{2}\, \partial^m h^{00} + \frac{1}{2}\, \partial^m h_n^n + M_2 \eta^m \ , & \pi_\eta^m = 0 \ , \\
\pi_2^{nm} = \frac{1}{2}\, \dot{h}^{nm} - \frac{1}{2}\, g^{nm} \dot{h}_k^k + \frac{1}{2}\, g^{nm} \partial_k h^{k0} \ , & \pi_\epsilon = 0 \ ,
\end{array}
\tag{9.5}
$$

from which we deduce the velocities

$$
\begin{aligned}
\dot{A}^n &= -\pi_1^n + \partial^n A^0 \ , \\
\dot{h}^{nm} &= 2\pi_2^{nm} - g^{nm} \pi_{2\,k}^{\ k} + \frac{1}{2}\, g^{nm} \partial_k h^{k0} \ , \\
\dot{h}_k^k &= -\pi_{2\,k}^{\ k} + \frac{3}{2}\, \partial_k h^{k0} \ .
\end{aligned}
\tag{9.6}
$$

These velocities are the same as in the previous chapter (see (8.7)). The primary constraints are

$$
\begin{array}{ll}
\theta_1^0 = \pi_1^0 - M_1 B \ , & \theta_B = \pi_B \ , \\
 & \\
\theta_{3/2}^0 = \pi_{3/2}^0 \ , & \theta_{3/2}^{0\ \ddagger} = \pi_{3/2}^{0\ \ddagger} \ , \\
\theta_{3/2}^n = \pi_{3/2}^n - \frac{i}{2}\, \psi_k^\dagger \sigma^{kn} \ , & \theta_{3/2}^{n\ \ddagger} = \pi_{3/2}^{n\ \ddagger} - \frac{i}{2}\, \sigma^{nk} \psi_k \ , \\
\theta_\chi = \pi_\chi \ , & \theta_\chi^\ddagger = \pi_\chi^\ddagger \ , \\
 & \\
\theta_2^{00} = \pi_2^{00} + \frac{1}{2}\, \partial_n h^{n0} - M_2 \eta^0 \ , & \theta_\eta^0 = \pi_\eta^0 \ , \\
\theta_2^{0m} = \pi_2^{0m} + \partial_n h^{nm} - \frac{1}{2}\, \partial^m h^{00} & \theta_\eta^m = \pi_\eta^m \ , \\
\qquad - \frac{1}{2}\, \partial^m h_n^n - M_2 \eta^m \ , & \theta_\epsilon = \pi_\epsilon \ .
\end{array}
\tag{9.7}
$$

Having determined the canonical momenta, the velocities and the primary constraints we determine the (strong) Hamiltonians to be

$$
\begin{aligned}
\mathcal{H}_{B,S} \ &= \ -\frac{1}{2}\, \pi_1^n \pi_{1,n} + \pi_1^n \partial_n A_0 + \frac{1}{2}\, \partial_m A_n \partial^m A^n - \frac{1}{2}\, \partial_m A_n \partial^n A^m - \frac{1}{2}\, M_1^2 A^0 A_0 \\
&\quad -\frac{1}{2}\, M_1^2 A^n A_n - M_1 B \partial^m A_m - \frac{1}{2}\, a M_1^2 B^2 + \lambda_{1,0} \theta_1^0 + \lambda_B \theta_B \ ,
\end{aligned}
$$

$$\begin{aligned}
\mathcal{H}_{\chi,S} &= \frac{1}{2}\,\epsilon^{\mu\nu\rho k}\bar{\psi}_\mu\gamma_5\gamma_\rho\left(\partial_k\psi_\nu\right) - \frac{1}{2}\,\epsilon^{\mu\nu\rho k}\left(\partial_k\bar{\psi}_\mu\right)\gamma_5\gamma_\rho\psi_\nu + M_{3/2}\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu \\
&\quad -M_{3/2}\bar{\chi}\gamma^\mu\psi_\mu - M_{3/2}\bar{\psi}_\mu\gamma^\mu\chi - bM_{3/2}\bar{\chi}\chi + \lambda_{3/2,0}\theta^0_{3/2} + \lambda_{3/2,n}\theta^n_{3/2} \\
&\quad +\lambda^\ddagger_{3/2,0}{\theta^0_{3/2}}^\ddagger + \lambda^\ddagger_{3/2,n}{\theta^n_{3/2}}^\ddagger + \lambda_\chi\theta_\chi + \lambda^\ddagger_\chi\theta^\ddagger_\chi\ , \\
\mathcal{H}_{\eta\epsilon,S} &= \pi_2^{nm}\pi_{2,nm} - \frac{1}{2}\,\pi_{2}{}^n_n\pi_{2}{}^m_m + \frac{1}{2}\,\pi_{2}{}^n_n\partial^m h_{m0} - \frac{1}{2}\,\partial^k h^{n0}\partial_k h_{n0} \\
&\quad -\frac{1}{4}\,\partial^k h^{nm}\partial_k h_{nm} + \frac{1}{8}\,\partial_n h^{n0}\partial^m h_{m0} + \frac{1}{2}\,\partial_n h^{nm}\partial^k h_{km} \\
&\quad +\frac{1}{2}\,\partial_m h^{00}\partial^m h^n_n + \frac{1}{4}\,\partial_m h^n_n\partial^m h^k_k - \frac{1}{2}\,\partial_n h^{nm}\partial_m h_{00} - \frac{1}{2}\,\partial_n h^{nm}\partial_m h^k_k \\
&\quad +\frac{1}{2}\,M_2^2 h^{n0}h_{n0} + \frac{1}{4}\,M_2^2 h^{nm}h_{nm} - \frac{1}{2}\,M_2^2 h^{00}h^m_m - \frac{1}{4}\,M_2^2 h^n_n h^m_m \\
&\quad -\frac{1}{2}\,cM_2^2\eta^\mu\eta_\mu - M_2\partial_n h^{n0}\eta_0 - M_2\partial_n h^{nm}\eta_m - M_2^2 h^0_0\epsilon - M_2^2 h^k_k\epsilon \\
&\quad +\lambda_{2,00}\theta_2^{00} + \lambda_{2,0m}\theta_2^{0m} + \lambda_{0,\eta}\theta^0_\eta + \lambda_{m,\eta}\theta^m_\eta + \lambda_\epsilon\theta_\epsilon\ . \qquad (9.8)
\end{aligned}$$

With this Hamiltonians (9.8) and with the definition of the Pb in (8.10) we impose the time-derivatives of the constraints (9.7) to be zero

$$\begin{aligned}
\left\{\theta^0_1(x), H_{B,S}\right\}_P &= \partial_n\pi^n_1 + M_1^2A^0 - M_1\lambda_B = 0\ , \qquad (9.9a) \\
\left\{\theta_B(x), H_{B,S}\right\}_P &= M_1\partial^m A_m + aM_1^2B + M_1\lambda_{1,0} = 0\ , \qquad (9.9b)
\end{aligned}$$

$$\begin{aligned}
\left\{\theta^0_{3/2}(x), H_{\chi,S}\right\}_P &= \epsilon^{\mu0\rho k}\left(\partial_k\bar{\psi}_\mu\right)\gamma_5\gamma_\rho - M_{3/2}\bar{\psi}_\mu\sigma^{\mu0} + M_{3/2}\bar{\chi}\gamma^0 \\
&= 0 \equiv -{\Phi^0_{3/2}}^\ddagger(x)\ , \qquad (9.10a) \\
\left\{{\theta^0_{3/2}}^\ddagger(x), H_{\chi,S}\right\}_P &= -\epsilon^{\mu0\rho k}\gamma^0\gamma_5\gamma_\rho\left(\partial_k\psi_\mu\right) + M_{3/2}\gamma^0\sigma^{0\mu}\psi_\mu - M_{3/2}\chi \\
&= 0 \equiv -\Phi^0_{3/2}(x)\ , \qquad (9.10b) \\
\left\{\theta^n_{3/2}(x), H_{\chi,S}\right\}_P &= \epsilon^{\mu n\rho k}\left(\partial_k\bar{\psi}_\mu\right)\gamma_5\gamma_\rho - M_{3/2}\bar{\psi}_\mu\sigma^{\mu n} + M\bar{\chi}\gamma^n \\
&\quad +i\lambda^\ddagger_{3/2,k}\sigma^{kn} = 0\ , \qquad (9.10c) \\
\left\{{\theta^n_{3/2}}^\ddagger(x), H_{\chi,S}\right\}_P &= -\epsilon^{\mu n\rho k}\gamma^0\gamma_5\gamma_\rho\left(\partial_k\psi_\mu\right) + M_{3/2}\gamma^0\sigma^{n\mu}\psi_\mu \\
&\quad -M\gamma^0\gamma^n\chi + i\sigma^{nk}\lambda_{3/2,k} = 0\ , \qquad (9.10d) \\
\left\{\theta_\chi(x), H_{\chi,S}\right\}_P &= M_{3/2}\bar{\psi}\cdot\gamma + bM_{3/2}\bar{\chi} = 0 \equiv -M_{3/2}\Phi^\ddagger_\chi\gamma^0\ , \qquad (9.10e) \\
\left\{\theta^\ddagger_\chi(x), H_{\chi,S}\right\}_P &= -M_{3/2}\gamma^0\gamma\cdot\psi - bM_{3/2}\gamma^0\chi = 0 \\
&\equiv -M_{3/2}\gamma^0\Phi_\chi\ , \qquad (9.10f)
\end{aligned}$$

$$\begin{aligned}
\left\{\theta_2^{00}(x), H_{\eta\epsilon,S}\right\}_P &= -M_2\lambda_\eta^0 + \frac{1}{2}\left(\partial^k\partial_k + M_2^2\right)h_m^m - \frac{1}{2}\partial_n\partial_m h^{nm} \\
&\quad + M_2^2\epsilon = 0 \ , &(9.11a)\\
\left\{\theta_2^{0m}(x), H_{\eta\epsilon,S}\right\}_P &= 2\partial_k\pi_2^{km} - \left(\partial^k\partial_k + M_2^2\right)h^{0m} - M_2\partial^m\eta^0 \\
&\quad - M_2\lambda_\eta^m = 0 \ , &(9.11b)\\
\left\{\theta_\eta^0(x), H_{\eta\epsilon,S}\right\}_P &= \partial_n h^{n0} + \lambda_2^{00} + cM_2\eta^0 = 0 \ , &(9.11c)\\
\left\{\theta_\eta^m(x), H_{\eta\epsilon,S}\right\}_P &= \partial_n h^{nm} + \lambda_2^{0m} + cM_2\eta^m = 0 \ , &(9.11d)\\
\left\{\theta_\epsilon(x), H_{\eta\epsilon,S}\right\}_P &= M_2^2\left[h_0^0 + h_n^n\right] = 0 \equiv M_2^2\Phi_\eta \ , &(9.11e)
\end{aligned}$$

Equations (9.9a), (9.9b), (9.10c), (9.10d) and (9.11a)-(9.11d) determine the Lagrange multipliers $\lambda_B, \lambda_{1,0}, \lambda_{3/2,k}^\ddagger, \lambda_{3/2,k}, \lambda_\eta^0, \lambda_\eta^m, \lambda_2^{00}, \lambda_2^{0m}$, respectively. All other equations in (9.9), (9.10) and (9.11) yield new (secondary) constraints. Imposing their time derivatives to be zero, yields

$$\begin{aligned}
\left\{\Phi_{3/2}^0(x), H_{\chi,S}\right\}_P &= \sigma^{nk} i\partial_n\lambda_k + M_{3/2}\gamma^k\lambda_{3/2,k} - M_{3/2}\lambda_\chi = 0 \ , \\
\left\{\Phi_{3/2}^{0\ \ddagger}(x), H_{\chi,S}\right\}_P &= i\partial_n\lambda_{3/2,k}^\ddagger\sigma^{kn} + M_{3/2}\lambda_{3/2,k}^\ddagger\gamma^k + M_{3/2}\lambda_\chi^\ddagger = 0 \ , \\
\left\{\Phi_\chi(x), H_{\chi,S}\right\}_P &= -b\lambda_\chi - \gamma^0\lambda_{3/2,0} - \gamma^n\lambda_{3/2,n} = 0 \ , \\
\left\{\Phi_\chi^\ddagger(x), H_{\chi,S}\right\}_P &= b\lambda_\chi^\ddagger + \lambda_{3/2,0}^\ddagger\gamma^0 - \lambda_{3/2,n}^\ddagger\gamma^n = 0 \ , &(9.12)
\end{aligned}$$

$$\left\{\Phi_\eta(x), H_{\eta\epsilon}\right\}_P = -\pi_{2k}^{\ k} + \frac{1}{2}\partial_n h^{n0} - cM_2\eta^0 = 0 = -\Phi_2^{(1)} \ . \qquad (9.13)$$

The equations in (9.12) determine the Lagrange multipliers λ_χ, $\lambda_\chi^\ddagger$, $\lambda_{3/2,0}$ and $\lambda_{3/2,0}^\ddagger$. Equation (9.13) yields yet another (tertiary) constraint. Imposing its time derivative to be zero

$$\begin{aligned}
\left\{\Phi_2^{(1)}(x), H_{\eta\epsilon}\right\}_P &= \partial^k\partial_k h^{00} + \frac{1}{2}\partial^k\partial_k h_m^m - \frac{1}{2}\partial_n\partial_m h^{nm} + \frac{3}{2}M_2^2 h^{00} + M_2^2 h_m^m \\
&\quad - M_2\partial^k\eta_k - \partial_m\lambda_2^{m0} + 3M_2^2\epsilon + cM_2\lambda_\eta^0 = 0 \ , &(9.14)
\end{aligned}$$

gives an equation for λ_η^0. Since we already had an equation determining λ_η^0 (9.11a) we combine both equations for consistency and use Φ_η as a weakly vanishing constraint. What we get is the last constraint

$$\begin{aligned}
\Phi_2^{(2)} &= -\partial_n\partial_m h^{nm} + \left(\partial^k\partial_k + M_2^2\right)h_m^m + 2M_2\partial^k\eta_k \\
&\quad -2\left(\frac{3+c}{1-c}\right)M_2^2\epsilon \ ,
\end{aligned}$$

$$\left\{\Phi_2^{(2)}(x), H_{\eta\epsilon,S}\right\}_P = -2\partial_n\partial_m\pi_2^{nm} - M_2^2\pi_{2k}^{k} + \left(\partial^k\partial_k + \frac{3}{2}M_2^2\right)\partial_n h^{n0} + 2M_2\partial_k\lambda_\eta^k - 2\left(\frac{3+c}{1-c}\right)M_2^2\lambda_\epsilon = 0 \ . \tag{9.15}$$

As can be seen in (9.15) imposing the time derivative of $\Phi_2^{(2)}$ to be zero determines the remaining Lagrange multiplier λ_ϵ.

All Lagrange multipliers are determined, which, again, means that all constraints are second class. So, every constraint has at least one non-vanishing Pb with another constraint. The complete set of constraints is

$$\begin{aligned}
&\theta_1^0 = \pi_1^0 - M_1 B \ , && \theta_B = \pi_B \ , \\
&\theta_{3/2}^0 = \pi_{3/2}^0 \ , && \theta_{3/2}^{0\ \ddagger} = \pi_{3/2}^{0\ \ddagger} \ , \\
&\theta_{3/2}^n = \pi_{3/2}^n - \tfrac{i}{2}\psi_k^\dagger\sigma^{kn} \ , && \theta_{3/2}^{n\ \ddagger} = \pi_{3/2}^{n\ \ddagger} - \tfrac{i}{2}\sigma^{nk}\psi_k \ , \\
&\theta_\chi = \pi_\chi \ , && \theta_\chi^\ddagger = \pi_\chi^\ddagger \ , \\
&\Phi_{3/2}^0 = -i\sigma^{kn}\partial_k\psi_n - M_{3/2}\left(\gamma^k\psi_k - \chi\right) \ , && \Phi_{3/2}^{0\ \ddagger} = -i\partial_k\psi_n^\dagger\sigma^{nk} - M_{3/2}\left(\psi_k^\dagger\gamma^k + \chi^\dagger\right) \ , \\
&\Phi_\chi = \gamma^0\psi_0 + \gamma^k\psi_k + b\chi \ , && \Phi_\chi^\ddagger = -\psi_0^\dagger\gamma^0 + \psi_k^\dagger\gamma^k - b\chi^\dagger \ , \\
&\theta_2^{00} = \pi_2^{00} + \tfrac{1}{2}\partial_n h^{n0} - M_2\eta^0 \ , && \theta_\eta^0 = \pi_\eta^0 \ , \\
&\theta_2^{0m} = \pi_2^{0m} + \partial_n h^{nm} - \tfrac{1}{2}\partial^m h^{00} - \tfrac{1}{2}\partial^m h_n^n - M_2\eta^m \ , && \theta_\eta^m = \pi_\eta^m \ , \\
& && \theta_\epsilon = \pi_\epsilon \ , \\
&\Phi_2^{(2)} = -\partial_n\partial_m h^{nm} + \left(\partial^k\partial_k + M_2^2\right)h_m^m \ , + 2M_2\partial^k\eta_k - 2\left(\tfrac{3+c}{1-c}\right)M_2^2\epsilon \ , && \Phi_\eta = h_0^0 + h_n^n \ , \\
& && \Phi_2^{(1)} = \pi_{2k}^{k} - \tfrac{1}{2}\partial_n h^{n0} + cM_2\eta^0 \ .
\end{aligned} \tag{9.16}$$

Again we make linear combinations of constraints in order to reduce the number of non-vanishing Pb's

$$\begin{aligned}
\tilde{\Phi}_\chi &= \Phi_\chi - \frac{b}{M_{3/2}}\Phi_{3/2}^0 = \gamma^0\psi_0 + (1+b)\gamma^k\psi_k + \frac{b}{M_{3/2}}i\partial_k\sigma^{kl}\psi_l \ , \\
\tilde{\theta}_{3/2}^n &= \theta_{3/2}^n - \theta_{3/2}^0\gamma_0\left[(1+b)\gamma^n - \frac{b}{M_{3/2}}i\overleftarrow{\partial_k}\sigma^{kn}\right] \\
&\quad + \frac{1}{M_{3/2}}\theta_\chi\left[M_{3/2}\gamma^n - i\overleftarrow{\partial_k}\sigma^{kn}\right] = \pi_{3/2}^n - (1+b)\pi_{3/2}^0\gamma_0\gamma^n \\
&\quad + \frac{bi\partial_k}{M_{3/2}}\pi_{3/2}^0\gamma_0\sigma^{kn} + \pi_\chi\gamma^n - \frac{i\partial_k}{M_{3/2}}\pi_\chi\sigma^{kn} - \frac{i}{2}\psi_k^\dagger\sigma^{kn} \ , \\
\tilde{\Phi}_\chi^\ddagger &= \Phi_\chi^\ddagger - \frac{b}{M_{3/2}}\Phi_{3/2}^{0\ \ddagger} = -\psi_0^\dagger\gamma^0 + (1+b)\psi_k^\dagger\gamma^k + \frac{b}{M_{3/2}}i\partial_k\psi_n\sigma^{nk} \ ,
\end{aligned}$$

$$
\begin{aligned}
\tilde{\theta}^{n\ddagger}_{3/2} &= {\theta^n_{3/2}}^\ddagger - \left[-(1+b)\gamma^n + \frac{b}{M_{3/2}}\,\sigma^{nk} i\partial_k\right]\gamma_0 {\theta^0_{3/2}}^\ddagger \\
&-\frac{1}{M_{3/2}}\left[M_{3/2}\gamma^n - \sigma^{nk} i\partial_k\right]\theta^\ddagger_\chi = {\pi^n_{3/2}}^\ddagger + (1+b)\gamma^n\gamma_0 {\pi^0_{3/2}}^\ddagger \\
&-\frac{bi\partial_k}{M_{3/2}}\,\sigma^{nk}\gamma_0 {\pi^0_{3/2}}^\ddagger - \gamma^n\pi^\ddagger_\chi + \frac{i\partial_k}{M_{3/2}}\,\sigma^{nk}\pi^\ddagger_\chi - \frac{i}{2}\,\sigma^{nk}\psi_k \ ,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\Phi}_\eta &= \Phi_\eta - \frac{1}{M_2}\,\theta^0_\eta = h^0_0 + h^k_k - \frac{1}{M_2}\,\pi^0_\eta \ , \\
\tilde{\Phi}^{(1)}_2 &= \Phi^{(1)}_2 + c\theta^{00}_2 + \frac{1}{2M^2}\left(\frac{1-c}{3+c}\right)\left(2\partial^k\partial_k + 3M^2\right)\theta_\epsilon \\
&= \pi^k_k + c\pi^{00} + \frac{1}{2M_2^2}\left(\frac{1-c}{3+c}\right)\left(2\partial^k\partial_k + 3M_2^2\right)\pi_\epsilon - \frac{1}{2}(1-c)\partial_n h^{n0} \ , \\
\tilde{\theta}^{0n}_2 &= \theta^{0n}_2 + \frac{1}{(3+c)}\,\partial^n\tilde{\Phi}_\eta \\
&= \pi^{0n}_2 - \frac{1}{(3+c)}\frac{\partial^n}{M_2}\,\pi^0_\eta + \partial_k h^{kn} - \frac{1}{2}\left(\frac{1+c}{3+c}\right)\left(\partial^n h^{00} + \partial^n h^k_k\right) - M_2\eta^n \ , \\
\tilde{\Phi}^{(2)}_2 &= \Phi^{(2)}_2 + 2\partial_k\tilde{\theta}^{0k}_2 = 2\partial_k\pi^{k0}_2 - \frac{2}{(3+c)M_2}\,\partial_k\partial^k\pi^0_\eta + \partial_n\partial_m h^{nm} \\
&+\frac{2}{(3+c)}\,\partial^k\partial_k h^n_n - \frac{1+c}{3+c}\,\partial^k\partial_k h^0_0 - 2\left(\frac{3+c}{1-c}\right)M_2^2\,\epsilon + M_2^2 h^k_k \ . \qquad (9.17)
\end{aligned}
$$

With these new constraints the remaining non-vanishing Pb's are

$$
\begin{aligned}
\left\{\theta^0_1(x), \theta_B(y)\right\}_P &= -M_1\delta^3(x-y) \ , \\
\left\{\theta^0_{3/2}(x), \tilde{\Phi}_\chi(y)\right\}_P &= \gamma_0\delta^3(x-y) = -\left\{{\theta^0_{3/2}}^\ddagger(x), \tilde{\Phi}^\ddagger_\chi(y)\right\}_P \ , \\
\left\{\theta_\chi(x), \Phi^0_{3/2}(y)\right\}_P &= M_{3/2}\,\delta^3(x-y) = -\left\{\theta^\ddagger_\chi(x), {\Phi^0_{3/2}}^\ddagger(y)\right\}_P \ , \\
\left\{\tilde{\theta}^n_{3/2}(x), \tilde{\theta}^{m\ddagger}_{3/2}(y)\right\}_P &= -i\sigma^{mn}\delta^3(x-y) \ , \\
\left\{\theta^{00}_2(x), \theta^0_\eta(y)\right\}_P &= -M_2\,\delta^3(x-y) \ , \\
\left\{\tilde{\theta}^{0n}_2(x), \theta^m_\eta(y)\right\}_P &= -M_2\,g^{nm}\,\delta^3(x-y) \ , \\
\left\{\theta_\epsilon(x), \tilde{\Phi}^{(2)}_2(y)\right\}_P &= 2\left(\frac{3+c}{1-c}\right)M_2^2\,\delta^3(x-y) \ , \\
\left\{\tilde{\Phi}^{(1)}_2(x), \tilde{\Phi}_\eta(y)\right\}_P &= -(3+c)\,\delta^3(x-y) \ . \qquad (9.18)
\end{aligned}
$$

The Db and the inverse functions that go with them are defined in (8.19) and (8.20), so we can immediately write down the ETC and ETAC relations

$$\begin{aligned}
\left[A^\mu(x), \dot{A}^\nu(y)\right]_0 &= -i\left(g^{\mu\nu} - (1-a)\delta_0^\mu\delta_0^\nu\right)\delta^3(x-y) \;, \\
\left[A^\mu(x), B(y)\right]_0 &= \frac{i}{M_1}\,\delta_0^\mu\delta^3(x-y) \;, \\
\left[A^\mu(x), \dot{B}(y)\right]_0 &= -\left[\dot{A}_\mu(x), B(y)\right]_0 = -i\delta_k^\mu \frac{\partial^k}{M_1}\,\delta^3(x-y) \;, \\
\left[B(x), \dot{B}(y)\right]_0 &= -i\delta^3(x-y) \;,
\end{aligned} \tag{9.19}$$

$$\begin{aligned}
\left\{\psi^n(x), \psi^{m\dagger}(y)\right\}_0 &= -\left[g^{nm} - \frac{1}{2}\,\gamma^n\gamma^m\right]\delta^3(x-y) \;, \\
\left\{\psi^0(x), \psi^{0\dagger}(y)\right\}_0 &= -\frac{3}{2}\,(1+b)^2\,\delta^3(x-y) \;, \\
\left\{\psi^0(x), \psi^{m\dagger}(y)\right\}_0 &= \left[\frac{b+1}{2}\,\gamma^m - b\,\frac{i\partial^m}{M_{3/2}}\right]\gamma_0\,\delta^3(x-y) \;, \\
\left\{\psi^n(x), \psi^{0\dagger}(y)\right\}_0 &= \left[\frac{b+1}{2}\,\gamma^n - b\,\frac{i\partial^n}{M_{3/2}}\right]\gamma_0\,\delta^3(x-y) \;, \\
\left\{\chi(x), \chi^\dagger(y)\right\}_0 &= -\frac{3}{2}\,\delta^3(x-y) \;, \\
\left\{\psi^0(x), \chi^\dagger(y)\right\}_0 &= \gamma_0\left[\frac{3(1+b)}{2} - \frac{1}{M_{3/2}}\,i\gamma^k\partial_k\right]\delta^3(x-y) \;, \\
\left\{\psi^n(x), \chi^\dagger(y)\right\}_0 &= -\left[\frac{1}{2}\,\gamma^n - \frac{i\partial^n}{M_{3/2}}\right]\delta^3(x-y) \;,
\end{aligned} \tag{9.20}$$

$$\begin{aligned}
\left[h^{00}(x), \eta^0(y)\right]_0 &= \frac{3}{M_2(3+c)}\,i\delta^3(x-y) \;, \\
\left[h^{0n}(x), \eta^m(y)\right]_0 &= \frac{1}{M_2}\,g^{nm}\,i\delta^3(x-y) \;, \\
\left[h^{0n}(x), \epsilon(y)\right]_0 &= -\frac{1}{M_2^2}\left(\frac{1-c}{3+c}\right)\partial^n i\delta^3(x-y) \;, \\
\left[h^{nm}(x), \eta^0(y)\right]_0 &= -\frac{1}{M_2(3+c)}\,g^{nm}\,i\delta^3(x-y) \;, \\
\left[\eta^0(x), \eta^m(y)\right]_0 &= \frac{1}{M_2^2(3+c)}\,\partial^m i\delta^3(x-y) \;, \\
\left[\eta^0(x), \epsilon(y)\right]_0 &= \frac{3}{2M_2}\,\frac{(1-c)}{(3+c)^2}\,i\delta^3(x-y) \;.
\end{aligned} \tag{9.21}$$

In principle there are also ETC relations among time derivatives of the fields in (9.21), that we have not shown for convenience. However, they are of importance when calculating the commutation relations for non-equal times, below.

9.3 Propagators

In order to get commutation and anti-commutation relations for non-equal times we first construct solutions to the EoMs ((9.2), (9.3) and (9.4)) based on the identities (8.22)

$$\begin{aligned}
B(x) &= \int d^3z \left[\partial_0^z \Delta(x-z;M_B^2)\cdot B(z) - \Delta(x-z;M_B^2)\cdot \partial_0^z B(z)\right] \ , \\
A_\mu(x) &= \int d^3z \left[\partial_0^z \Delta(x-z;M_1^2)\cdot A_\mu(z) - \Delta(x-z;M_1^2)\cdot \partial_0^z A_\mu(z)\right] \\
&\quad + \frac{1}{(1-a)M_1^2}\int d^3z \left[\Big(\partial_0^z\Delta(x-z;M_B^2) - \partial_0^z\Delta(x-z;M_1^2)\Big)\right. \\
&\qquad\qquad \left. - \left(\Delta(x-z;M_B^2) - \Delta(x-z;M_1^2)\right)\partial_0^z\right] \\
&\qquad\qquad \times (\Box + M_1^2)A_\mu(z) \ ,
\end{aligned}$$

$$\begin{aligned}
\chi(x) &= i\int d^3z (i\partial\!\!\!/_x + M_\chi)\gamma^0\Delta(x-z;M_\chi^2)\chi(z) \ , \\
\psi_\mu(x) &= i\int d^3z (i\partial\!\!\!/_x + M_{3/2})\gamma^0\Delta(x-z;M_{3/2}^2)\psi_\mu(z) \\
&\quad + \frac{2i}{3(b+2)M_{3/2}}\int d^3z \Big[(i\partial\!\!\!/_x + M_{3/2})\Delta(x-z;M_{3/2}^2) \\
&\qquad - (i\partial\!\!\!/_x - M_\chi)\Delta(x-z;M_\chi^2)\Big]\gamma^0(i\partial\!\!\!/_z - M_{3/2})\psi_\mu(z) \\
&\quad + \frac{2i}{(3b+2)M_{3/2}}\int d^3z \left\{\Delta(x-z;M_\chi^2) - \frac{2}{3(b+2)M_{3/2}}\Big[\right. \\
&\qquad \left.\times (i\partial\!\!\!/_x + M_{3/2})\Delta(x-z;M_{3/2}^2) - (i\partial\!\!\!/_x - M_\chi)\Delta(x-z;M_\chi^2)\Big]\right\} \\
&\qquad \times\gamma^0(i\partial\!\!\!/_z + M_\chi)(i\partial\!\!\!/_z - M_{3/2})\psi_\mu(z) \ ,
\end{aligned}$$

$$\begin{aligned}
\epsilon(x) &= \int d^3z \left[\partial_0^z \Delta(x-z;M_\epsilon^2)\cdot\epsilon(z) - \Delta(x-z;M_\epsilon^2)\cdot\partial_0^z\epsilon(z)\right] \ , \\
\eta^\mu(x) &= \int d^3z \left[\partial_0^z \Delta(x-z;M_\eta^2)\cdot\eta^\mu(z) - \Delta(x-z;M_\eta^2)\cdot\partial_0^z\eta^\mu(z)\right] \\
&\quad + \frac{1}{M_\eta^2 - M_\epsilon^2}\int d^3z \left[\partial_0^z\left(\Delta(x-z;M_\epsilon^2) - \Delta(x-z;M_\eta^2)\right)\right. \\
&\qquad \left. - \left(\Delta(x-z;M_\epsilon^2) - \Delta(x-z;M_\eta^2)\right)\cdot\partial_0^z\right] \\
&\qquad \times (\Box + M_\eta^2)\eta^\mu(z) \qquad (9.22)
\end{aligned}$$

$$\begin{aligned}
& h^{\mu\nu}(x) = \\
= & \int d^3z \left[\partial_0^z \Delta(x-z;M_2^2)\cdot h^{\mu\nu}(z) - \Delta(x-z;M_2^2)\cdot\partial_0^z h^{\mu\nu}(z)\right] \\
& + \frac{1}{M_2^2 - M_\eta^2}\int d^3z \left[\partial_0^z\left(\Delta(x-z;M_\eta^2) - \Delta(x-z;M_2^2)\right)\right. \\
& \qquad \left. - \left(\Delta(x-z;M_\eta^2) - \Delta(x-z;M_2^2)\right)\partial_0^z\right] \\
& \qquad \times (\Box + M_2^2)h^{\mu\nu}(z) \\
& + \frac{1}{(M_\eta^2 - M_\epsilon^2)(M_2^2 - M_\eta^2)(M_2^2 - M_\epsilon^2)}\int d^3z \Big[\\
& \partial_0^z\Big((M_2^2 - M_\eta^2)\Delta(x-z;M_\epsilon^2) - (M_2^2 - M_\epsilon^2)\Delta(x-z;M_\eta^2) \\
& \qquad + (M_\eta^2 - M_\epsilon^2)\Delta(x-z;M_2^2)\Big) \\
& - \Big((M_2^2 - M_\eta^2)\Delta(x-z;M_\epsilon^2) - (M_2^2 - M_\epsilon^2)\Delta(x-z;M_\eta^2) \\
& \qquad + (M_\eta^2 - M_\epsilon^2)\Delta(x-z;M_2^2)\Big)\partial_0^z\Big]\left(\Box + M_\eta^2\right)\left(\Box + M_2^2\right)h^{\mu\nu}(z) \ . \\
& \qquad (9.23)
\end{aligned}$$

Using these equations (9.23) and the ETC and ETAC relations of (9.19), (9.20) and (9.21) we obtain the following commutation and anti-commutation relations

$$
\begin{aligned}
[B(x), B(y)] &= -i\Delta(x-y, M_B^2)\ , \\
[A^\mu(x), B(y)] &= -i\frac{\partial^\mu}{M_1}\,\Delta(x-y, M_B^2)\ , \\
[A^\mu(x), A^\nu(y)] &= -i\left(g^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{M_1^2}\right)\Delta(x-y; M_1^2) + i\frac{\partial^\mu\partial^\nu}{M_1^2}\Delta(x-y; M_B^2) \\
&= P_1^{\mu\nu} i\Delta(x-y; M_1^2) + P_B^{\mu\nu} i\Delta(x-y; M_B^2)\ , \qquad (9.24)
\end{aligned}
$$

$$
\begin{aligned}
\{\chi(x), \bar\chi(y)\} &= -\frac{3}{2}\, i\,(i\partial\!\!\!/ + M_\chi)\,\Delta(x-y; M_\chi^2)\ , \\
\{\psi^\mu(x), \bar\chi(y)\} &= -\frac{1}{2}\left[\gamma^\mu - \frac{2i\partial^\mu}{M_{3/2}}\right] i\,(i\partial\!\!\!/ + M_\chi)\,\Delta(x-y; M_\chi^2)\ , \\
\left\{\psi^\mu(x), \bar\psi^\nu(y)\right\} &= -i\left(i\partial\!\!\!/ + M_{3/2}\right)\Bigg[g^{\mu\nu} - \frac{1}{3}\,\gamma^\mu\gamma^\nu + \frac{2\partial^\mu\partial^\nu}{3M_{3/2}^2} \\
&\qquad - \frac{1}{3M_{3/2}}\,(\gamma^\mu i\partial^\nu - \gamma^\nu i\partial^\mu)\Bigg]\Delta(x-y; M_{3/2}^2) \\
&\quad -\frac{1}{6}\left[\gamma^\mu - \frac{2i\partial^\mu}{M_{3/2}}\right] i\,(i\partial\!\!\!/ + M_\chi)\left[\gamma^\nu - \frac{2i\partial^\nu}{M_{3/2}}\right]\Delta(x-y; M_\chi^2) \\
&= \left(i\partial\!\!\!/ + M_{3/2}\right) P_{3/2}^{\mu\nu} i\Delta(x-y; M_{3/2}^2) \\
&\quad + P_\chi^{\mu\nu} i\Delta(x-y; M_\chi^2)\ , \qquad (9.25)
\end{aligned}
$$

$$
\begin{aligned}
[\epsilon(x), \epsilon(y)] &= -\frac{3}{4}\,\frac{c(1-c)^2}{(3+c)^3}\, i\Delta(x-y; M_\epsilon^2)\ , \\
[\eta^\mu(x), \epsilon(y)] &= -\frac{3}{2}\,\frac{(1-c)}{(3+c)^2}\,\frac{\partial^\mu}{M_2}\, i\Delta(x-y; M_\epsilon^2)\ , \\
[\eta^\mu(x), \eta^\nu(y)] &= \left[g^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{M_\eta^2}\right] i\Delta(x-y; M_\eta^2) \\
&\quad - \frac{3}{(3+c)}\,\frac{\partial^\mu\partial^\nu}{M_\eta^2}\, i\Delta(x-y; M_\epsilon^2)\ , \\
[\epsilon(x), h^{\mu\nu}(y)] &= \frac{(1-c)}{(3+c)}\left[\frac{\partial^\mu\partial^\nu}{M_2^2} - \frac{1}{2}\,\frac{c}{(3+c)}\, g^{\mu\nu}\right] i\Delta(x-y; M_\epsilon^2)\ , \\
[\eta^\alpha(x), h^{\mu\nu}(y)] &= \frac{1}{M_2}\left[\partial^\mu g^{\alpha\nu} + \partial^\nu g^{\alpha\mu} + \frac{2}{M_\eta^2}\,\partial^\alpha\partial^\mu\partial^\nu\right] i\Delta(x-y; M_\eta^2) \\
&\quad - \frac{1}{M_2}\left[\frac{1}{(3+c)}\,\partial^\alpha g^{\mu\nu} + \frac{2}{M_\eta^2}\,\partial^\alpha\partial^\mu\partial^\nu\right] i\Delta(x-y; M_\epsilon^2)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\left[h^{\mu\nu}(x), h^{\alpha\beta}(y)\right] &= \Bigg[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - \frac{2}{3}\,g^{\mu\nu}g^{\alpha\beta} \\
&\quad + \frac{1}{M_2^2}\left(\partial^\mu\partial^\alpha g^{\nu\beta} + \partial^\nu\partial^\alpha g^{\mu\beta} + \partial^\mu\partial^\beta g^{\nu\alpha} + \partial^\nu\partial^\beta g^{\mu\alpha}\right) \\
&\quad - \frac{2}{3M_2^2}\left(\partial^\mu\partial^\nu g^{\alpha\beta} + g^{\mu\nu}\partial^\alpha\partial^\beta\right) + \frac{4}{3M_2^4}\,\partial^\mu\partial^\nu\partial^\alpha\partial^\beta\Bigg] \\
&\qquad \times i\Delta(x-y;M_2^2) \\
&\quad - \frac{1}{M_\eta^2}\Bigg[\partial^\mu\partial^\alpha g^{\nu\beta} + \partial^\nu\partial^\alpha g^{\mu\beta} + \partial^\mu\partial^\beta g^{\nu\alpha} + \partial^\nu\partial^\beta g^{\mu\alpha} \\
&\qquad + \frac{4}{M_\eta^2}\,\partial^\mu\partial^\nu\partial^\alpha\partial^\beta\Bigg] i\Delta(x-y;M_\eta^2) \\
&\quad - \Bigg[\frac{1}{3}\frac{c}{3+c}\,g^{\mu\nu}g^{\alpha\beta} - \frac{2}{3M_2^2}\left(\partial^\mu\partial^\nu g^{\alpha\beta} + g^{\mu\nu}\partial^\alpha\partial^\beta\right) \\
&\qquad + \frac{4(3+c)}{3cM_2^4}\,\partial^\mu\partial^\nu\partial^\alpha\partial^\beta\Bigg] i\Delta(x-y;M_\epsilon^2) \\
&= 2P_2^{\mu\nu\alpha\beta}(\partial)i\Delta(x-y;M_2^2) + P_\eta^{\mu\nu\alpha\beta}(\partial)i\Delta(x-y;M_\eta^2) \\
&\quad + P_\epsilon^{\mu\nu\alpha\beta}(\partial)i\Delta(x-y;M_\epsilon^2)\ . \qquad (9.26)
\end{aligned}
$$

From the overall minus signs in the (anti-) commutation relations of the auxiliary fields in (9.26) we conclude that all auxiliary fields are ghost, except for the ϵ-field. There the choice of the gauge parameter c determines whether it is ghost-like or not.

Having obtained these (anti-) commutation relations we calculate the propagators

$$
\begin{aligned}
& D_{F,a}^{\mu\nu}(x-y) = \\
&= -i < 0|T\left[A^\mu(x), A^\nu(y)\right]|0 > \\
&= -i\theta(x_0-y_0)\left[P_1^{\mu\nu}(\partial)\Delta^{(+)}(x-y;M_1^2) + P_B^{\mu\nu}(\partial)\Delta^{(+)}(x-y;M_B^2)\right] \\
&\quad -i\theta(x_0-y_0)\left[P_1^{\mu\nu}(\partial)\Delta^{(-)}(x-y;M_1^2) + P_B^{\mu\nu}(\partial)\Delta^{(-)}(x-y;M_B^2)\right] \\
&= P_1^{\mu\nu}(\partial)\Delta_F(x-y;M_1^2) + P_B^{\mu\nu}(\partial)\Delta_F(x-y;M_B^2)\ . \qquad (9.27)
\end{aligned}
$$

We see that this propagator is explicitly covariant, independent of the choice of the gauge parameter. Choosing $a = 1$ we see that the terms containing derivatives cancel and that only the $g^{\mu\nu}$ term remains. It can be seen as the massive photon propagator. For $a = \infty$ we re-obtain the massive spin-1 field, like in (8.24). Except in the above derivation it is obtained without non-covariant terms in the propagator. The choice $a = 0$ is particularly interesting, because then still the spin-1 condition $\partial \cdot A = 0$ holds (text below (9.2)), but the propagator is covariant. In momentum space it looks like

$$D^{\mu\nu}_{F,0}(P) = \frac{-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}}{p^2 - M_1^2 + i\varepsilon} \ . \tag{9.28}$$

The spin-3/2 propagator is

$$\begin{aligned} & S^{\mu\nu}_{F,b}(x-y) = -i < 0|T\left[\psi^\mu(x), \bar{\psi}^\nu(y)\right]|0> \\ = \ & -i\theta(x_0-y_0)\Big[\left(i\partial\!\!\!/ + M_{3/2}\right) P^{\mu\nu}_{3/2}(\partial)\Delta^{(+)}(x-y;M^2_{3/2}) \\ & \qquad + P^{\mu\nu}_\chi(\partial)\Delta^{(+)}(x-y;M^2_\chi)\Big] \\ & -i\theta(x_0-y_0)\Big[\left(i\partial\!\!\!/ + M_{3/2}\right) P^{\mu\nu}_{3/2}(\partial)\Delta^{(-)}(x-y;M^2_{3/2}) \\ & \qquad + P^{\mu\nu}_\chi(\partial)\Delta^{(-)}(x-y;M^2_\chi)\Big] \\ = \ & \left(i\partial\!\!\!/ + M_{3/2}\right) P^{\mu\nu}_{3/2}(\partial)\Delta_F(x-y;M_1^2) + P^{\mu\nu}_\chi(\partial)\Delta_F(x-y;M_B^2) \\ & + \frac{b}{M_{3/2}}\,\delta^\mu_0\delta^\nu_0\,\delta^4(x-y) \ . \end{aligned} \tag{9.29}$$

Only for $b = 0$ we have an explicitly covariant propagator. This result was also obtained in [59]. From the text below (9.3) we see that the choice $b = 0$ means that we have only one of the two spin-3/2 conditions or, to put it in a different way, we have added an extra spin-1/2 piece to make the RS propagator explicitly covariant.

For $b = -\frac{4}{3}$ and $b = -1$ we have that $i\partial \cdot \psi = 0$ (, but $\gamma \cdot \psi \neq 0$), but then the propagator is not covariant anymore.

The spin-2 propagator is

$$\begin{aligned} & D^{\mu\nu\alpha\beta}_{F,c}(x-y) = -i < 0|T\left[h^{\mu\nu}(x)h^{\alpha\beta}(y)\right]|0> \\ = \ & -i\theta(x^0-y^0)\Big[2P^{\mu\nu\alpha\beta}_2(\partial)\Delta^{(+)}(x-y;M^2) + P^{\mu\nu\alpha\beta}_\eta(\partial)i\Delta^{(+)}(x-y;M^2_\eta) \\ & \qquad + P^{\mu\nu\alpha\beta}_\epsilon(\partial)i\Delta^{(+)}(x-y;M^2_\epsilon)\Big] \\ & -i\theta(y^0-x^0)\Big[2P^{\mu\nu\alpha\beta}_2(\partial)\Delta^{(-)}(x-y;M^2) + P^{\mu\nu\alpha\beta}_\eta(\partial)i\Delta^{(-)}(x-y;M^2_\eta) \\ & \qquad + P^{\mu\nu\alpha\beta}_\epsilon(\partial)i\Delta^{(-)}(x-y;M^2_\epsilon)\Big] \\ = \ & 2P^{\mu\nu\alpha\beta}_2(\partial)\Delta_F(x-y;M^2) + P^{\mu\nu\alpha\beta}_\eta(\partial)\Delta_F(x-y;M^2_\eta) \\ & + P^{\mu\nu\alpha\beta}_\epsilon(\partial)\Delta_F(x-y;M^2_\epsilon) \ . \end{aligned} \tag{9.30}$$

We see that this propagator (9.30) does not contain local, non-covariant terms independent of the choice of the gauge parameter. The first part of (9.30) ($P_2^{\mu\nu\alpha\beta}(\partial)$-part) is pure spin-2 [1]. The nature of the other parts depends on the free gauge parameter.

Since c is still a free parameter it is interesting to look at several gauges. But before that, we exclude $c = 1$ and $c = -3$. In these cases the ϵ-field vanishes and the EoM are quite different. Also the quantization procedure runs differently.

An interesting gauge which we want to discuss here is $c = -1$. From (9.4) we see that all fields become free Klein-Gordon fields of mass M_2. As a result of this choice all derivative terms disappear in (9.30) and what is left is

$$D_{F,-1}^{\mu\nu\alpha\beta}(x-y) \;=\; \left[g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - \frac{1}{2}\,g^{\mu\nu}g^{\alpha\beta}\right]\Delta_F(x-y;M^2)\ . \tag{9.31}$$

In contrast to the spin-1 case, discussed above, equation (9.31) is not the massive version of the massless spin-2 propagator.

Making the choice $c = 0$ in (9.30) is easily done except for the $\frac{1}{c}$ terms, with which we deal explicitly

$$\begin{aligned}
& \underset{c\to 0}{Lim}\,\frac{1}{M_2^4}\left[\frac{1}{3}\,\frac{1}{p^2-M_2^2+i\varepsilon} + \frac{1}{c}\,\frac{1}{p^2-M_\eta^2+i\varepsilon} - \frac{3+c}{3c}\,\frac{1}{p^2-M_\epsilon^2+i\varepsilon}\right] \\
=\;& \underset{c\to 0}{Lim}\left(\frac{(1+c)^2}{3+c}\right) \\
\times\;& \left[\frac{1}{p^6 + \left(\frac{c^2+4c-3}{3+c}\right)p^4M_2^2 + \left(\frac{c(c-5)}{3+c}\right)p^2M_2^4 - \left(\frac{2c^2}{3+c}\right)M_2^6 + i\varepsilon}\right] \\
=\;& \frac{1}{3}\,\frac{1}{p^6 - p^4M_2^2 + i\varepsilon}\ , \\
\to\;& \frac{1}{3M_2^4}\left[\Delta_F(x-y;M_2^2) - \Delta_F(x-y) + M_2^2\tilde{\Delta}_F(x-y)\right]\ .
\end{aligned} \tag{9.32}$$

Using (9.32) we get for $c \to 0$

$$\begin{aligned}
D_{F,0}^{\mu\nu\alpha\beta}(x-y) \;=\;& 2P_2^{\mu\nu\alpha\beta}(\partial)\Delta_F(x-y;M_2^2) - \frac{1}{M_2^2}\Big[\partial^\mu\partial^\alpha g^{\nu\beta} + \partial^\nu\partial^\alpha g^{\mu\beta} \\
& + \partial^\mu\partial^\beta g^{\nu\alpha} + \partial^\nu\partial^\beta g^{\mu\alpha} - \frac{2}{3}\left(\partial^\mu\partial^\nu g^{\alpha\beta} + g^{\mu\nu}\partial^\alpha\partial^\beta\right) \\
& + \frac{4\partial^\mu\partial^\nu\partial^\alpha\partial^\beta}{3M_2^2}\Big]\Delta_F(x-y) + \frac{4}{3M_2^2}\,\partial^\mu\partial^\nu\partial^\alpha\partial^\beta\tilde{\Delta}_F(x-y)\ ,
\end{aligned}$$

[1] The factor 2 can again be transformed away by redefining all fields as in (8.23)

$$
\begin{aligned}
D_{F,0}^{\mu\nu\alpha\beta}(p) \;=\; & \Bigg[g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} - \frac{2}{3}\, g^{\mu\nu} g^{\alpha\beta} + \frac{2}{3p^2} \left(p^\mu p^\nu g^{\alpha\beta} + g^{\mu\nu} p^\alpha p^\beta \right) \\
& - \frac{1}{p^2} \left(p^\mu p^\alpha g^{\nu\beta} + p^\nu p^\alpha g^{\mu\beta} + p^\mu p^\beta g^{\nu\alpha} + p^\nu p^\beta g^{\mu\alpha} \right) \\
& + \frac{4}{3p^4}\, p^\mu p^\nu p^\alpha p^\beta \Bigg] \frac{1}{p^2 - M_2^2 + i\varepsilon} \;. \qquad (9.33)
\end{aligned}
$$

As in the spin-1 case this propagator satisfies the field equations (and is therefore pure spin-2) and is explicitly covariant. This result is also obtained by ignoring the c term in the Lagrangian (9.1a) from the outset.

9.4 Massless limit

It is most easy to study the massless limits of the propagators obtained in the previous section in momentum space

$$
\begin{aligned}
\underset{M_1 \to 0}{Lim}\, D_{F,a}^{\mu\nu}(p) \;&=\; \underset{M_1 \to 0}{Lim} \left[- \frac{g^{\mu\nu}}{p^2 - M_1^2 + i\varepsilon} + \frac{\frac{p^\mu p^\nu}{M_1^2}}{p^2 - M_1^2 + i\varepsilon} - \frac{\frac{p^\mu p^\nu}{M_1^2}}{p^2 - aM_1^2 + i\varepsilon} \right] \\
&=\; \underset{M_1 \to 0}{Lim} \left[- \frac{g^{\mu\nu}}{p^2 - M_1^2 + i\varepsilon} + \frac{(1-a) p^\mu p^\nu}{p^4 - (1+a) p^2 M_1^2 + a M_1^4 + i\varepsilon'} \right] \\
&=\; \left[-g^{\mu\nu} + (1-a)\, \frac{p^\mu p^\nu}{p^2} \right] \frac{1}{p^2 + i\varepsilon} \;. \qquad (9.34)
\end{aligned}
$$

Although we have not presented the massless case, it is done rather easily. The quantization procedure runs very similar to what is presented in section 9.2, contrary to the case without an auxiliary field (section 8.2), only the equations like in (9.23) are a bit different. It should be noticed that it is sufficient in the massless case to ignore the mass term of the spin-1 field in (9.1a), only. So, even though allowing for a mass term for the auxiliary field, both A^μ and B turn out to be massless. Therefore the freedom in choosing the gauge parameter is still present. In the massless case the exact same result as (9.34) is obtained, so the massless limit connects smoothly with the massless case and is explicitly covariant. In fact this line of reasoning is valid for all three spin cases with auxiliary fields. Having mentioned this, we will not come back to this when discussing the massless limits of the spin-3/2 and spin-2 cases below.

The massless limit of the spin-3/2 field is

$$\begin{aligned}
&\underset{M_{3/2}\to 0}{Lim}\, S^{\mu\nu}_{F,0}(p) = \\
= \; & \left[\gamma^\mu p^\nu - \not{p}\left(g^{\mu\nu} - \frac{1}{2}\,\gamma^\mu\gamma^\nu\right)\right]\frac{1}{p^2+i\varepsilon} \\
&+2p^\mu p^\nu \not{p}\,\underset{M_{3/2}\to 0}{Lim}\left[\frac{1}{3M^2_{3/2}}\left(\frac{1}{p^2-M^2_{3/2}+i\varepsilon} - \frac{1}{p^2-4M^2_{3/2}+i\varepsilon}\right)\right] \\
&+\left(2p^\mu p^\nu + \not{p}\left(\gamma^\mu p^\nu - p^\mu\gamma^\nu\right)\right) \\
&\quad\times\underset{M_{3/2}\to 0}{Lim}\left[\frac{1}{3M_{3/2}}\left(\frac{1}{p^2-M^2_{3/2}+i\varepsilon} - \frac{1}{p^2-4M^2_{3/2}+i\varepsilon}\right)\right] \\
= \; & -\not{p}\left[g^{\mu\nu} - \frac{1}{2}\,\gamma^\mu\gamma^\nu\right]\frac{1}{p^2+i\varepsilon} + \gamma^\mu p^\nu\frac{1}{p^2+i\varepsilon} - 2p^\mu p^\nu\not{p}\,\frac{1}{p^4+i\varepsilon}\,. \qquad (9.35)
\end{aligned}$$

We notice that when this propagator (9.35) is coupled to conserved currents only the first two parts contribute. These parts form exactly the massless spin-3/2 propagator with only the helicities $\lambda = \pm 3/2$ ([68]). When we couple the (massive) RS-propagator (8.25) to conserved currents and take the massless limit [2] we see that it is different from the one in (9.35) because of the factor in front of the $\gamma^\mu\gamma^\nu$ term.

The massless limit of the spin-2 propagator is

$$\begin{aligned}
&\underset{M_2\to 0}{Lim}\, D^{\mu\nu\alpha\beta}_{F,c}(p) = \\
= \; & \left(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}\right)\frac{1}{p^2+i\varepsilon} \\
&-\frac{1}{3}\,g^{\mu\nu}g^{\alpha\beta}\,\underset{M_2\to 0}{Lim}\left[\frac{2}{p^2-M_2^2+i\varepsilon} + \frac{c}{3+c}\frac{1}{p^2-M_\epsilon^2+i\varepsilon}\right] \\
&-\left(p^\mu p^\alpha g^{\nu\beta} + p^\nu p^\alpha g^{\mu\beta} + p^\mu p^\beta g^{\nu\alpha} + p^\nu p^\beta g^{\mu\alpha}\right) \\
&\quad\times\underset{M_{3/2}\to 0}{Lim}\left[\frac{1}{M_2^2}\left(\frac{1}{p^2-M_2^2+i\varepsilon} - \frac{1}{p^2-M_\eta^2+i\varepsilon}\right)\right] \\
&+\frac{2}{3}\left(p^\mu p^\nu g^{\alpha\beta} + g^{\mu\nu}p^\alpha p^\beta\right) \\
&\quad\times\underset{M_{3/2}\to 0}{Lim}\left[\frac{1}{M_2^2}\left(\frac{1}{p^2-M_2^2+i\varepsilon} - \frac{1}{p^2-M_\epsilon^2+i\varepsilon}\right)\right]
\end{aligned}$$

[2] Terms in the massive RS propagator that do not have a proper massless limit do not contribute since we couple to conserved currents

$$
\begin{aligned}
&+4\,p^\mu p^\nu p^\alpha p^\beta \underset{M_{3/2}\to 0}{Lim}\left[\frac{1}{M_2^4}\left(\frac{1}{3}\frac{1}{p^2-M_2^2+i\varepsilon}+\frac{1}{c}\frac{1}{p^2-M_\eta^2+i\varepsilon}\right.\right.\\
&\qquad\qquad\left.\left.-\frac{3+c}{3c}\frac{1}{p^2-M_\epsilon^2+i\varepsilon}\right)\right]\\
= &\left[g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha}-\frac{2+c}{3+c}\,g^{\mu\nu}g^{\alpha\beta}\right]\frac{1}{p^2+i\varepsilon}\\
&-(1+c)\frac{1}{p^2}\left[p^\mu p^\alpha g^{\nu\beta}+p^\nu p^\alpha g^{\mu\beta}+p^\mu p^\beta g^{\nu\alpha}+p^\nu p^\beta g^{\mu\alpha}\right.\\
&\qquad\left.-\frac{2}{3+c}\left(p^\mu p^\nu g^{\alpha\beta}+g^{\mu\nu}p^\alpha p^\beta\right)\right]\frac{1}{p^2+i\varepsilon}\\
&+\frac{4(1+c)^2}{3+c}\,\frac{p^\mu p^\nu p^\alpha p^\beta}{p^4}\frac{1}{p^2+i\varepsilon}\ .
\end{aligned}
\tag{9.36}
$$

Making the choice of the gauge parameter $c \to \pm\infty$ we see that (9.36) becomes the massless spin-2 propagator plus terms proportional to p. In physical processes these terms do not contribute when current conservation is demanded

$$
D^{\mu\nu\alpha\beta}_{F,\pm\infty}(p) \;=\; \left[g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha}-\,g^{\mu\nu}g^{\alpha\beta}\right]\frac{1}{p^2+i\varepsilon}+O(p)\ . \tag{9.37}
$$

Again, this is different from taking the massive spin-2 propagator (8.26), couple it to conserved currents and taking the massless limit, as is mentioned in [69].

Having obtained the correct massless spin-2 propagator (9.36) it is particularly interesting to see how this limit comes about. Considering the propagator (9.30) (coupled to conserved currents) with a small non-zero mass and requiring that it is a mixture of pure spin-2 and spin-0 (so no ghosts or tachyons) in order to have a kind of massive Brans-Dicke [85] theory, this would imply that $-3 < c < 0$. However with this restriction we cannot take the mass smoothly to zero in order to have a pure massless spin-2 propagator, because this requires $c \to \pm\infty$ as mentioned before.

The above situation of a pure massive spin-2 and spin-0 propagator limiting smoothly to a pure massless spin-2 propagator can be obtained in [60], but there the set-up is quite different as well as the original goal.

9.5 Momentum Representation

To finalize the description of the higher spin fields coupled to auxiliary fields we give the momentum representation of these fields in this section. Also, we give the relations which hold for the various creation and annihilation operators.

A solution to the EoM of the fields in (9.2), (9.3) and (9.4) in terms of the auxiliary fields is

$$\begin{aligned} A_\mu &= V_\mu + \frac{\partial_\mu}{M_1} B \ , \\ \psi_\mu &= \Psi_\mu + \frac{1}{3}\left(\gamma_\mu - \frac{2i\partial_\mu}{M_{3/2}}\right)\chi \ , \\ \eta_\mu &= \Phi_{1,\mu} + \frac{2(3+c)}{c(1-c)}\frac{\partial_\mu}{M_2}\epsilon \ , \\ h_{\mu\nu} &= \Phi_{2,\mu\nu} - \frac{1}{M_2}\left(\partial_\mu \Phi_{1,\nu} + \partial_\nu \Phi_{1,\mu}\right) \\ &\quad + \frac{2}{3}\frac{3+c}{1-c}\left(g_{\mu\nu} - \frac{2(3+c)}{c}\frac{\partial_\mu\partial_\nu}{M_2^2}\right)\epsilon \ , \end{aligned} \tag{9.38}$$

where

$$\begin{array}{lll} (\Box + M_1^2)V_\mu = 0 \ , & \partial \cdot V = 0 \ , & \\ (i\partial\!\!\!/ - M_{3/2})\Psi_\mu = 0 \ , & \gamma\cdot\Psi = 0 \ , & i\partial\cdot\Psi = 0 \ , \\ (\Box + M_2^2)\Phi_{2,\mu\nu} = 0 \ , & \partial^\mu \Phi_{2,\mu\nu} = 0 \ , & \Phi^\mu_{2,\mu} = 0 \ , \end{array} \tag{9.39}$$

and are therefore free spin-1, spin-3/2 and spin-2 fields, respectively. The field $\Phi_{1,\mu}$ also satisfies the free spin-1 equations, but is of negative norm as we will see below.

Since the anti-commutator of the χ-field (9.25) and the commutator of the ϵ-field (9.26) contain constants we redefine these fields for convenience

$$\begin{aligned} \chi &= \sqrt{\frac{3}{2}}\chi' \\ \epsilon &= \frac{\sqrt{3}(1-c)}{2(3+c)}\epsilon' \ . \end{aligned} \tag{9.40}$$

[3] Therefore (9.38) becomes

$$\begin{aligned} \psi_\mu &= \Psi_\mu + \frac{1}{\sqrt{6}}\left(\gamma_\mu - \frac{2i\partial_\mu}{M_{3/2}}\right)\chi' \ , \\ \eta_\mu &= \Phi_{1,\mu} + \frac{\sqrt{3}}{c}\frac{\partial_\mu}{M_2}\epsilon' \ , \end{aligned}$$

[3] The part in the commutator of the ϵ-field that determines whether ϵ is ghostlike or not is not taken in the redefinition.

$$\begin{aligned} h_{\mu\nu} &= \Phi_{2,\mu\nu} - \frac{1}{M_2}\left(\partial_\mu \Phi_{1,\nu} + \partial_\nu \Phi_{1,\mu}\right) \\ &\quad + \frac{1}{\sqrt{3}}\left(g_{\mu\nu} - \frac{2(3+c)}{c}\frac{\partial_\mu\partial_\nu}{M_2^2}\right)\epsilon' \ . \end{aligned} \tag{9.41}$$

The momentum representation of the fields is

$$\begin{aligned} B(x) &= \int \frac{d^3p}{(2\pi)^3 2E_B}\left[a_B(p)e^{-ipx} + a_B^\dagger(p)e^{ipx}\right]_{p^0=E_B} \ , \\ V_\mu(x) &= \sum_{\lambda=-1}^{1}\int \frac{d^3p}{(2\pi)^3 2E_V}\left[a_{V,\mu}(p\lambda)e^{-ipx} + a_{V,\mu}^\dagger(p\lambda)e^{ipx}\right]_{p^0=E_V} \ , \end{aligned}$$

$$\begin{aligned} \chi'(x) &= \sum_{s=-\frac{1}{2}}^{\frac{1}{2}}\int \frac{d^3p}{(2\pi)^3 2E_\chi}\left[b_\chi(ps)u_\chi(ps)e^{-ipx} + d_\chi^\dagger(ps)v_\chi(ps)e^{ipx}\right]_{p^0=E_\chi} \ , \\ \Psi_\mu(x) &= \sum_{s=-\frac{3}{2}}^{\frac{3}{2}}\int \frac{d^3p}{(2\pi)^3 2E_\Psi}\left[b_\Psi(ps)u_\mu(ps)e^{-ipx} + d_\Psi^\dagger(ps)v_\mu(ps)e^{ipx}\right]_{p^0=E_\Psi} \ , \end{aligned}$$

$$\begin{aligned} \epsilon'(x) &= \int \frac{d^3p}{(2\pi)^3 2E_\epsilon}\left[a_\epsilon(p)e^{-ipx} + a_\epsilon^\dagger(p)e^{ipx}\right]_{p^0=E_\epsilon} \ , \\ \Phi_{1,\mu}(x) &= \sum_{\lambda=-1}^{1}\int \frac{d^3p}{(2\pi)^3 2E_1}\left[a_{1,\mu}(p\lambda)e^{-ipx} + a_{1,\mu}^\dagger(p\lambda)e^{ipx}\right]_{p^0=E_1} \ , \\ \Phi_{2,\mu\nu} &= \sum_{\lambda=-2}^{2}\int \frac{d^3p}{(2\pi)^3 2E_2}\left[a_{2,\mu\nu}(p\lambda)e^{-ipx} + a_{2,\mu\nu}^\dagger(p\lambda)e^{ipx}\right]_{p^0=E_2} \ , \end{aligned} \tag{9.42}$$

where $E_i = \sqrt{|\vec{p}|^2 + M_i^2}$. In (9.42) the spin-3/2 spinor $u_\mu(ps)$ is a tensor product of a spin-1 polarization vector and a spin-1/2 spinor: $u_\mu = \epsilon_\mu \otimes u$. The normalization of this (spin-1/2) spinor, as well as that of u_χ, is $\bar{u}(ps)u(ps') = 2M\delta_{ss'}$ and of course something similar for the v-spinors. With this normalization the creation and annihilation operators satisfy the following (commutation) relations

$$\begin{aligned} \left[a_B(p), a_B^\dagger(p')\right] &= -(2\pi)^3 2E_B\, \delta^3(p-p') \ , \\ \left[a_{V,\mu}(p\lambda), a_{V,\nu}^\dagger(p'\lambda')\right] &= \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M_1^2}\right)(2\pi)^3 2E_V\, \delta^3(p-p')\delta_{\lambda\lambda'} \ , \end{aligned}$$

$$
\begin{aligned}
\left\{b_\chi(ps), b^\dagger_\chi(p's')\right\} &= \left\{d_\chi(ps), d^\dagger_\chi(p's')\right\} = -(2\pi)^3 2E_\chi\, \delta^3(p-p')\delta_{ss'}\ , \\
\left\{b_\Psi(ps), b^\dagger_\Psi(p's')\right\} &= \left\{d_\Psi(ps), d^\dagger_\Psi(p's')\right\} = (2\pi)^3 2E_\Psi\, \delta^3(p-p')\delta_{ss'}\ ,
\end{aligned}
$$

$$
\begin{aligned}
\left[a_\epsilon(p), a^\dagger_\epsilon(p')\right] &= -\frac{c}{3+c}(2\pi)^3 2E_\epsilon\, \delta^3(p-p')\ , \\
\left[a_{1,\mu}(p\lambda), a^\dagger_{1,\nu}(p'\lambda')\right] &= -\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M_\eta^2}\right)(2\pi)^3 2E_1\, \delta^3(p-p')\delta_{\lambda\lambda'}\ , \\
[a_{2,\mu\nu}(p\lambda), a_{2,\alpha\beta}(p'\lambda')] &= \Bigg[g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - \frac{2}{3}\, g_{\mu\nu}g_{\alpha\beta} \\
&\quad -\frac{1}{M_2^2}\left(p_\mu p_\alpha g_{\nu\beta} + p_\nu p_\alpha g_{\mu\beta} + p_\mu p_\beta g_{\nu\alpha} + p_\nu p_\beta g_{\mu\alpha}\right) \\
&\quad +\frac{2}{3M_2^2}\left(p_\mu p_\nu g_{\alpha\beta} + g_{\mu\nu} p_\alpha p_\beta\right) + \frac{4}{3M_2^4}\, p_\mu p_\nu p_\alpha p_\beta\Bigg] \\
&\quad \times(2\pi)^3 2E_2\, \delta^3(p-p')\delta_{\lambda\lambda'}\ . \qquad (9.43)
\end{aligned}
$$

All other (anti-) commutation relations vanish. These (anti-) commutation relations are such that the relations in (9.24), (9.25) and (9.26) remain valid.

To complete the properties of the fields in momentum space there still are the following relations

$$
\begin{aligned}
&p^\mu a_{V,\mu}(p\lambda) = 0\ , \\
&p^\mu u_\mu(ps) = 0\ , \qquad \gamma^\mu u_\mu(ps) = 0\ , \\
&p^\mu a_{1,\mu}(p\lambda) = 0\ , \\
&p^\mu a_{2,\mu\nu}(p\lambda) = 0\ , \qquad a^\mu_{2,\mu}(p\lambda) = 0\ . \qquad (9.44)
\end{aligned}
$$

Appendices

Appendix D

Δ Propagators

A few definitions of on mass-shell propagators, according to [12], are

$$\begin{aligned}
\Delta(x;m^2) &= \frac{-i}{(2\pi)^3}\int d^4p\epsilon(p_0)\delta(p^2-m^2)e^{-ipx}\ , \\
\Delta^{\pm}(x;m^2) &= (2\pi)^{-3}\int d^4p\theta(\pm p_0)\delta(p^2-m^2)e^{-ipx}\ , \\
\Delta^{(1)}(x;m^2) &= \frac{1}{(2\pi)^3}\int d^4p\,\delta(p^2-m^2)e^{-ipx}\ , \qquad \text{(D.1)}
\end{aligned}$$

which satisfy the relations amongst each other

$$\begin{aligned}
i\Delta(x;m^2) &= \Delta^+(x;m^2)-\Delta^-(x;m^2)\ , \\
\Delta^+(-x;m^2) &= \Delta^-(x;m^2)\ , \\
\Delta^{(1)}(x;m^2) &= \Delta^+(x;m^2)+\Delta^-(x;m^2)\ . \qquad \text{(D.2)}
\end{aligned}$$

Furthermore, there are the following Green functions

$$\begin{aligned}
-\Delta_F(x;m^2) &= i\left[\theta(x_0)\Delta^+(x;m^2)+\theta(-x_0)\Delta^-(x;m^2)\right]\ , \\
\Delta_{ret}(x;m^2) &= -\theta(x^0)\Delta(x;m^2)\ , \\
\Delta_{adv}(x;m^2) &= \theta(-x^0)\Delta(x;m^2)\ , \\
\bar{\Delta}(x;m^2) &= -\frac{1}{2}\,\epsilon(x-y)\Delta(x;m^2)\ , \qquad \text{(D.3)}
\end{aligned}$$

where the Green function of the last line of (D.3) is defined in the book of Nakanishi and Ojima (see [58]). A well known form the the Feynman propagator is

$$\Delta_F(x;m^2) = \frac{1}{(2\pi)^4}\int d^4p\,\frac{e^{-ipx}}{p^2-m^2+i\varepsilon}\ . \qquad \text{(D.4)}$$

Furthermore we define the following Δ propagators

$$\begin{aligned}
\tilde{\Delta}(x) &= -\frac{\partial}{\partial m^2}\,\Delta(x;m^2)|_{m^2=0}\ , \\
\tilde{\tilde{\Delta}}(x) &= \left(\frac{\partial}{\partial m^2}\right)^2 \Delta(x;m^2)|_{m^2=0}\ .
\end{aligned} \tag{D.5}$$

Since the last two lines of (D.5) are also valid for Feynman function we can, by using the integral representation of the Feynman function (D.3), give integral representations for $\tilde{\Delta}_F(x)$ and $\tilde{\tilde{\Delta}}_F(x)$

$$\begin{aligned}
\tilde{\Delta}_F(x;m^2) &= -\frac{1}{(2\pi)^4}\int d^4p\,\frac{e^{-ipx}}{p^4+i\varepsilon}\ , \\
\tilde{\tilde{\Delta}}_F(x;m^2) &= \frac{1}{(2\pi)^4}\int d^4p\,\frac{e^{-ipx}}{p^6+i\varepsilon}\ .
\end{aligned} \tag{D.6}$$

Furthermore we have the important relations

$$\begin{aligned}
\left(\Box+m^2\right)\Delta(x;m^2) &= 0\ , \\
\Delta(x;m^2)|_0 &= 0\ , \\
\left[\partial_0\Delta(x;m^2)\right]|_0 &= -\delta(\vec{x})\ ,
\end{aligned}$$

$$\begin{aligned}
\Box\tilde{\Delta}(x) &= \Delta(x)\ , \\
\tilde{\Delta}(x)|_0 &= \partial_0\tilde{\Delta}(x)|_0 = \partial_0^2\tilde{\Delta}(x)|_0 = 0\ , \\
\partial_0^3\tilde{\Delta}(x)|_0 &= -\delta(\vec{x})\ ,
\end{aligned}$$

$$\begin{aligned}
\Box\tilde{\tilde{\Delta}}(x) &= \tilde{\Delta}(x)\ , \\
\tilde{\tilde{\Delta}}(x)|_0 &= \partial_0\tilde{\tilde{\Delta}}(x)|_0 = \ldots = \partial_0^4\tilde{\tilde{\Delta}}(x)|_0 = 0\ , \\
\partial_0^5\tilde{\tilde{\Delta}}(x)|_0 &= -\delta(\vec{x})\ ,
\end{aligned}$$

$$\left[\partial_0\Delta^{(1)}(x;m^2)\right]|_0 = 0\ . \tag{D.7}$$

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Summary

This thesis contains two parts, the first part deals with pion-nucleon/meson-baryon scattering in the Kadyshevsky formalism and the second one with higher spin field quantization in the framework of Dirac's Constraint analysis.

In the first part we have presented the Kadyshevsky formalism in chapter 2. Main (new) contributions here, are the study of the frame dependence, i.e. n-dependence, of the integral equation and the second quantization.

Couplings containing derivatives and higher spin fields may cause differences and problems as far as the results in the Kadyshevsky formalism and the Feynman formalism are concerned. This is discussed in chapter 3 by means of an example. After a second glance the results in both formalisms are the same, however, they contain extra frame dependent contact terms. Two methods are introduced, which discuss a second source extra terms: the Takahashi-Umezawa (TU) and the Gross-Jackiw (GJ) method. The extra terms coming from this second source cancel the former ones exactly. We have discussed and extended both TU and GJ formalisms: the TU method is a more fundamental one, which makes use of an auxiliary field and the GJ method is a more systematic and pragmatic method. It is particularly useful for studying the frame dependence. Both formalisms, however, yield the same results. With the use of (one of) these methods the final results for the S-matrix or amplitude are covariant and frame independent (n-independent). At the end of chapter 3 we have introduced and discussed the $\bar{P}$-method and last but nog least we have shown that the TU method can be derived from the BMP theory.

After discussing the Kadyshevsky formalism in great detail we have applied it to the pion-nucleon system, although we have presented it in such a way that it can easily be extended to other meson-baryon systems. The results for meson exchange are given in chapter 4 and those for baryon exchange in chapter 5.

Chapter 5 also contains a formal introduction and detail discussion of

so-called pair suppression. We have formally implemented "absolute" pair suppression and applied it to the baryon exchange processes, although it is in principle possible to also allow for some pair production. For the resulting amplitudes, we have shown, to our knowledge for the first time, that they are causal, covariant and n-independent. Moreover, the amplitudes are just a factor 1/2 of the usual Feynman expressions. This could be intercepted by rescaling the coupling constants in the interaction Lagrangian. The amplitudes contain only positive energy (or if one wishes, only negative energy) initial and final states. This is particularly convenient for the Kadyshevsky integral equation. It should be mentioned that negative energy is present inside an amplitude via the $\Delta(x-y)$ propagator. This is, however, also the case in the academic example of the infinite dense anti-neutron star.

The last chapter of part I (chapter 6) contains the partial wave expansion. This is used for solving the Kadyshevsky integral equation and to introduce the phase-shifts.

In the second part we have quantized the (massive) higher spin fields $j = 1, 3/2, 2$ both in the situation where they are free (chapter 8) and where they are coupled to auxiliary fields (chapter 9). We have done this using Dirac's prescription. For the first time a full constraint analysis and quantization is presented by determining and discussing all constraints and Lagrange multipliers and by giving all equal times (anti) commutation relations. Using free field identities we have come to (anti) commutation relations for unequal times, from which the propagators are determined. In the free fields case (chapter 8) it is explicitly shown that they are non-covariant, as is well known.

In chapter 9 we have coupled auxiliary fields to gauge conditions of the free, massless systems. Introducing mass terms for these auxiliary fields in the Lagrangian brings about free (gauge) parameters. The requirement of explicit covariant propagators only determines the gauge parameter in the spin-3/2 case.

After obtaining all the various (covariant) propagators we have discussed several choices of the parameters and the massless limits of these propagators. We have shown that the propagators do not only have a smooth massless limit but that they also connect to the ones obtained in the massless case (including (an) auxiliary field(s)).

When coupled to conserved currents we have seen that it is possible to obtain the correct massless spin-j propagators carrying only the helicities $\lambda = \pm j_z$. This does not require a choice of the parameter in the spin-1 case, but in the spin-3/2 and in the spin-2 case we have had to make the choices $b = 0$ and $c = \pm\infty$, respectively. We stress however, that in the spin-3/2 and the spin-2 case this limit is only smooth if the massive propagator contains ghosts.

Samenvatting

Dit proefschrift bevat twee delen, het eerste deel handelt over pion-nucleon/meson baryon verstrooiïng in het Kadyshevsky formalisme en het tweede over hogere spin-velden quantizatie in het kader van "Dirac's Constraint" analyse.

In het eerste deel hebben wij het Kadyshevsky formalisme gepresenteerd in hoofdstuk 2. De belangrijkste (nieuwe) bijdragen hierin zijn de bestudering van de stelselafhankelijkheid, in andere woorden de n-afhankelijkheid, van de integraal vergelijking en de tweede quantizatie.

Koppelingen die afgeleiden en hogere spin velden bevatten, kunnen verschillen en problemen veroorzaken voor wat betreft de resultaten in het Kadyshevsky en het Feynman formalisme. Dit is besproken in hoofdstuk 3 door middel van een voorbeeld. Na een tweede blik zijn de resultaten in beide formalismen wel gelijk, maar bevatten ze extra stelselafhankelijke contact termen. Twee methodes zijn geïntroduceerd, welke een tweede bron van extra termen bespreken: de Takahashi-Umezawa (TU) en de Gross-Jackiw (GJ) methode. De extra termen die van deze tweede bron komen vallen precies weg tegen de eerdere genoemde extra termen. We hebben zowel het TU, als het GJ formalisme besproken en uitgebreid: de TU methode is een meer fundamentele methode, welke gebruik maakt van een hulpveld en de GJ methode is meer systematische en pragmatische methode. Het is met name handig voor het bestuderen van de stelselafhankelijkheid. Beide formalismes geven echter hetzelfde resultaat. Met behulp van (een van) deze methoden is het uiteindelijke resultaat voor de S-matrix of de amplitude covariant en stelselonafhankelijk (n-onafhankelijk). Aan het einde van hoofdstuk 3 hebben we de $\bar{P}$ geïntroduceerd en bediscussieerd en als laatste, maar niet als minst belangrijke, hebben we laten zien dat de TU methode kan worden afgeleid vanuit de BMP theorie.

Nadat we het Kadyshevksy formalism in veel detail hebben besproken, hebben we het toegepast op het pion-nucleon systeem, alhoewel we het op zo'n manier hebben gepresenteerd dat het eenvoudig kan worden uitgebreid naar andere meson-baryon systemen. De resultaten voor mesonuitwisseling zijn gegeven in hoofdstuk 4 en die voor baryonuitwisseling in hoofdstuk 5.

Hoofdstuk 5 bevat ook een formele introductie en gedetailleerde discussie van zogenoemde paar-onderdrukking. We hebben "absolute" paaronderdrukking formeel geïmplementeerd en toegepast op de baryon uitwisselings processen, alhoewel het in principe mogelijk is om een beetje paarproductie toe te staan. Voor de resulterende amplitudes hebben we, naar onze kennis voor de eerste keer, laten zien dat ze causaal, covariant en n-onafhankelijk zijn. Sterker, de amplitudes verschillen een factor 1/2 van de normale Feynman uitdrukkingen. Dit kan worden ondervangen door de koppelingsconstantes in de interactie Lagrangiaan te herschalen. De amplitudes bevatten alleen begin en eindtoestanden met positieve energie (of, mocht dat gewenst zijn, alleen negatieve energie). Dit is met name handig voor de Kadyshevsky integraal vergelijking. Het moet worden genoemd dat negatieve energie aanwezig is in een amplitude via de $\Delta(x-y)$ propagator. Dit is echter ook het geval in het academisch voorbeeld van de oneindig dichte anti-neutron ster.

Het laatste hoofdstuk van deel 1 (hoofdstuk 6) bevat de partiële golf ontwikkeling. Dit wordt gebruikt voor het oplossen van de Kadyshevksy integraal vergelijking en om de fase-verschuivingen te introduceren.

In het tweede deel hebben we de (massieve) hogere spin-velden $j = 1,\ 3/2,\ 2$ gequantizeerd in zowel de situatie waar ze vrij zijn (hoofdstuk 8), als waar ze gekoppeld zijn aan hulpvelden (hoofdstuk 9). We hebben dit gedaan gebruikmakende van Dirac zijn voorschrift. Voor de eerste keer is een volledige restrictie analyse en quantizatie gepresenteerd door alle restricties en Lagrange multiplicatoren en door alle gelijke tijd (anti-) commutatie relaties te bepalen en te bediscussiëren. Door gebruik te maken van vrije veld identiteiten, zijn we gekomen tot (anti-) commutatie relaties voor niet gelijke tijden, waaruit de propagatoren zijn bepaald. In het geval van het vrije veld (hoofdstuk 8) is expliciet aangetoond dat ze niet covariant zijn, zoals wel bekend is.

In hoofdstuk 9 hebben we hulpvelden gekoppeld aan ijkkondities van de vrije, massaloze systemen. Het introduceren van massa termen voor deze hulpvelden in de Lagrangiaan brengt vrije (ijk-)parameters met zich mee. De vereiste dat de propagatoren expliciet covariant zijn, legt alleen de ijkparameter in het geval van spin-3/2 vast.

Nadat alle verschillende (covariante,) propagatoren zijn bepaald, hebben we verschillende keuzes van de parameters bestudeerd en de massaloze limieten van de propagatoren. We hebben laten zien dat de propagatoren niet alleen een gladde massaloze limiet hebben, maar dat ze ook aansluiten op diegenen die zijn bepaald in het massaloze geval (inclusief (een) hulpveld(en)).

Wanneer de propagatoren gekoppeld zijn aan behouden stromen, hebben we laten zien dat het mogelijk is om de correcte, massaloze spin-j propagatoren te verkrijgen met alleen de heliciteiten $\lambda = \pm j_z$. Dit vereist niet het maken van een keuze voor de parameter in het geval van spin-1, maar in het geval van spin-3/2 en spin-2 moeten we respectievelijk de keuzes $b = 0$ and $c = \pm\infty$ maken. We benadrukken echter, dat in het geval van spin-3/2 en spin-2 de limiet alleen glad is als de massieve propagator "ghosts" bevat.