Vector-meson–baryon coupling constants in QCD Sum Rules*

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Abstract

The external-field QCD Sum Rules method is used to evaluate the coupling constants of the vector mesons ρ and ω to the nucleon and the Λ, Σ, and Ξ baryons. It is shown that these coupling constants as calculated from QCD Sum Rules are consistent with $SU(3)$-flavor relations. By assuming ideal mixing, this leads to a determination of the $F/(F+D)$ ratio of the vector-meson octet: we find $\alpha_v = 1$ and $\alpha_m = 0.18$ for the vector and the magnetic $F/(F+D)$ ratios, respectively. The sensitivity of the results to the unknown vacuum susceptibility ζ is discussed. The coupling constants with $SU(3)$-breaking effects taken into account are also calculated.

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I. INTRODUCTION

An important ingredient of the baryon-baryon interactions is the exchange of the members of vector-meson nonet (ρ, ϕ, ω, K^*). Vector mesons play also a special role in the electromagnetic interactions of hadrons. The Vector-Meson Dominance (VMD) model [1] relates the hadronic electromagnetic current to the neutral vector-meson fields $V_\mu = \rho^0_\mu$, ω_μ, and ϕ_μ. In this context, the vector-meson–baryon coupling constants are fundamental quantities that one would like to compute from QCD. The Lagrangian density for the interaction of a vector meson with a spin-1/2 baryon is given by

$$\mathcal{L}_{VBB} = -ig_B^V\,\bar{\psi}\,\gamma_\mu\,\psi V^\mu - \frac{f_B^V}{4m}\bar{\psi}\sigma_{\mu\nu}\,\psi\,(\partial^\mu V^\nu - \partial^\nu V^\mu)\,, \tag{1}$$

where $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. The first term ($g_B^V$) is called the vector (electric) coupling and the second one (f_B^V) the tensor (magnetic) coupling; m is a scaling mass to make f_B^V dimensionless, conventionally taken to be equal to the proton mass.

The physical states ϕ and ω are mixtures of the unitary singlet and octet states. We will assume ideal mixing with the mixing angle $\theta_v = 35.3°$, which is close to the experimental value $\theta_v = 37.5°$ [2]. This means that the ϕ-meson is a pure $\bar{s}s$ state, and hence does not couple to the nucleon (in the absence of a strangeness content). The couplings of the vector mesons to the baryon octet can be written in terms of the $NN\rho$ coupling constant and $\alpha_{v,m}$ [3], where α_v (α_m) is the $F/(F+D)$ ratio of the vector (magnetic) coupling constants. VMD predicts $\alpha_v = 1$ via the universal coupling of the ρ-meson to the isospin current [4].

Our aim in this paper is to calculate the vector and the tensor coupling constants of the vector mesons ρ and ω to the N, Λ, Ξ, and Σ baryons using the external-field QCD Sum Rules (QCDSR) [5], which is a powerful tool [6, 7] to extract qualitative and quantitative information about hadron properties [8, 9]. For this purpose, we assume a constant background tensor field $Z_{\mu\nu}$ and evaluate the vacuum-to-vacuum transition matrix element of the two-baryon interpolating fields to construct the sum rules. We define the external vector-meson field as

$$Z_\mu = -\frac{1}{2}\,Z_{\mu\nu}\,x^\nu\,. \tag{2}$$

This background field can be decomposed into symmetric ($Z^S_{\mu\nu}$) and antisymmetric ($Z^A_{\mu\nu}$) parts. The antisymmetric part has been used to calculate the baryon magnetic moments [5, 10–12], while the symmetric part was used in Ref. [13] to determine the vector-meson couplings g_N^ρ and f_N^ω. In this work, we use a similar method to calculate the vector-meson–baryon coupling constants. The sum rules for the antisymmetric part of the external field can be obtained from the sum rules for the baryon magnetic moments in Refs. [5, 10–12], but the numerical results for the couplings cannot be obtained trivially, since they need an independent analysis that takes the sum rules for the symmetric part of the external field into account as well. This analysis was made in Ref. [13] with the aim to calculate the $NN\rho$ and $NN\omega$ couplings. We find it useful to revisit these calculations for a couple of reasons. First, we make a more systematic analysis of the sum rules that includes the single-pole contributions, which were not taken into account in Ref. [13]. Moreover, we extend the calculations to hyperons as well by calculating terms involving the quark mass in the sum rules. We compare our results with VMD and with a successful one-boson-exchange (OBE) model of the NN and YN interaction, the Nijmegen soft-core potential (NSC) [14–19], which was originally derived from Regge-pole theory. The coupling constants obtained from the external-field QCDSR method are defined at $t = 0$, and therefore the comparison to the OBE model is appropriate.

We follow an analysis similar to the one in our earlier work on scalar-meson–baryon coupling constants [20, 21]. We shall first consider the sum rules in the $SU(3)$-flavor symmetric limit to see if the predicted values for the meson-baryon coupling constants from the sum rules are consistent with $SU(3)$ relations. We show that this is indeed the case, which leads to a determination of the $F/(F+D)$ ratio of the vector-meson octet. Furthermore, keeping track of these coupling constants with the $SU(3)$ relations, we obtain the values of the other vector-meson–baryon coupling constants, where we assume ideal mixing. When we extend the calculations from the $S = 0$ to the $S = -1$ and $S = -2$ sectors, flavor-$SU(3)$ breaking occurs due to the s-quark mass and the physical masses of the baryons and mesons. We also consider the $SU(3)$-breaking effects for the sum rules to estimate the amount of breaking, individually for each coupling.

We have organized our work as follows: in section II we present the formulation of QCDSR with an external tensor field and construct the relevant sum rules. We give the numerical analysis of the sum rules and discuss the results in section III. Finally, we arrive at our conclusions for this chapter in section IV.

II. CONSTRUCTION OF THE SUM RULES

We start with the correlation function of the baryon interpolating fields in the presence of a constant background tensor field $Z_{\mu\nu}$, defined by

$$i\int d^4x\ e^{iq\cdot x}\left\langle 0\middle|\mathcal{T}[\eta_B(x)\bar{\eta}_B(0)]\middle|0\right\rangle_Z = \Pi(q) + g_q^V\ Z_{\mu\nu}\ \Pi_Z^{\mu\nu}(q)\ , \tag{3}$$

where g_q^V is the vector-meson–quark coupling constant and η_B are the baryon interpolating fields which are chosen as [8]

$$\eta_N = \epsilon_{abc}[(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu d_c]\,, \tag{4}$$

$$\eta_\Xi = \epsilon_{abc}[(s_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c]\,, \tag{5}$$

$$\eta_\Sigma = \epsilon_{abc}[(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu s_c]\,, \tag{6}$$

$$\eta_\Lambda = (2/3)^{1/2}\epsilon_{abc}[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c - (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c]\,, \tag{7}$$

for N, Ξ, Σ, and Λ, respectively; a, b, c are color indices, and T and C denote transposition and charge conjugation, respectively.

The external field contributes to the correlation function in Eq. (3) in two ways: First, it directly couples to the quark field in the baryon current. Second, it induces the following vacuum condensates:

$$\langle\bar{q}\sigma_{\mu\nu}q\rangle_Z = g_q^V\chi Z_{\mu\nu}^A\,\langle\bar{q}q\rangle\,, \tag{8}$$

$$g_c\langle\bar{q}G_{\mu\nu}q\rangle_Z = g_q^V\kappa Z_{\mu\nu}^A\,\langle\bar{q}q\rangle\,, \tag{9}$$

$$g\,\epsilon_{\mu\nu\alpha\beta}\langle\bar{q}\gamma_5 G_{\alpha\beta}q\rangle_Z = i\,g_q^V\xi Z_{\mu\nu}^A\,\langle\bar{q}q\rangle\,, \tag{10}$$

$$\langle\bar{q}\frac{1}{2}(\gamma_\mu\nabla_\nu + \gamma_\nu\nabla_\mu)q\rangle_Z = g_q^V\zeta Z_{\mu\nu}^S\,\langle\bar{q}q\rangle\,, \tag{11}$$

$$\langle\bar{q}\frac{1}{2}(\nabla_\mu\nabla_\nu + \nabla_\nu\nabla_\mu)q\rangle_Z = -\frac{g_q^V}{8}\langle\bar{q}\boldsymbol{\sigma}\cdot\boldsymbol{G}q\rangle\,g_{\mu\nu} + g_q^V\frac{i}{2}Z_{\mu\nu}^S\,\langle\bar{q}q\rangle + i\,g_q^V\,\phi\,Z_{\mu\nu}^S\langle\bar{q}q\rangle\,, \tag{12}$$

where (χ, κ, ξ) and (ζ, ϕ) are the susceptibilities related to $Z^A_{\mu\nu}$ and $Z^S_{\mu\nu}$, respectively. These susceptibilities are defined in terms of the vector-meson–quark coupling constants g_q^V, where we assume

$$g_u^\omega = g_d^\omega = g_q^V \,, \tag{13}$$

for the isospin $I = 0$ ω-current, and

$$g_u^\rho = -g_d^\rho = g_q^V \,, \tag{14}$$

for the isospin $I = 1$ ρ-current. For the couplings of the external field to the s-quark we assume

$$g_s^\omega = g_s^\rho = 0 \,. \tag{15}$$

Eq. (13) and Eq. (14) can be justified from the degeneracy and the equal decay constants of the ρ- and the ω-mesons [6–8] by using the Current Field Identities and VMD.

At the quark level, we obtain for the correlation functions:

$$\left\langle 0 \middle| \mathcal{T}[\eta_N(x)\bar{\eta}_N(0)] \middle| 0 \right\rangle_Z = 2i\epsilon^{abc}\epsilon^{a'b'c'} Tr\{S_u^{aa'}(x)\gamma_\nu C[S_u^{bb'}(x)]^T C\gamma_\mu\}\gamma_5\gamma^\mu S_d^{cc'}(x)\gamma^\nu\gamma_5 \,, \tag{16}$$

$$\left\langle 0 \middle| \mathcal{T}[\eta_\Xi(x)\bar{\eta}_\Xi(0)] \middle| 0 \right\rangle_Z = 2i\epsilon^{abc}\epsilon^{a'b'c'} Tr\{S_s^{aa'}(x)\gamma_\nu C[S_s^{bb'}(x)]^T C\gamma_\mu\}\gamma_5\gamma^\mu S_u^{cc'}(x)\gamma^\nu\gamma_5 \,, \tag{17}$$

$$\left\langle 0 \middle| \mathcal{T}[\eta_\Sigma(x)\bar{\eta}_\Sigma(0)] \middle| 0 \right\rangle_Z = 2i\epsilon^{abc}\epsilon^{a'b'c'} Tr\{S_u^{aa'}(x)\gamma_\nu C[S_u^{bb'}(x)]^T C\gamma_\mu\}\gamma_5\gamma^\mu S_s^{cc'}(x)\gamma^\nu\gamma_5 \,, \tag{18}$$

$$\begin{aligned}\left\langle 0 \middle| \mathcal{T}[\eta_\Lambda(x)\bar{\eta}_\Lambda(0)] \middle| 0 \right\rangle_Z = \frac{2i}{3}\epsilon^{abc}\epsilon^{a'b'c'} \Big(& Tr\{S_u^{aa'}(x)\gamma_\nu C[S_s^{bb'}(x)]^T C\gamma_\mu\}\gamma_5\gamma^\mu S_d^{cc'}(x)\gamma^\nu\gamma_5 \\ & + Tr\{S_d^{cc'}(x)\gamma_\nu C[S_s^{bb'}(x)]^T C\gamma_\mu\}\gamma_5\gamma^\mu S_u^{aa'}(x)\gamma^\nu\gamma_5 \\ & - \gamma_5\gamma_\mu S_d^{cc'}(x)\gamma_\nu C[S_s^{bb'}(x)]^T C\gamma^\mu S_u^{aa'}(x)\gamma^\nu\gamma_5 \\ & - \gamma_5\gamma_\mu S_u^{aa'}(x)\gamma_\nu C[S_s^{bb'}(x)]^T C\gamma^\mu S_d^{cc'}(x)\gamma^\nu\gamma_5 \Big) \,, \end{aligned} \tag{19}$$

where S_q represents the quark propagator in the presence of the external field and we use the quark propagator given in Ref. [13].

Lorentz covariance and parity conservation implies that the correlation function can be written in terms of different Lorentz-Dirac structures, *viz.*

$$\begin{aligned} g_q^V\, \Pi_Z^{\mu\nu}(q) = {} & \Pi_1^S(p_\mu\gamma_\nu + p_\nu\gamma_\mu) + \Pi_2^S \hat{p} p_\mu p_\nu + \Pi_3^S p_\mu p_\nu + \Pi_4^S \hat{p}(p_\mu\gamma_\nu + p_\nu\gamma_\mu) \\ & + \Pi_1^A(\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}) + \Pi_2^A \hat{p}(p_\mu\gamma_\nu - p_\nu\gamma_\mu) + \Pi_3^A \sigma_{\mu\nu} \,, \end{aligned} \tag{20}$$

where Π_1^S, Π_2^S, Π_3^S, and Π_4^S are related to the symmetric part of the external field, and Π_1^A, Π_2^A, and Π_3^A are related to the antisymmetric part of the external field. For the antisymmetric part of the external field, we construct the sum rules at the structure $\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}$, which have also been used for the determination of the baryon magnetic moments [5, 10–12]. For the symmetric part of the external field, we construct the sum rules at the structures $p_\mu\gamma_\nu + p_\nu\gamma_\mu$

(hereafter structure I) and $\hat{p} p_\mu p_\nu$ (hereafter structure II), for reasons that will become clear below.

In order to construct the hadronic side, we saturate the correlator in Eq.(3) with baryon states,

$$\Pi_Z^{\mu\nu}(q) = \frac{\langle 0|\eta_B|B\rangle}{q^2 - m_B^2} \, \langle B|V\,B\rangle \, \frac{\langle B|\bar{\eta}_B|0\rangle}{q^2 - m_B^2} \ , \tag{21}$$

and define the vector-meson–baryon interaction by the following vertices:

$$\Gamma_{\omega BB} \equiv \langle B|\omega\, B\rangle = \bar{\upsilon}\Big(g_B^\omega\, \gamma_\mu + f_B^\omega\, \frac{i}{2m}\sigma_{\mu\nu}\, q^\nu\Big)\upsilon \cdot \omega^\mu \, , \tag{22}$$

$$\Gamma_{\rho BB} \equiv \langle B|\rho\, B\rangle = \bar{\upsilon}\Big(g_B^\rho\, \gamma_\mu + f_B^\rho\, \frac{i}{2m}\sigma_{\mu\nu}\, q^\nu\Big)\boldsymbol{\tau}\upsilon \cdot \boldsymbol{\rho}^\mu \, , \tag{23}$$

where υ is the Dirac spinor for the baryon, which is normalized as $\bar{\upsilon}\upsilon = 2m_B$. In Eq. (21) we defined the overlap amplitude of the baryons as $\lambda_B = \langle 0|\eta_B|B\rangle$.

The sum rules are obtained by matching the operator product expansion (OPE) side with the hadronic side and applying the Borel transformation. The sum rules for N, Σ, Ξ, and Λ are given as follows at structure I:

$$\Big[M^6\, E_1^N\, L^{-4/9}(2g_u^V + g_d^V) + \frac{8M^4}{3}\, E_0^N\, L^{2/9}\, \zeta\, a_q\, (4g_u^V + g_d^V) + \frac{4}{3}a_q^2\, L^{4/9}(2g_u^V + g_d^V)\Big]\frac{e^{m_N^2/M^2}}{\tilde{\lambda}_N^2} = g_N^V + C_N\, M^2 \, , \tag{24}$$

$$g_u^V\Big[2\, M^6\, E_1^\Sigma\, L^{-4/9} + \frac{32M^4}{3}\, E_0^N\, L^{2/9}\, \zeta\, a_q\, g_u^V + \frac{8}{3}a_q^2\, L^{4/9} - 4m_s\,(f+1)\, a_q\, M^2\Big]\frac{e^{m_\Sigma^2/M^2}}{\tilde{\lambda}_\Sigma^2} = g_\Sigma^V + C_\Sigma\, M^2 \, , \tag{25}$$

$$g_u^V\Big[M^6\, E_1^\Xi\, L^{-4/9} + \frac{8M^4}{3}\, E_0^N\, L^{2/9}\, \zeta\, a_q\, g_d^V + \frac{4}{3}\,(f+1)^2 a_q^2\, L^{4/9}\Big]\frac{e^{m_\Xi^2/M^2}}{\tilde{\lambda}_\Xi^2} = g_\Xi^V + C_\Xi\, M^2 \, , \tag{26}$$

$$\begin{aligned}(g_u^V + g_d^V)\Big[M^6\, E_1^\Lambda\, L^{-4/9} &+ \frac{32}{9}M^4\, E_0^N\, L^{2/9}\, \zeta\, a_q + \frac{4}{9}\,(4f+3)a_q^2\, L^{4/9} \\ &+ \frac{2}{3}\, m_s\,(1-3f)\, a_q\, M^2\Big]\frac{e^{m_\Lambda^2/M^2}}{\tilde{\lambda}_\Lambda^2} = g_\Lambda^V + C_\Lambda\, M^2 \, ,\end{aligned} \tag{27}$$

and at structure II:

$$\Big[M^6\, E_0^N\, L^{-4/9}(2g_u^V + g_d^V) + \frac{8M^4}{3}\zeta\, L^{2/9}\, a_q\, (g_u^V + g_d^V) + \frac{4}{3}a_q^2\, L^{4/9}(2g_u^V + g_d^V)\Big]\frac{e^{m_N^2/M^2}}{\tilde{\lambda}_N^2} = g_N^V + C_N\, M^2 \tag{28}$$

$$g_u^V\Big[2\, M^6\, E_0^\Sigma\, L^{-4/9} + \frac{8M^4}{3}\zeta\, L^{2/9}\, a_q\, g_u^V + \frac{8}{3}a_q^2\, L^{4/9} + 4m_s\,(f+1)\, a_q\, M^2\Big]\frac{e^{m_\Sigma^2/M^2}}{\tilde{\lambda}_\Sigma^2} = g_\Sigma^V + C_\Sigma\, M^2 \, , \tag{29}$$

$$g_u^V\left[M^6\,E_0^{\Xi}\,L^{-4/9}\;+\;\frac{8M^4}{3}\zeta\,L^{2/9}\,a_q\,g_d^V\;+\;\frac{4}{3}\,(f+1)^2 a_q^2\,L^{4/9}\;\right]\frac{e^{m_\Xi^2/M^2}}{\tilde\lambda_\Xi^2}\;=g_\Xi^V\;+\;C_\Xi\,M^2 \tag{30}$$

$$(g_u^V+g_d^V)\left[M^6\,E_0^{\Lambda}\,L^{-4/9}\;+\;\frac{10}{9}M^4\,L^{2/9}\zeta\,a_q\;+\;\frac{4}{9}\,(4f+3)a_q^2\,L^{4/9}\right.$$
$$\left.+\;\frac{2}{3}\,m_s\,(1-3f)\,a_q\,M^2\right]\frac{e^{m_\Lambda^2/M^2}}{\tilde\lambda_\Lambda^2}\;=g_\Lambda^V\;+\;C_\Lambda\,M^2\,, \tag{31}$$

where $a_q=-(2\pi)^2\langle\bar qq\rangle$, M is the Borel mass, and we incorporated the effects of the anomalous dimensions of various operators through the factor $L=\ln(M^2/\Lambda_{QCD}^2)/\ln(\mu^2/\Lambda_{QCD}^2)$, where μ is the renormalization scale and Λ_{QCD} is the QCD scale parameter. We have defined $f=\langle\bar qq\rangle/\langle\bar ss\rangle-1$, which is a parameter that quantifies $SU(3)$ breaking in the vacuum condensates. We use these sum rules for the determination of the vector couplings, g.

The sum rules involving the antisymmetric part of the external field can easily be derived from the magnetic-moment sum rules in Refs. [5, 10–12]. We use Eqs. (13)-(15) with the sum rules at the structure $\hat p\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat p$, which were also used for the determination of the magnetic moments. We obtain:

$$\left\{4M^6\,E_1^N\,L^{-4/9}g_u^V\;+\;\frac{4}{9}a_q^2\,L^{4/9}[-(2g_u^V+3g_d^V)+g_u^V\,(2\kappa-\xi)]+\frac{b}{6}M^2\,L^{-4/9}(4g_u^V+g_d^V)\right.$$
$$\left.-\;\frac{8}{3}\chi a_q^2\,L^{-4/27}g_u^V\left[M^2-\frac{m_0^2L^{-4/9}}{8}\right]\right\}\frac{e^{m_N^2/M^2}}{\tilde\lambda_N^2}\;=(g_N^V+f_N^V)\;+\;C_N'\,M^2\,, \tag{32}$$

$$g_u^V\left\{4M^6\,E_1^{\Sigma}\,L^{-4/9}\;+\;\frac{4}{9}a_q^2\,L^{4/9}[-(2+(2\kappa-\xi)]+\frac{2b}{3}M^2\,L^{-4/9}\right.$$
$$\left.-\;\frac{8}{3}\chi a_q^2\,L^{-4/27}\left[M^2-\frac{m_0^2L^{-4/9}}{8}\right]\right\}\frac{e^{m_\Sigma^2/M^2}}{\tilde\lambda_\Sigma^2}\;=(g_\Sigma^V+f_\Sigma^V)\;+\;C_\Sigma'\,M^2\,, \tag{33}$$

$$g_u^V\left\{-\;\frac{4}{3}a_q^2\,L^{4/9}\;+\;\frac{b}{6}M^2\,L^{-4/9}+24\,m_s\,a_q(1+f)\,M^2\right\}\frac{e^{m_\Xi^2/M^2}}{\tilde\lambda_\Xi^2}\;=(g_\Xi^V+f_\Xi^V)\;+\;C_\Xi'\,M^2 \tag{34}$$

$$(g_u^V+g_d^V)\left\{\frac{2}{3}M^6\,E_1^{\Lambda}\,L^{-4/9}\;-\;\frac{4}{27}a_q^2\,L^{4/9}[7+8f-\frac{1}{2}(2\kappa-\xi)(1+2f)]-\frac{4}{9}\chi a_q^2\,L^{-4/27}\left[M^2-\frac{m_0^2L^{-4/9}}{8}\right]\right.$$
$$\left.+\;\frac{2b}{9}M^2\,L^{-4/9}-\frac{2}{9}m_s\,M^2\,a_q\,[19L^{-4/9}-8(2\kappa-\xi)]\right\}\frac{e^{m_\Lambda^2/M^2}}{\tilde\lambda_\Lambda^2}\;=(g_\Lambda^V+f_\Lambda^V)\;+\;C_\Lambda'\,M^2\,.$$

In the above sum rules, the continuum contributions are included by the factors

$$E_n^B\;\equiv\;1-(1+x_B+...+\frac{x_B^n}{n!})e^{-x_B}\,, \tag{36}$$

with $x_B=s_0^B/M^2$, where s_0^B is the continuum threshold. We have included the single-pole contributions with the factors $C_B^{(\prime)}$.

III. ANALYSIS OF THE SUM RULES

In order to proceed to the numerical analysis, we arrange the RHS of the sum rules in the form

$$f(M^2) \equiv F + C_B^{(\prime)} M^2 \ , \tag{37}$$

and fit the LHS to $f(M^2)$. We determine the g couplings from the sum rules in Eqs. (24)-(27), while we subtract these sum rules from the ones in Eqs. (32)-(35) to obtain the sum rules for the f couplings. For the vacuum parameters, we adopt standard values that have been used in QCDSR; for a review and discussion of QCD parameters see *e.g.* Refs. [22, 23]. The quark condensate a_q can be estimated using the Gell-Mann–Oakes–Renner relation,

$$m_\pi^2 f_\pi^2 = -(m_u + m_d) \langle \bar{q}q \rangle \ , \tag{38}$$

which gives $\langle \bar{q}q \rangle = -(0.243)^3$ GeV^3 for pion mass $m_\pi = 138$ MeV, pion decay constant $f_\pi = 93$ MeV, and quark masses $m_u = 4.2$ MeV and $m_d = 7.5$ MeV. Taking into account the uncertainties in the quark masses, we adopt

$$a_q = 0.51 \pm 0.03 \text{ GeV}^3 \, . \tag{39}$$

For the gluon condensate, we use the value

$$b = 0.47 \text{ GeV}^4 \, , \tag{40}$$

as determined from the charmonium sum rules [6, 7]. The value of the parameter m_0^2 has been taken in early baryon sum rules [24, 25] and heavy-light quark system analyses [26] as

$$m_0^2 = 0.8 \text{ GeV}^2 \, . \tag{41}$$

The commonly accepted value of the overlap amplitude of the nucleon as $\tilde{\lambda}_N^2 = 2.1$ GeV^6 is taken from Ref. [5] and the continuum threshold for the nucleon case is taken in the region 2.0 GeV$^2 \leq s_0^N \leq 2.5$ GeV^2. The values of the susceptibilities have been estimated in early magnetic-moment calculations [5, 10]. In this work, we adopt the average values of these susceptibilities as $\chi = -4.5$ GeV^{-2}, $\kappa = 0.4$ and $\xi = -0.8$ [27]. Finally, we use $\mu = 0.5$ GeV for the renormalization scale and $\Lambda_{QCD} = 0.1$ GeV for the QCD scale parameter.

We shall first consider the sum rules in the $SU(3)$-flavor symmetric limit, where we take $m_q = m_s = 0$ and $f = 0$. In this limit we also set the physical parameters of all the baryons equal to the ones of the nucleon: $m_B = m_N = 0.94$ GeV, $\tilde{\lambda}_B^2 = \tilde{\lambda}_N^2 = 2.1$ GeV^6, $s_0^B \equiv s_0^N$. In this $SU(3)$ limit we choose the Borel window 0.8 GeV$^2 \leq M^2 \leq 1.4$ GeV^2 which is commonly identified as the fiducial region for the nucleon mass sum rules. For the vector-meson–quark coupling constant we adopt the value

$$g_q^V = g_q^\rho = 3.7 \, , \tag{42}$$

as estimated from Nambu-Jona-Lasinio model of Ref. [28], which was used to successfully reproduce the $\rho\pi\pi$ coupling constant.

In order to determine the values of the vector couplings from the sum rules in Eqs. (24)-(31), one needs to know the value of the susceptibility ζ, which is unknown. We note, however, that if ζ is negligibly small, then the sum rules at structures I and II are consistent

TABLE I: The vector-meson–baryon coupling constants in the $SU(3)$ limit for the average values of the vacuum parameters.

M		NNM	$\Lambda\Lambda M$	$\Xi\Xi M$	$\Sigma\Sigma M$	$\Lambda\Sigma M$	$\Sigma N M$	$\Lambda N M$
ω	g	6.9	4.6	2.3	4.6			
	f	−2.2	−5.7	−4.9	2.7			
ϕ	g	0	−3.3	−6.5	−3.3			
	f	0	−4.8	−3.8	6.9			
ρ	g	2.3		2.3	4.6	0		
	f	7.6		−4.9	2.7	6.7		
K^*	g						−2.3	−4.0
	f						4.9	−5.9

with each other and have the nice feature that $g_N^\rho/g_N^\omega = 1/3$, which agrees well with the OBE potential model [29] and the VMD model [1] results. Therefore, we first analyze the sum rules for $\zeta = 0$ and then discuss the deviations for arbitrary ζ values. We present the Borel mass dependence of vector and tensor coupling constants of ρ and ω to the nucleon in Fig. 1 and to the hyperons in Fig. 2, for the average values of the vacuum parameters. The single-pole contributions (*cf.* the slopes in Figs. 1 and 2) are quite important especially in the case of the sum rules for the tensor couplings. Taking into account the uncertainties in s_0^B and a_q, the predicted values for the coupling constants of the ρ- and ω-mesons to the baryons read:

$$\begin{aligned}
&g_N^\omega = 7.2 \pm 1.8,\\
&g_\Sigma^\omega \equiv g_\Sigma^\rho \equiv g_\Lambda^\omega = 4.8 \pm 1.2,\\
&g_N^\rho \equiv g_\Xi^\omega \equiv g_\Xi^\rho = 2.4 \pm 0.6,\\
&g_\Lambda^\rho \equiv f_\Lambda^\rho = 0,\\
&f_N^\rho = 7.7 \pm 1.9,\\
&f_N^\omega = -2.2 \pm 0.6,\\
&f_\Sigma^\rho \equiv f_\Sigma^\omega = 2.3 \pm 0.4,\\
&f_\Xi^\rho \equiv f_\Xi^\omega = -5.0 \pm 1.0,\\
&f_\Lambda^\omega = -5.7 \pm 1.0.
\end{aligned} \tag{43}$$

Our next concern is to investigate the $SU(3)$ relations for the vector-meson–baryon interactions and see if the coupling constants above as obtained from QCDSR are consistent with these relations. For this purpose, we calculate the coupling constants in Eq. (43) with the central values of the parameters: $a_q = 0.51$ GeV^2 and $s_0^B = 2.3$ GeV^2. We assume the ideal mixing angle $\theta_s \simeq 35.3°$. The $F/(F+D)$ ratios, α_v and α_m, can directly be calculated via the relations

$$\frac{g_\Xi^\omega - g_N^\omega}{g_\Sigma^\omega - g_N^\omega} = \frac{-2\alpha_v}{1-2\alpha_v}\,, \tag{44}$$

$$\frac{f_\Xi^\omega - f_N^\omega}{f_\Sigma^\omega - f_N^\omega} = \frac{-2\alpha_m}{1-2\alpha_m}\,. \tag{45}$$

With straightforward algebra, the values of the $F/(F+D)$ ratios $\alpha_{v,m}$, and the octet, $g^{v,m}$, and the singlet couplings, $g_1^{v,m}$, are then determined as

$$\alpha_v = 1\,,\ \alpha_m = 0.18\,,\ g^v \equiv g_N^\rho = 2.3\,,\ g^m \equiv f_N^\rho = 7.6\,,\ g_1^v = 5.6\,,\ g_1^m = -1.8\,. \quad (46)$$

Inserting $\alpha_{v,m}$, $g^{v,m}$ and $g_1^{v,m}$ into the $SU(3)$ relations we observe that the coupling constants as determined from QCDSR are consistent with $SU(3)$. This also gives $g_N^\phi = f_N^\phi = 0$, which is justified by the zero strangeness content of the nucleon and by the ideal-mixing scheme. In Table I we give all the vector-meson–baryon coupling constants, obtained from these relations.

In Fig. 3, we present the dependence of $\alpha_v = F/(F+D)$ on the susceptibility ζ for the sum rules at structures I and II, at $M^2 = 1$ GeV^2 and for the average values of the other vacuum parameters. The sum rules are rather sensitive to a change in the value of ζ, since it appears in the coefficient of a dimension-3 operator. The sum rule at structure I shows a more reliable behavior. In Fig. 4, the dependence of g_N^ρ/g_N^ω on the susceptibility ζ for the sum rules at structures I and II is given. For $|\zeta| > 1$, the terms in the sum rules involving ζ dominate. In order to avoid the pole in α_v (for structure I) on the negative ζ-plane, we concentrate on the region $0 \le \zeta \le 1$ GeV^{-1}, where we obtain $5.2 \le g_N^\omega \le 12.7$ and $1.7 \le g_N^\rho \le 5.2$. This implies that away from $\zeta = 0$, g_N^ρ/g_N^ω tends to increase for the sum rules at structure I, and gets as high as 0.5, while the value of α_v gets as low as 0.8 . These results disagree with the ones from the OBE potential model [29] and the VMD model [1], which give $g_N^\rho/g_N^\omega = 1/3$ and $\alpha_v = 1$.

Next, we turn to the effect of $SU(3)$-flavor breaking, where we allow $m_s = 0.15$ GeV and $f = -0.2$, keeping $m_u = m_d \equiv 0$. We also restore the physical values for the masses and the other parameters of the baryons [30, 31]:

$$\begin{aligned}\tilde\lambda_\Lambda^2 &= 3.3\ \text{GeV}^6\,, \quad \tilde\lambda_\Xi^2 = 4.6\ \text{GeV}^6\,, \quad \tilde\lambda_\Sigma^2 = 3.3\ \text{GeV}^6\,,\\ s_0^\Lambda &= 3.1 \pm 0.3\ \text{GeV}^2\,, \quad s_0^\Xi = 3.6 \pm 0.4\ \text{GeV}^2\,, \quad s_0^\Sigma = 3.2 \pm 0.3\ \text{GeV}^2\,. \end{aligned} \quad (47)$$

The corresponding Borel windows are chosen as follows:

$$\begin{aligned}&\text{for } \Lambda\,, \quad 1.0\ \text{GeV}^2 \le M^2 \le 1.4\ \text{GeV}^2\,, \quad (48)\\ &\text{for } \Xi\,, \quad 1.5\ \text{GeV}^2 \le M^2 \le 1.9\ \text{GeV}^2\,,\\ &\text{for } \Sigma\,, \quad 1.2\ \text{GeV}^2 \le M^2 \le 1.6\ \text{GeV}^2\,.\end{aligned}$$

We follow a procedure similar to the one in the $SU(3)$-flavor conserving case and fit the LHS's of the sum rules to the function in Eq. (37) in the Borel windows specified in Eq. (48). Taking into account the uncertainties in s_0^B and a_q, the predicted values for the coupling constants of the ρ- and ω-mesons to Λ, Ξ, and Σ with the $SU(3)$-flavor breaking effects read:

$$\begin{aligned}&g_\Lambda^\omega = 2.9 \pm 1.1\,,\ g_\Xi^\omega \equiv g_\Xi^\rho = 1.1 \pm 0.7\,,\ g_\Sigma^\omega \equiv g_\Sigma^\rho = 3.1 \pm 1.2\,,\\ &f_\Lambda^\omega = -4.0 \pm 0.8\,,\ f_\Xi^\omega \equiv g_\Xi^\rho = 2.4 \pm 0.6\,,\ f_\Sigma^\omega \equiv g_\Sigma^\rho = 7.0 \pm 1.6\,. \end{aligned} \quad (49)$$

We observe that the $SU(3)$-breaking effects modify the couplings by 30-50%, which indicates a large breaking. While the $\Sigma\Sigma\omega$ and $\Sigma\Sigma\rho$ coupling constants increase with $SU(3)$-breaking effects, the other coupling constants tend to decrease.

IV. DISCUSSION AND CONCLUSIONS

In this work, we have calculated the vector-meson–baryon coupling constants, which are important quantities in OBE models of the YN and YY interactions, employing the external-field QCDSR method. The main uncertainties in the results stem from the undetermined

QCD parameters. Although the values of the susceptibilities χ, ξ, and κ are relatively better-known from magnetic-moment calculations, ζ is undetermined. We have first made the analysis by taking ζ negligibly small, which produces couplings in agreement with the ones from the literature. Then, we have analyzed the sum rules for arbitrary ζ and observed that the results are sensitive to a change in this susceptibility. In this respect, an independent determination of the susceptibility ζ is desirable.

The coupling constants can be determined in terms of vector-meson–quark coupling constant in this method. In order to compare our results with the others in the literature and to remain as model-independent as possible, we find it useful to give the following ratios of the coupling constants in the $SU(3)$ limit for the average values of the vacuum parameters,

$$\frac{f_N^\rho}{g_N^\rho} = 3.8\,, \qquad \frac{f_N^\omega}{g_N^\omega} = -0.3\,, \tag{50}$$

which compares well with the results from VMD,

$$\frac{f_N^\rho}{g_N^\rho} \equiv \frac{F_2^v}{F_1^v} = 3.3\ , \ \frac{f_N^\omega}{g_N^\omega} \equiv \frac{F_2^s}{F_1^s} = -0.1\,, \tag{51}$$

where F_1^s (F_1^v) and F_2^s (F_2^v) are the isoscalar (isovector) electric and magnetic form factors of the nucleon, respectively, at zero momentum transfer. This result is not totally surprising, since a similar scheme to the one of electromagnetic coupling has been assumed for vector-meson–baryon interaction. These ratios are close to the ones from the NSC NN potential model [29], which are $f_N^\rho/g_N^\rho = 4.2$ and $f_N^\omega/g_N^\omega = 0.3$. Our value for the vector $NN\rho$ coupling constant, with the choice of the quark-ρ coupling constant in Eq. (42), agrees with the one from the recent Nijmegen extended-soft-core (ESC) potential model [19], which is $g_N^\rho = 2.8$. The ESC model gives $g_N^\rho/g_N^\omega = 1/4$, that is, a value for the $NN\omega$ coupling constant larger than what we have obtained from QCDSR. Since from $SU(3)$ symmetry, ideal mixing, and $\alpha_v = 1$ it follows that

$$g_N^\omega + \sqrt{2} g_N^\phi = 3 g_N^\rho\,, \tag{52}$$

the main reason for this is the sizable $NN\phi$ coupling in NSC potential models, which is simply $g_N^\phi = 0$ in the QCDSR. Such a large value for the $NN\omega$ coupling constant as in the ESC or $^3P_0$ models [19] requires a quark-ω coupling constant that is about 50% larger than what we have adopted in Eq. (42). Our value of the $F/(F+D)$ ratio for the vector coupling, which is $\alpha_v = 1$, agrees with the value given in NSC89 [17]. Our value for α_m, which is $\alpha_m = 0.18$, is about half of the values obtained in NSCa-f [18] and NSC89 [17], which are $0.37 \leq \alpha_m \leq 0.45$ and $\alpha_m = 0.28$, respectively.

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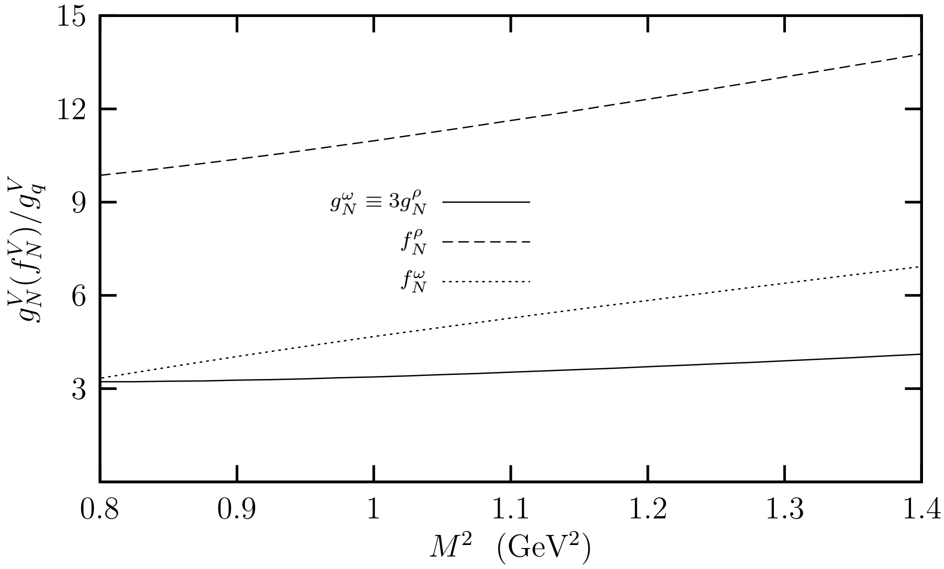

FIG. 1: The Borel mass dependence of the vector and tensor coupling constants of ρ and ω to the nucleon for average values of the vacuum parameters and the susceptibilities.

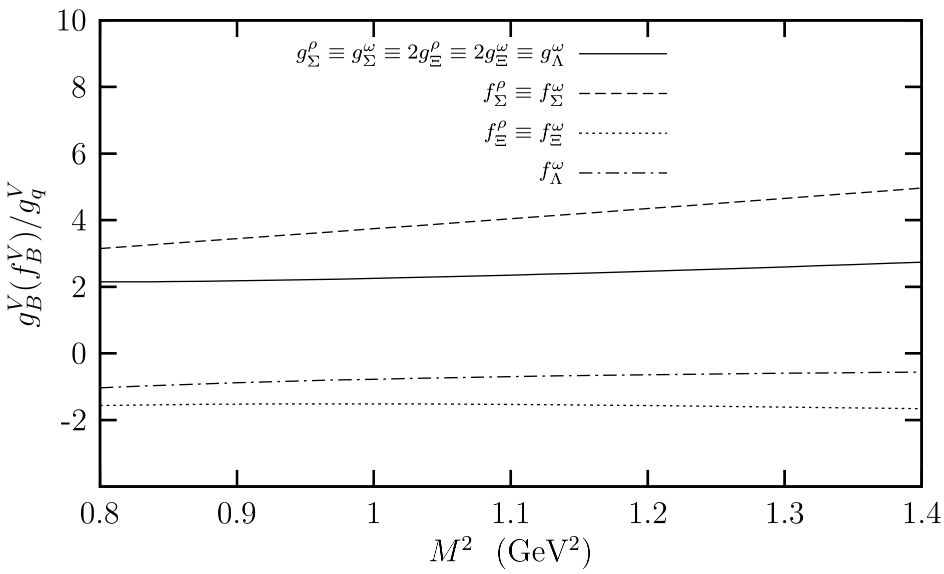

FIG. 2: Same as Fig. 1 but for the vector and tensor coupling constants of ρ and ω to the hyperons in the $SU(3)$ limit.

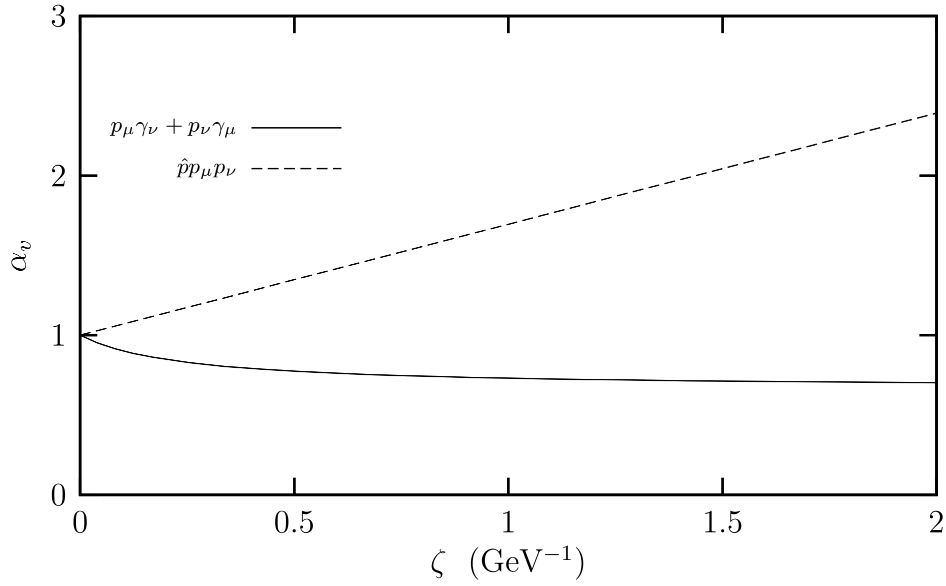

FIG. 3: The dependence of $\alpha_v = F/(F + D)$ on the susceptibility ζ for the sum rules at structures I and II, at $M^2 = 1$ GeV^2 and for the average values of the other vacuum parameters.

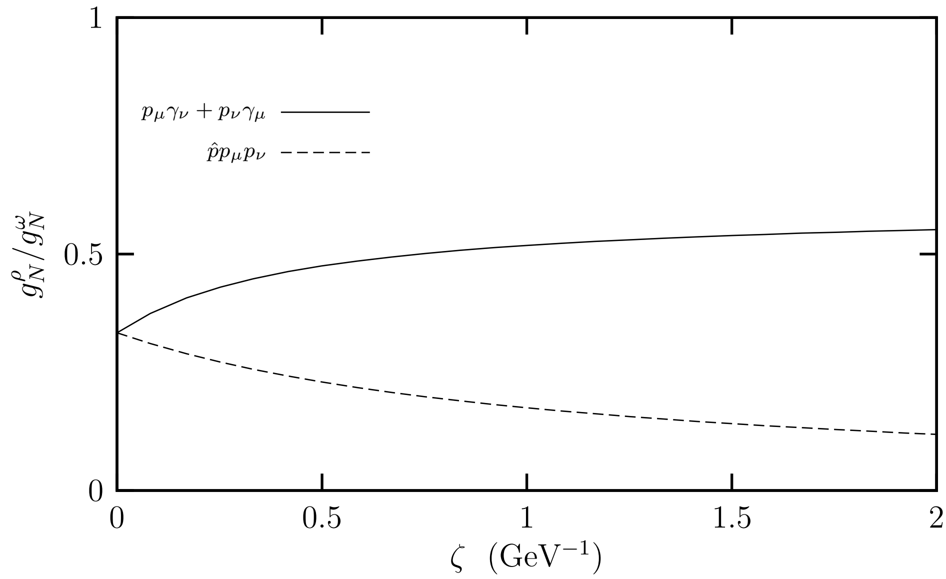

FIG. 4: Same as Fig. 3 but for the dependence of g_N^ρ/g_N^ω.