The Nucleon-Mass Difference in Chiral Perturbation Theory and Nuclear Forces*

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Abstract

A new method is developed for treating the effect of the neutron-proton mass difference in isospin-violating nuclear forces. Previous treatments utilized an awkward subtraction scheme to generate these forces. A field redefinition is used to remove that mass difference from the Lagrangian (and hence from asymptotic nucleon states) and replace its effect by effective interactions. Previous calculations of static Class II charge-independence-breaking and Class III charge-symmetry-breaking potentials are verified using the new scheme, which is also used to calculate Class IV nuclear forces. Two-body forces of the latter type are found to be identical to previously obtained results. A novel three-body force is also found. Problems involving Galilean invariance with Class IV one-pion-exchange forces are identified and resolved.

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I. INTRODUCTION

Although isospin violation in nuclear physics is a rather mature topic[1, 2], it has recently undergone a renaissance because of Chiral Perturbation Theory (χPT)[3, 4]. Many of the phenomenology-based mechanisms that underlie the traditional approach to isospin violation in nuclear forces have been rederived in χPT[5–12]. Most of the results of this reanalysis are the same as that of the traditional approach, which should be no surprise. There have nevertheless been several mechanisms that had been incompletely calculated using older techniques and have been recently completed in χPT, such as the static π–γ exchange force[7], the two-pion-exchange charge-independence-breaking (CIB) potential[8], and the two-pion-exchange charge-symmetry-breaking (CSB) potential[12]. The primary innovation of χPT, however, is the use of power counting to order the sizes of interactions and (Lagrangian) building blocks in a well-defined way[3, 13] so that it is apparent which interactions and mechanisms are dominant. In some cases this leads to the identification of important contributions that had not been considered before, which in turn give results that are significantly different from traditional approaches. An example is charge-symmetry breaking in $pn \to d\pi^0$, where previously-ignored contributions required by chiral symmetry change the sign of the predicted front-back asymmetry[14], in agreement with subsequent data[15].

The most important attribute of effective field theories is the underlying power counting that allows a systematic organization of calculations. In the case of χPT, which is the low-energy effective field theory based on the symmetries and scales of QCD[3], the relevant scales for constructing nuclear potentials (using Weinberg power counting[3, 5]) include the pion decay constant, $f_\pi \sim 93$ MeV, which sets the scale for pion emission or absorption, the pion mass, m_π, which sets the scale for chiral-symmetry breaking, the typical nucleon momentum, $Q \sim m_\pi$, which is an inverse correlation length in nuclei, and the characteristic QCD scale, $\Lambda \sim m_\rho$, which is the scale of QCD bound states appropriate for heavy mesons, nucleon resonances, etc. The latter are frozen out and do not explicitly appear, although their effect is present in the counter terms of the effective interactions. The resulting field theory is a power series in Q/Λ, and the number of powers of $1/\Lambda$ (e.g., n) is used to label individual terms in the Lagrangian (viz., $\mathcal{L}^{(n)}$). In this way higher powers denote smaller terms, and this is an integral part of the organizing principle of χPT.

Chiral Perturbation Theory was originally applied[3, 5, 16] to ordinary strong forces (Class I in the terminology of Ref.[1]) and, for the two-nucleon potential, these calculations have now been completed at the two-loop level[17]. A major success of the program has been the numerical determination of the coefficients of several counter terms in the χPT Lagrangian whose role had previously been restricted to pion-nucleon scattering. This determination used partial-wave analysis of nucleon-nucleon scattering data to isolate the contributions proportional to those counter terms[18].

The χPT formalism was extended in Ref.[5] to incorporate isospin violation in nuclear forces. The extended theory has now been applied to charge-independence-breaking forces[6–10] (Class II forces) and ordinary charge-symmetry-breaking forces[6, 9–12] (Class III forces). The latter are determined by differences between "mirror" forces in a given multiplet, such as the difference between pp ($T_3 = +1$) and nn ($T_3 = -1$) forces within the $T = 1$ isomultiplet (for later notational consistency we will uniformly use "3" rather than "z" to refer to the third component of an isospin vector). In this work we will complete the list by treating Class IV charge-symmetry-breaking two-nucleon forces[1], which lead to transitions (only) between the $T = 0$ to $T = 1$ isomultiplets in the np system. We also note that the scales of isospin violation in χPT were used in the past[5] to prove that these forces satisfy (in magnitude) Class I > Class II > Class III > Class IV.

While electromagnetic interactions break charge independence in general, the up-down quark-mass difference breaks charge symmetry specifically. CSB observables can, therefore, be linearly sensitive to the up-down quark-mass difference, while CIB observables that are charge symmetric at best depend quadratically on the quark-mass difference. Since the quark-mass difference is small on a typical hadronic scale, CIB is for all practical purposes dominated by electromagnetism. Interest in quark masses takes us to CSB.

At low energies, CSB originates from a variety of sources, but the terms favored by power counting are associated with the nucleon mass difference. In general, in order to understand CSB at low energies we need to include the effects of the nucleon mass difference. In Sect. II we invent a field redefinition that removes the nucleon-mass-difference term from the low-energy effective Lagrangian at the expense of new interactions. In Sect. III we show that the previous calculations of Class II and III forces are very easily reproduced in the new field basis. The implications for Class IV forces in χPT are discussed in Sects. IV and V.

II. THE NUCLEON-MASS DIFFERENCE

The mass difference between the proton and neutron, $\delta M_{\rm N} = m_p - m_n$, plays an important role in charge-symmetry breaking. This mass difference arises from two separate physical mechanisms. One of these is the up-down quark-mass difference, which dominates and makes the neutron heavier than the proton. The other mechanism is hard electromagnetic (EM) interactions at the quark level, which tends to make the proton heavier than the neutron. The dimensionless parameter associated with up-down quark-mass-difference isospin violation is $\epsilon\, m_\pi^2/\Lambda^2 \sim 1\%$, where $\epsilon = \frac{m_d - m_u}{m_d + m_u} \sim 0.3$ and we have chosen Λ to be the mass of the ρ meson. The parameter associated with hard EM interactions is $\alpha/\pi \sim \frac{1}{4}\%$, where α is the fine-structure constant. In addition to these mechanisms, which have an origin in short-distance physics, there are also important soft-photon contributions (such as the Coulomb interaction between protons) that dominate isospin violation in nuclei. All three of these mechanisms contribute to Class IV forces.

Because asymptotic nuclear states individually reflect the appropriate nucleon masses, previous work on Class III forces noted that only those nuclear intermediate states where $Z - N$ changes will contribute to isospin violation. An example would be pp scattering with the emission of two π^+ mesons (creating an nn intermediate nucleon configuration with a different mass) and subsequent reabsorption of the pions. In Ref.[12] we adopted a subtraction procedure that accomplished the necessary bookkeeping, although it was somewhat awkward and would have been difficult to generalize to more complicated operators (such as three-body forces). In what follows below we will use a field redefinition procedure that simply removes the $n - p$ mass difference from the asymptotic states (in favor of an average nucleon mass, $M_{\rm N} = \frac{1}{2}(M_n + M_p)$) and compensates for this by introducing new effective interactions determined by $\delta M_{\rm N}$ that must be treated in perturbation theory.

We illustrate the method in the lowest chiral orders, in which case only the lowest orders in $\delta M_{\rm N}$ appear. In addition, for the sake of simplicity, we display in the equations below only those few terms of most interest for the nuclear potential. It should of course be kept in mind that the χPT Lagrangian includes all terms allowed by QCD symmetries, and that at each chiral order all powers of pion fields are required by chiral symmetry.

The leading-order Lagrangian in χPT is

$$\mathcal{L}^{(0)} = \frac{1}{2}[\dot{\boldsymbol{\pi}}^2 - (\vec{\nabla}\boldsymbol{\pi})^2 - m_\pi^2\boldsymbol{\pi}^2] + N^\dagger[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})]N + \frac{g_A}{2f_\pi}N^\dagger\vec{\sigma}\cdot\vec{\nabla}(\boldsymbol{\tau}\cdot\boldsymbol{\pi})N + \ldots\,, \quad (1)$$

while the sub-leading-order Lagrangian is given by

$$\mathcal{L}^{(1)} = \frac{g_A}{4f_\pi\, M_{\rm N}}N^\dagger\{\vec{\sigma}\cdot\vec{p},\boldsymbol{\tau}\cdot\dot{\boldsymbol{\pi}}\}N \,+\, \frac{\tilde{c}_2}{f_\pi^2}N^\dagger N\,\dot{\boldsymbol{\pi}}^2 \,+\ldots\,. \quad (2)$$

In these equations $g_A = O(1)$ ($g_A \simeq 1.26$) and $\tilde{c}_2 = O(1/\Lambda)$ ($\tilde{c}_2 \sim -2\ \mathrm{GeV}^{-1}$) are parameters not determined by chiral symmetry, and "$\ldots$" denote terms that we do not require[19]. There are three $\mathcal{L}^{(2)}$ terms with one pion interacting with a single nucleon; we will comment further on them below.

In addition to these Class I interactions we have isospin-violating interactions, a comprehensive list of which can be found in Ref.[5]. We are here particularly interested in the interactions generated by the quark-mass ($\delta M_{\rm N}^{\rm qm} = O(\epsilon\, m_\pi^2/\Lambda)$) and hard-photon ($\delta M_{\rm N}^{\rm em} = O(\alpha\Lambda/\pi)$) contributions to the nucleon mass ($\delta M_{\rm N} = \delta M_{\rm N}^{\rm qm} + \delta M_{\rm N}^{\rm em}$),

$$\begin{aligned}\mathcal{L}_{\rm iv} \;=\; & -\frac{\delta M_{\rm N}}{2}\; N^\dagger\tau_3 N + \frac{\delta M_{\rm N}^{\rm qm}}{4f_\pi^2}\; N^\dagger\boldsymbol{\tau}\cdot\boldsymbol{\pi}\pi_3 N + \frac{\delta M_{\rm N}^{\rm em}}{4f_\pi^2}\; N^\dagger(\tau_3\boldsymbol{\pi}^2 - \boldsymbol{\tau}\cdot\boldsymbol{\pi}\pi_3)N \\ & -\frac{1}{2}\delta m_\pi^2\left(\boldsymbol{\pi}^2 - \pi_3^2\right) + \frac{\bar{\beta}_1}{4f_\pi^2}\left(\boldsymbol{\pi}^2 - \pi_3^2\right)\; N^\dagger N + \frac{\bar{\beta}_2}{4f_\pi^2}\left(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}\right)_3\; N^\dagger N + \ldots\,. \end{aligned} \quad (3)$$

For reasons that will soon become obvious we have also shown explicitly the pion-mass-splitting term and two pion-nucleon seagulls. The pion-mass-splitting term is dominated by the electromagnetic contribution, $\delta m_\pi^2 \simeq (\delta m_\pi^2)^{\rm em} = O(\alpha\Lambda^2/\pi)$ ($\delta m_\pi^2 \simeq (38\,\mathrm{MeV})^2$), since the contribution from the quark masses is small, $(\delta m_\pi^2)^{\rm qm} = O(\epsilon^2 m_\pi^4/\Lambda^2)$. Because of the quark-mass contribution, $\delta M_{\rm N}$ counts formally as chiral order $n = 1$. (See, however, the discussion in Sect. V.) Noting that α/π is numerically comparable to $\epsilon m_\pi^3/\Lambda^3$ and adjusting our power counting of EM terms accordingly, the pion-mass splitting term then counts as $n = 1$, and all other isospin-violating interactions are of higher order[5]. (For example, $\bar{\beta}_1$ is $O(\alpha\Lambda/\pi)$ and $n = 2$, while $\bar{\beta}_2$ is $O(\alpha/\pi)$ and $n = 3$.)

The average nucleon mass $M_{\rm N}$ has already been removed from consideration by means of the time-dependent transformation $N = e^{-iM_{\rm N}t}N'$, which uses the fact that only the second term in Eqn. (1) contains a time derivative of a nucleon field, while the exponential multiplying N' commutes with everything else. That procedure will not work straightforwardly for the $\delta M_{\rm N}$ term because $\delta M_{\rm N}\,\tau_3$ does not commute with other nucleon isospin operators in $\mathcal{L}^{(n)}$. One can eliminate the first term in Eqn. (3) by an appropriate redefinition of the nucleon field,

$$N \rightarrow e^{-i\frac{1}{2}\delta M_{\rm N}\, t\,\tau_3}\; N \;\equiv \cos\left(\tfrac{1}{2}\delta M_{\rm N}\, t\right) - i\tau_3 \sin\left(\tfrac{1}{2}\delta M_{\rm N}\, t\right). \quad (4)$$

In the process, however, we create interactions that are explicitly dependent on the time t, unless we also redefine the pion fields. Using Eqn. (4) we find

$$e^{i\frac{1}{2}\delta M_{\rm N}\, t\,\tau_3}\;\tau_i\; e^{-i\frac{1}{2}\delta M_{\rm N}\, t\,\tau_3} \;= A(\delta M_{\rm N}\, t)\,\tau_i + B(\delta M_{\rm N}\, t)\,\epsilon_{ij3}\tau_j + C(\delta M_{\rm N}\, t)\,\delta_{i3}\tau_3\,, \quad (5)$$

where

$$\begin{aligned} A(z) \;&=\; \cos(z) \\ B(z) \;&=\; -\sin(z) \\ C(z) \;&=\; 1 - \cos(z)\,. \end{aligned} \quad (6)$$

The transformations for the Cartesian components of τ_i show that they are identical to those of a coordinate rotation about the z-axis in isospin space by an angle $-\delta M_{\rm N} t$. This immediately suggests the corresponding form for the pion transformation:

$$\pi_i \longrightarrow A(\delta M_{\rm N}\, t)\, \pi_i + B(\delta M_{\rm N}\, t)\, \epsilon_{ij3}\pi_j + C(\delta M_{\rm N}\, t)\, \delta_{i3}\pi_3\,. \tag{7}$$

To leading order in $\delta M_{\rm N}\, t$ this pair of transformations is nothing more than the usual $SU(2)_{\rm V}$ generators for (electric) charge conservation. Application of these transformations demonstrates that $\boldsymbol{\pi}^2$, π_3, $\boldsymbol{\tau}\cdot\boldsymbol{\pi}$, and τ_3 are invariant, as one expects. Only terms that involve a time derivative in the Lagrangian are not invariant, and these will generate new Lagrangian terms[20] that compensate for the rotating isospin coordinate system, each of them modifying the isospin-violating Lagrangian. Each time derivative can introduce one power of $\delta M_{\rm N}$ into the final result in Eqn. (8). Because $\delta M_{\rm N}$ is order $n = 1$, a new term generated by an isospin-symmetric term of order n will have order $n + 1$ or higher. Note that terms with an even number of time derivatives can generate new interactions with even powers of $\delta M_{\rm N}$. Although the original nucleon-mass-difference term in Eqn. (3) is charge-symmetry breaking, some of the new interactions will be charge symmetric.

Since the maximum number of derivatives at order n is $n-f/2+2$, where f is the number of fermion fields, the above field redefinition generates a finite number of new terms at each chiral order. Four new terms arise from transforming $\mathcal{L}^{(0)}$. One of them comes from the nucleon kinetic term, and is equal in magnitude and opposite in sign to the first term in $\mathcal{L}_{\rm iv}$. Another new term comes from the Weinberg-Tomozawa interaction (the chiral partner of the nucleon kinetic term), and has the form of the third term in $\mathcal{L}_{\rm iv}$ (the chiral partner of the nucleon EM mass-difference term). The third and fourth terms come from the pion kinetic term. In addition, two new terms are generated by $\mathcal{L}^{(1)}$, and so on.

The sum of the new isospin-violating contributions to our Lagrangian together with the surviving terms from Eqn. (3) is:

$$\begin{aligned}
\mathcal{L}'_{\rm iv} \;=\;& \delta M_{\rm N}\, (\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})_3 + \frac{\delta M_{\rm N}^{\rm qm}}{4f_\pi^2} N^\dagger \left[\boldsymbol{\tau}\cdot\boldsymbol{\pi}\pi_3 + ((\boldsymbol{\tau}\times\boldsymbol{\pi})\,\times\boldsymbol{\pi})_3\right] N \\
& -\tfrac{1}{2}\left(\delta m_\pi^2 - \delta M_{\rm N}^2\right)\left(\boldsymbol{\pi}^2 - \pi_3^2\right) - \tfrac{g_A}{4f_\pi}\tfrac{\delta M_{\rm N}}{M_{\rm N}} N^\dagger \{\vec{\sigma}\cdot\vec{p}, (\boldsymbol{\tau}\times\boldsymbol{\pi})_3\} N \\
& + \frac{1}{4f_\pi^2}\left(\bar{\beta}_2 + 8\,\tilde{c}_2\,\delta M_{\rm N}\right)(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})_3\, N^\dagger N + \frac{1}{4f_\pi^2}\left(\bar{\beta}_1 + 4\,\tilde{c}_2\,\delta M_{\rm N}^2\right)(\boldsymbol{\pi}^2 - \pi_3^2)\, N^\dagger N \\
& + \ldots\,.
\end{aligned} \tag{8}$$

Because the quark-mass difference part of $\delta M_{\rm N}$ counts like two derivatives[12], the first and second terms in Eqn. (8) are of order $n = 1$, the second part of the third, the fourth, and the second part of the fifth term are of order $n = 2$, while the second part of the sixth term is of order $n = 3$. The $\mathcal{L}^{(2)}$ interactions generate one single-nucleon contribution proportional to $\delta M_{\rm N}\, \dot{\boldsymbol{\pi}}/M_{\rm N}^2$, which has $n = 3$ (plus another of order $\delta M_{\rm N}^2$ with $n = 4$). Note, however, that in the nuclear potential the energy transferred by pions is $O(Q^2/M_{\rm N})$, and a time derivative produces contributions that are effectively the size of contributions with two space derivatives. Thus the OPEP (one-pion-exchange potential) derived from this interaction effectively contributes at order $n = 4$. The fifth and sixth terms in Eqn. (8) and terms stemming from $\mathcal{L}^{(n\geq 2)}$ produce a higher-order potential than we wish to consider.

Our new Lagrangian is $\mathcal{L}^{(0)}+\mathcal{L}^{(1)}+\mathcal{L}'_{\rm iv}+\ldots$. The nucleon-mass difference has been entirely removed from the asymptotic states and now resides only in the new effective interactions (see, however, the discussion below Eqn. (22)). Among the latter we find novel two-pion seagull terms. The field redefinition presented here is thus particularly suited to the study of nuclear processes.

III. CLASS II AND III FORCES

Like any other field redefinition, Eqns. (4) and (7) do not introduce any new physics; they only produce a new —in this case, useful— bookkeeping of various contributions. We can check this result by repeating previous calculations of isospin-violating forces. Three vertices corresponding to the various terms in Eqn. (8) are illustrated in Figs. (1a), (1b), and (1c). Figure (1d) depicts the usual isospin-conserving OPEP (which is Class I), while (1e) is generated by vertex (1b) (and corresponds to Class IV) and (1f) is generated by vertex (1a). The latter includes a term that is proportional to the energy transfer (q^0, or the time component of the four-momentum transfer, q^μ) between the two nucleons and hence vanishes in the center-of-mass (CM) frame. It has a Class IV type of isospin structure, and we will treat both OPEP graphs (i.e., Figs. (1e) and (1f)) in the next section.

Fig. (1a) also contains the pion-mass splitting and generates well-known, relatively-large Class II forces. The new $\delta M_{\rm N}^2$ term in the pion-mass splitting results in small Class II forces. For example, it generates a small Class II OPEP that has been obtained before[5]. However, the field redefinition above makes it obvious that the contribution of this $\delta M_{\rm N}^2$ term to higher-order Class II forces can also be obtained from the corresponding δm_π^2 contribution by the straightforward substitution $\delta m_\pi^2 \to \delta m_\pi^2 - \delta M_N^2$. In particular, this remark holds for the two-pion-exchange potential of Ref.[8]. These new terms are all expected to be small because formally $\delta M_{\rm N}^2$ is the size of the expected small quark-mass contribution to δm_π^2, $O(\epsilon^2 m_\pi^4/\Lambda^2)$. In addition, the discussion in Sect. V suggests that $\delta M_{\rm N}^2$ in pion-mass splitting should be treated as if it were $n = 4$, rather than $n = 2$, since it is approximately $\frac{1}{8}$% of the usual pion-mass difference.

We can also reproduce the calculation of static Class III two-pion-exchange potentials that was performed in Ref.[12]. The remaining graphs to consider are two-pion-exchange graphs such as those in Fig. (2), which must be modified by introducing Fig. (1a) into pion propagators, Fig. (1b) into single-pion vertices, or Fig. (1c) into two-pion seagull vertices. We will ignore the modifications from Fig. (1b) because they are non-static, and for this reason are higher order in power counting than was calculated in Ref.[12]. Likewise, the $\tilde{c}_2$ interaction in Fig. (1c) contributes to the potential at higher order.

The remaining terms in the seagull, Fig. (1c), consist of the original seagull (that in Eqn. (3)) plus the $\delta M_{\rm N}$ modification induced by the transformations (4) and (7). Like the original seagull, the seagull modification vanishes in Fig. (2d) to order $\delta M_{\rm N}$ because of isospin symmetry. The seagull terms in Fig. (2c) give Eqns. (9b) and (9c) of Ref.[12]; the original seagull gave Eqn. (9c), while the seagull modification reproduces Eqn. (9b). If one ignores the energy transfer between nucleons and other nuclear-energy dependence (which is a higher-order correction), the graphs that result from pion-propagator modification by Fig. (1a) are greatly simplified by a symmetry that develops. The integral over the loop four-momentum (k^ν) then has a simplified time component (i.e., the integral over the loop energy, k^0), which can be classified according to the parity ($k^0 \to -k^0$) of the k^0-factors. The $\dot{\boldsymbol{\pi}}$ factors are odd, since each generates one factor of k^0. Each inverse pion propagator becomes proportional to $(k^0)^2$ and is therefore even under a sign change, while each nucleon

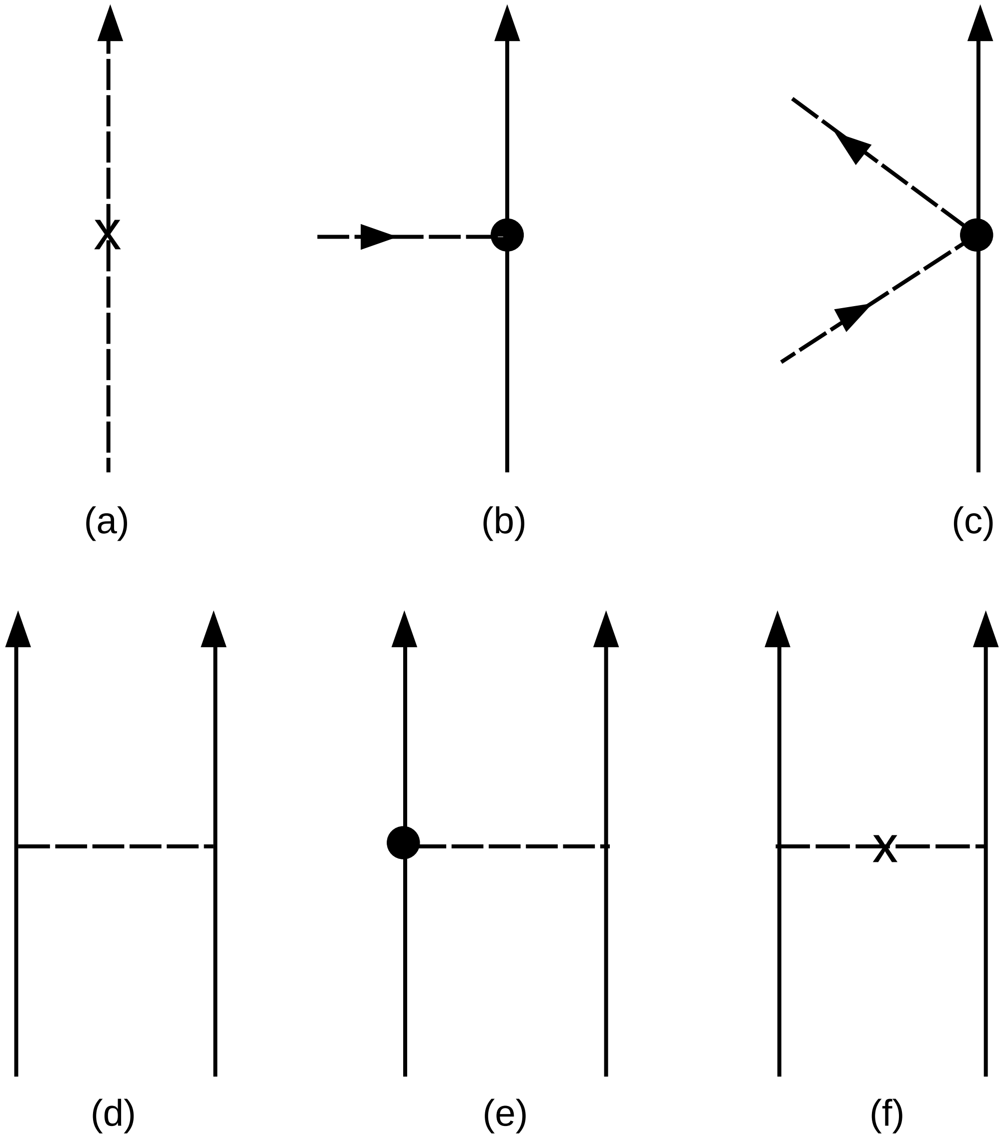

FIG. 1: Vertices created by removal of the nucleon-mass difference from the basis states of our Hilbert space are shown in (a), (b), and (c), while the usual one-pion-exchange graph is shown in (d) and additional graphs generated by the interactions (a) and (b) are illustrated in (f) and (e). Pions are depicted as dashed lines and nucleons as solid lines.

propagator becomes

$$\frac{1}{\pm k^0 + i\,\epsilon} = \pm \mathcal{P}\frac{1}{k^0} - i\,\pi\,\delta(k^0)\,, \tag{9}$$

where $\mathcal{P}$ denotes a principal-value integral (odd in k^0), while the δ-function part ($\delta(k^0)$) is an even function of k^0. All modifications of Fig. (2) produced by inserting Fig. (1a) only once are found to contain an odd number of k^0-factors, and have at most one surviving nucleon propagator. Thus if we use Eqn. (9) the k^0-factors all vanish upon (symmetric) k^0 integration except for the δ-function part. In this way only the modification of the crossed-box graph in Fig. (2b) contributes (the remaining graphs vanish, as they did in Ref.[12]).

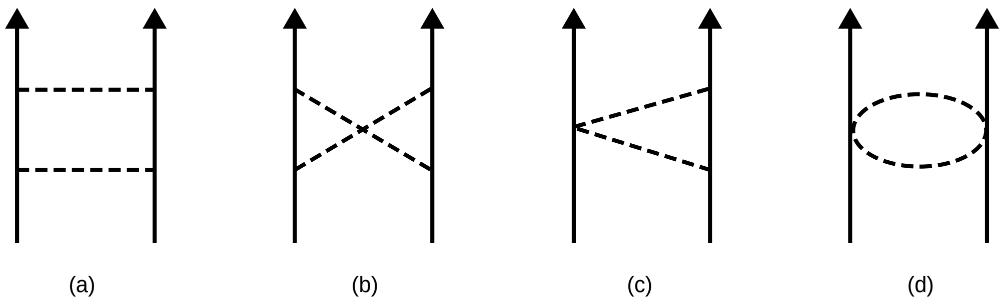

FIG. 2: Two-pion-exchange graphs that contribute to isospin-conserving nucleon-nucleon scattering.

Performing the trivial integral over the δ-function leads directly to Eqn. (9a) of Ref.[12].

Therefore, the formalism for treating isospin violation from $\delta M_{\rm N}$ using Eqn. (8) reproduces previous results but is much more direct and transparent. Although we have not calculated the corresponding three-nucleon isospin-violating forces, it should prove much easier with the new approach. We turn now to the remaining component (Class IV) of the two-nucleon potential.

IV. CLASS IV FORCES

Two-body Class IV forces have traditionally been classified into two types with the generic forms in the CM frame

$$V_a^{\rm IV}(\vec{r}) = (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_3 \, (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{L} \, w_a(r) \tag{10}$$

and

$$V_b^{\rm IV}(\vec{r}) = (\tau_1 - \tau_2)_3 \, (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} \, w_b(r) \tag{11}$$

(where $\vec{r} = \vec{r}_1 - \vec{r}_2$). These forms have been simplified by ignoring possible factors of $\vec{p}^{\,2}$, the square of the common CM nucleon momentum, $\vec{p}$, and thus correspond only to the lowest order in power counting. Given an isospin operator that is antisymmetric under the interchange of the two nucleons, parity conservation (requiring symmetric radial forms) then dictates an antisymmetric combination for the spin vector. We note, however, that since antisymmetric isospin vectors can only induce transitions between $T = 0$ and $T = 1$ (two-nucleon) states, the two forms in Eqns. (10) and (11) are proportional and effectively equivalent, as are the two spin-vector forms. Thus in an operational sense there is only a single Class IV type, either (10) or (11), even though the two isospin (spin) forms have different time-reversal properties.

The dominant Class IV force ($n = 2$) is generated by one-pion exchange using the fourth term in Eqn. (8) in Fig. (1e). A simple calculation in configuration space leads to

$$V_{\pi;1e}^{\rm IV} = -\frac{\delta M_{\rm N}\, g_A^2}{8 f_\pi^2\, M_{\rm N}} \sum_{i\neq j} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_3 \left\{ \vec{\sigma}_i \cdot \vec{p}_i \, , \vec{\sigma}_j \cdot \vec{\nabla}_{ij} \, h_0(r_{ij}) \right\} , \tag{12}$$

where

$$h_0(z) = \frac{1}{4\pi z} e^{-m_\pi z} . \tag{13}$$

We have chosen to write the complete frame-dependent form of $V^{\rm IV}_{\pi;1e}$ for reasons that will become obvious. If one now writes the mass of the $i^{\rm th}$ nucleon in isospin notation (which is implicit in Eqn. (3)) as

$$M_i = M_{\rm N} + \tfrac{1}{2}\tau_i^3\,\delta M_{\rm N}\,, \tag{14}$$

which expresses the total mass in terms of the 3-component of the total isospin

$$M_t = \sum_{i=1}^{A} M_i = A\,M_{\rm N} + \tfrac{1}{2}\delta M_{\rm N}\tau_3\,, \tag{15}$$

we can separate each nucleon's momentum into a CM part ($\vec{P}$) and an internal part ($\vec{K}$) using the usual relations

$$\vec{p}_i = \vec{K}_i + \frac{M_i}{M_t}\vec{P}\,. \tag{16}$$

Using Eqns. (14)—(16) we decompose $V^{\rm IV}_{\pi;1e}$ into the form (10) for the internal part,

$$w_a(r) = \frac{\delta M_{\rm N}\,g_A^2}{4f_\pi^2\,M_{\rm N}}\frac{h_0'(r)}{r}\,, \tag{17}$$

plus a frame-dependent part

$$V^{\rm IV}_{\pi;1e}(\vec{P}) = -\frac{\delta M_{\rm N}\,g_A^2}{4f_\pi^2\,M_{\rm N}}\sum_{i\neq j}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_3\,\vec{\sigma}_i\cdot\vec{P}\,\vec{\sigma}_j\cdot\vec{\nabla}_{ij}\,h_0(r_{ij})\,. \tag{18}$$

Although this form resembles frame-dependent relativistic corrections to nuclear potentials, which were exhaustively treated in the past[21], it has too few powers of $1/M_{\rm N}$ to be a relativistic correction to OPEP.

To clarify the role this term plays it is necessary to determine the contribution of Fig. (1f), which also has $n = 2$ but vanishes in the two-nucleon CM frame (and hence is usually ignored). That contribution is

$$V^{\rm IV}_{\pi;1f} = \frac{\delta M_{\rm N}\,g_A^2}{32f_\pi^2\,M_{\rm N}}\sum_{i\neq j}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_3\,\vec{\sigma}_i\cdot\vec{\nabla}_{ij}\,\vec{\sigma}_j\cdot\vec{\nabla}_{ij}\,\{\vec{p}_i+\vec{p}_j\,,\cdot\vec{r}_{ij}\,h_0(r_{ij})\}\,. \tag{19}$$

The decomposition of this potential into internal and CM parts leads to

$$V^{\rm IV}_{\pi;1f}(\vec{P}) = \frac{\delta M_{\rm N}\,g_A^2}{8f_\pi^2\,M_{\rm N}}\sum_{i\neq j}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_3\,(2\,\vec{\sigma}_i\cdot\vec{P}\,\vec{\sigma}_j\cdot\vec{\nabla}_{ij}\,h_0(r_{ij}) + \vec{P}\cdot\vec{r}_{ij}\,\vec{\sigma}_i\cdot\vec{\nabla}_{ij}\,\vec{\sigma}_j\cdot\vec{\nabla}_{ij}\,h_0(r_{ij})), \tag{20}$$

for the CM part, while the internal part is obtained by replacing $\vec{p}_i$ and $\vec{p}_j$ by $\vec{K}_i$ and $\vec{K}_j$, respectively. Since the sum of all $\vec{K}_i$ in any system vanishes, this force vanishes in a two-body system. In a three-body system, however, $\vec{K}_i + \vec{K}_j = -\vec{K}_k$ (i, j, k all different), and this force does not vanish. The OPEP from Fig. (1f) is therefore a peculiar three-body force that violates isospin conservation. Although it has Class IV isospin dependence, this force does not mix spin representations in the manner of two-body Class IV forces. Note that this effect is present in *any* three-or-more-body system where momentum is transferred to the two-nucleon system.

Adding the $\vec{P}$-dependent terms in Eqns. (20) and (18) together we arrive at a relatively simple form

$$V_\pi^{\rm IV}(\vec{P}) = \frac{\delta M_{\rm N}}{2M_{\rm N}} \sum_{i\neq j} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_3 \, \vec{P} \cdot \vec{r}_{ij} \, v_\pi^{ij} \, , \tag{21}$$

whereas the usual (Class I) OPEP is given by

$$V_\pi = \frac{1}{2} \sum_{i\neq j} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \, v_\pi^{ij} \, . \tag{22}$$

The origin of this unusual force can be understood in simple terms. Consider a neutron and a proton placed some distance apart, and place the origin of coordinates on the neutron (for simplicity). The center-of-mass of the system is slightly closer to the neutron than the proton because the neutron is heavier. The exchange of a charged pion interchanges the neutron and the proton, which causes the CM to move (slightly) further from the origin. Thus with differing neutron and proton masses the **usual** CM does not move in a straight line in the absence of an external force. This problem is Galilean in origin (see Refs. [22]) and is unrelated to the specific problems that arise from special relativity (such as the Thomas precession and Lorentz contraction).

Forming the usual CM coordinate vector

$$\vec{R}_{\rm CM} = \sum_{i=1}^{A} \frac{M_i \, \vec{r}_i}{M_t} = \frac{A \, M_{\rm N}}{M_t} \vec{R}_0 + \frac{\delta M_{\rm N}}{2M_t} \sum_{i=1}^{A} \tau_i^3 \vec{r}_i \, , \tag{23}$$

with $\vec{R}_0 = \sum_{i=1}^{A} \vec{r}_i / A$, it then follows that

$$i \, \vec{P} \cdot \, [\vec{R}_{\rm CM}, V_\pi \,] = V_\pi^{\rm IV}(\vec{P}) \, , \tag{24}$$

where the latter quantity ($V_\pi^{\rm IV}(\vec{P})$) was derived in Eqn. (21) and therefore reflects the fact that OPEP and the usual non-relativistic CM coordinate do not commute. Note that M_t commutes with V_π, and the non-vanishing commutator is generated by the $\delta M_{\rm N}$ term in Eqn. (23).

The presence of the term $V_\pi^{\rm IV}(\vec{P})$ in the potential is required in order to preserve the Galilean invariance of the matrix element of the Hamiltonian, H. Galilean invariance requires that in an arbitrary frame of reference we have

$$\langle \vec{P} | H(\vec{P}) | \vec{P} \rangle = \frac{\vec{P}^2}{2 \, M_t} + E \, , \tag{25}$$

where the constant E is the useful part of the matrix element (nuclear binding energy, for example). The presence of $V_\pi^{\rm IV}(\vec{P})$ in $H(\vec{P})$ would ordinarily spoil Eqn. (25), but the wave function $|\vec{P}\rangle$ is defined as $|\vec{P}\rangle = \exp{(i\vec{P} \cdot \vec{R}_{\rm CM})}|0\rangle$, and we recall that $\vec{R}_{\rm CM}$ does not commute with V_π. Expanding the plane wave to first order in $\delta M_{\rm N}$ we find

$$\langle \vec{P} | V_\pi + V_\pi^{\rm IV}(\vec{P}) | \vec{P} \rangle \cong \langle \vec{P}' | V_\pi + V_\pi^{\rm IV}(\vec{P}) - i \, \vec{P} \cdot \, [\vec{R}_{\rm CM}, V_\pi(\vec{P}) \,] | \vec{P}' \rangle \equiv \langle \vec{P}' | V_\pi | \vec{P}' \rangle \, , \tag{26}$$

where $|\vec{P}'\rangle = \exp{(i\vec{P} \cdot \vec{R}_0)}|0\rangle$. This cancellation of terms proportional to $\delta M_{\rm N}$ therefore preserves the Galilean structure of the matrix element of the Hamiltonian. In other words the formalism we have developed remembers that we have removed $\delta M_{\rm N}$ from asymptotic states, and corrects for this change by introducing $V_\pi^{\rm IV}(\vec{P})$. The corresponding Lorentz case (treating relativity properly in the matrix element in Eqn. (26)) is considerably more complicated.

What other Class IV forces are expected to be significant? Other forces arise from short-range CSB mechanisms in higher orders. We note that there are no $n = 3$ terms. The leading-order short-range interaction is of order $n = 4$ and has the form

$$\mathcal{L}^{\rm IV} = \frac{i\delta_1}{2f_\pi^2}(N^\dagger\sigma^i\tau_\alpha N)\,\nabla^l\left(N^\dagger\sigma^j\tau_\beta(\overleftarrow{\nabla} - \overrightarrow{\nabla})^m N\right)\epsilon_{\alpha\beta 3}\,\epsilon^{ijk}\epsilon^{klm}\,, \tag{27}$$

with $\delta_1 = O(\epsilon\, m_\pi^2/\Lambda^4)$. All other possibilities can be manipulated into this form. The origin of this interaction cannot be asserted from the symmetries of QCD, and therefore depends on the details of the QCD short-range dynamics. In the existing literature, this interaction has been modeled by various mechanisms involving meson exchange. When the mesons are frozen out, Eqn. (27) results. An example of this type of interaction is provided by $\rho - \omega$ mixing, which is usually constructed by imitating one-photon exchange[23]. As demonstrated in Ref.[6] the usual form of the Class III ρ–ω-mixing force has "natural" size. We will comment below on the corresponding Class IV form. Note that in addition to this short-range interaction, at $n = 4$ there exist also loop diagrams that give rise to Class IV forces. For example, we have one-loop graphs involving the fourth term in Eqn.(8); however, because they should be suppressed by $\sim m_\pi^2/(4\pi f_\pi)^2$ with respect to the OPEP term above, the discussion in the next Section suggests that these graphs might contribute little. It is likely that the most important loop diagrams involve the fifth term in Eqn.(8), since $\tilde{c}_2$ is relatively large due to contributions from the delta-isobar.

In addition to these short-range CSB mechanisms, there exist Class IV forces from photon exchange. The dominant soft EM interaction is the Breit interaction produced by one-photon exchange. Since the only two-nucleon system with a Class IV interaction is the np system, only the spin-orbit and spin-other-orbit[24] parts of the Breit interaction are of this type, and they correspond to the magnetic moment of the neutron interacting with the charge of the proton. This produces a Class IV interaction of the type (11) with

$$w_b^\gamma(r) = \frac{\alpha\kappa_n}{4\,M_{\rm N}^2\,r^3}\,, \tag{28}$$

where $\kappa_n = O(1)$ ($\kappa_n \simeq -1.91$) is the neutron anomalous magnetic moment. This interaction is $O(Q^2/M_{\rm N}^2)$ smaller than Coulomb exchange. If one takes α/π as $\epsilon m_\pi^3/\Lambda^3$, this interaction counts as $n = 3$.

V. COMMENTS AND CONCLUSIONS

Much of the recent interest in Class IV CSB forces has centered around two sets of very different experiments. The first set of three experiments measured the difference in neutron and proton analyzing powers in elastic np scattering at 183 MeV[25], 347 MeV[26], and 477 MeV[27] neutron (lab) energies. Some recent reviews of CSB that discuss these measurements are listed in Ref.[28]. Agreement between theory and experiment is quite good. Three dominant mechanisms contribute to the theoretical description: (a) the EM Breit interaction between the neutron magnetic moment and the proton charge (given by Eqn. (28)); (b) the Class IV OPEP given by Eqn. (17); (c) the short-range ρ–ω-mixing force.

Additional small contributions from ρ-exchange and 2π-exchange are sometimes included. Our χPT derivation agrees with the previously obtained results for these forces.

The Breit-interaction Class IV force was first mentioned in the context of Class IV experimental tests by Refs.[1, 29]. It is an important contribution and is included in all comprehensive calculations.

The importance of the nucleon-mass difference in the presence of one-pion exchange in a relativistic model was emphasized by Gersten[30], who did not calculate a potential. A potential was calculated in Ref.[31], which verified that both pseudovector and pseudoscalar (relativistic) coupling of a pion to a nucleon gave identical results for the Class IV OPEP, presumably because the overall momentum dependence of the force is determined by Galilean invariance. We note, however, that other terms would not be the same; pseudoscalar coupling is very dangerous to use if one wishes to preserve chiral symmetry, and for this reason can lead to anomalous results. The Class IV OPEP corresponds to $n = 2$ in power counting.

Calculations also include short-range forces from ρ–ω mixing. Although the ρ–ω-mixing force is part of the short-range χPT counter term (and hence of undetermined size) in Eqn. (27), its coefficient in the traditional approach is fixed by $\rho-\omega$-mixing experiments[28]. Thus there are no adjustable constants in the dominant contributions to the traditional theory of Class IV forces, and this leads to impressive agreement with experiment.

Other ingredients have been used in calculations, including two-pion exchange forces[32] and heavy-meson exchange modified by $\delta M_{\rm N}$[33]. Reference [33] has a particularly useful catalog of forces based on the exchange of different types of particles. These mechanisms are smaller than the ones given above. In χPT two-pion exchange can be calculated explicitly at $n = 4$, and all heavy-meson-exchange contributions are subsumed in contact interactions to be fitted to experiment.

Recent calculations typically combine the dominant forces with a subset of the smaller ones[32–38]. These recent numerical calculations point out a potentially serious problem with the power counting. The three dominant mechanisms (Breit interaction, OPEP, and meson mixing) are all approximately the same size. The power counting would suggest that the OPEP should dominate the meson-mixing potential by a factor of roughly 30. To understand this discrepancy it is useful to substitute the estimate of $Q \sim m_\pi$ for $|\vec{q}|$ and $|\vec{p}|$ in the momentum-space expressions for these three forces, while ignoring the spin and isospin factors. Doing this reveals that all three forces are within a factor of two of each other in size. The contradiction with naive power counting arises from the smaller than normal OPEP (by a factor of more than 5) and the larger than normal meson-mixing force (by a factor of about 3). The reason for the former is that the OPEP isospin violation is proportional to $\delta M_{\rm N} \simeq -1.3$ MeV, while the dimensional estimate for the quark-mass component of this is $\epsilon\, m_\pi^2/\Lambda \sim 7.6$ MeV. The physical mass difference is the result of cancellation between the quark-mass-difference effect and the EM contribution (of opposite sign), and is fine tuned to the correct physical value. Its size is therefore anomalously small and more typical of $n = 3$ terms in the power counting.

The large Class IV meson-mixing force is primarily the result of the large $\rho-$nucleon tensor coupling ($\sim f_\rho$) that has been used historically, although this coupling plays only a minor role in Class III forces. To see this we strip the dimensional factors from the $\rho - \omega$-mixing force in momentum space and compare the result to Eqn. (27):

$$\delta_1^{\rho\omega} = f_\pi^2 g_\rho \kappa_\rho g_\omega \langle \rho | H | \omega \rangle / m_{\rm v}^4 M_{\rm N}^2, \tag{29}$$

where g_ρ and g_ω are the usual ρ- and ω-nucleon coupling constants, $\kappa_\rho \equiv f_\rho / g_\rho$ determines the strength of the ρ-nucleon tensor-coupling term, $\langle \rho | H | \omega \rangle$ is the $\rho - \omega$-mixing matrix element, and $m_{\rm v}$ is the common value chosen for the mass of these two mesons. On the basis of arguments given in Ref.[6] we expect that $c_{\rm v} = f_\pi g_{\rm v}/m_{\rm v}$ is the natural dimensionless coupling strength of any vector meson to the nucleon. We similarly expect that $\langle\rho|H|\omega\rangle = -c_{\rho\omega}\,\epsilon\, m_\pi^2$, where $c_{\rho\omega}$ should be natural. This leads to $\delta_1^{\rho\omega} = c_\rho c_\omega \kappa_\rho c_{\rho\omega}[-\epsilon\, m_\pi^2/m_{\rm v}^2 m_{\rm N}^2]$. Using a typical set of values for the coupling constants used in Class IV calculations (see Table I of Ref. [33]) we find $c_\rho = 0.42, c_\omega = 1.9, c_{\rho\omega} = 0.6$, and $\kappa_\rho = 6.1$, and the product of these factors is 2.9, which is large but natural. Using the vector-dominance value for κ_ρ (i.e., 3.7) would lead to a smaller value, as would a smaller c_ω[39]. Even larger values of these coupling constants have been occasionally used in Class IV calculations.

The fact that $\rho - \omega$ mixing seems to provide the necessary additional ingredient for conventional calculations to agree with experiment suggests that a χPT calculation at $n = 4$ will also be successful. At this order, χPT includes a contact interaction of the appropriate form, and the previous discussion implies that a relatively large, but not unnatural, coefficient would suffice.

Note that this argument does not rely on $\rho - \omega$ mixing providing the correct short-range force. For example, an alternative short-range force from isospin violation in the coupling constants of vector mesons has been proposed by Ref.[40]. That result is compatible in sign and magnitude with the $\rho - \omega$-mixing force. The sum of the two mechanisms is too large to reproduce the experimental data, if the above values for ρ and ω parameters are used. In fact, these two mechanisms cannot be distinguished at low energies: only their sum, together with an infinite number of other CSB short-range interactions, can be determined. All short-range mechanisms are subsumed in δ_1, and a δ_1 of about 3 times its natural size seems to be appropriate. How much each short-range mechanism contributes to δ_1 can only be decided at higher energies than those accessible to χPT.

Of course, the above arguments are purely suggestive. A consistent, model-independent calculation is required before more definitive statements can be made. A framework for such a calculation is provided by the Nijmegen partial-wave analysis (PWA)[18, 41]. In this PWA long-range forces, including Eqns. (12) and (28), are used as input, and a general boundary condition at a certain radius, which represents short-range forces, is adjusted until it reproduces data. The IUCF and TRIUMF data have not been analyzed in detail yet. It will be very interesting to see to what extent a short-range parameter equivalent to a natural-sized δ_1 can reproduce the available data, in particular their energy dependence[42]. Preliminary estimates suggest that the long-range parts of the OPEP and Breit interactions alone account for about half of the experimental values at all three energies.

Finally we recall that the original version of the proof[5] that isospin-dependent forces satisfy (in magnitude) Class I > Class II > Class III > Class IV took into account explicitly the structure of Class IV short-range forces, but not the corresponding OPEP. (The latter is momentum dependent and suppressed by one power of M_N. In Ref.[5], a power counting was used in which Q/M_N was counted as $(Q/\Lambda)^2$, rendering this force $n = 3$.) Although the size of the latter estimated from the present power counting ($n = 2$) is nominally the same as that of Class III forces, its suppression due to cancellations and fine tuning (to reproduce the physical nucleon mass) makes the Class IV OPEP more typical of $n = 3$ size, and therefore the results of the proof are not altered.

The second set of two CSB experiments measured π^0 production: $n + p \to d + \pi^0$[15] and $d + d \to {}^4\mathrm{He} + \pi^0$[43]. The front-back asymmetry is the CSB signal in the first reaction, while the cross-section of the second reaction vanishes in the absence of isospin mixing. The effect of the second and third terms in Eqn. (3) on the $n + p \to d + \pi^0$ front-back asymmetry was calculated in Ref.[14]. It was found to be relatively large, and of opposite sign to other mechanisms. This prediction is in good agreement with the experimental result[15]. The situation is considerably more complicated for $d + d \to {}^4\mathrm{He} + \pi^0$. A preliminary, simplified calculation[44] suggests that various mechanisms contribute significantly. Both reactions should be further studied. The field redefinitions that were invented in Eqns. (4) and (7) and lead to Eqn. (8) could prove useful in this regard.

In summary, in this paper we have presented a convenient framework in which to analyze nuclear effects of the nucleon-mass difference. We examined in some detail the Class IV force in the context of χPT, stressing its similarities and differences with respect to conventional approaches.

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