THEF-NIJM 02.03
nucl-th/0201020

ESC NN-Potentials in Momentum Space
II. Meson-Pair Exchange Potentials

Th.A. Rijken and H. Polinder
Institute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands

J. Nagata*
Venture Business Laboratory, Hiroshima University, Kagamiyama 2-313, Higashi-Hiroshima, Japan

The partial wave projection of the Nijmegen soft-core potential model for Meson-Pair-Exchange (MPE) for NN-scattering in momentum space is presented. Here, nucleon-nucleon momentum space MPE-potentials are NN-interactions where either one or both nucleons contains a meson-pair vertex. Dynamically, the meson-pair vertices can be viewed as describing in an effective way (part of) the effects of heavy-meson exchange and meson-nucleon resonances. From the point of view of "duality," these two kinds of contribution are roughly equivalent. Part of the MPE-vertices can be found in the chiral-invariant phenomenological Lagrangians that have a basis in spontaneous broken chiral symmetry. It is shown that the MPE-interactions are a very important component of the nuclear force, which indeed enables a very succesful description of the low and medium energy NN-data. Here we present a precise fit to the NN-data with the extended-soft-core (ESC) model containing OBE-, PS-PS-, and MPE-potentials. An excellent description of the NN-data for $T_{Lab} \leq 350$ MeV is presented and discussed. Phase shifts are given and a $\chi^2_{p.d.p.} = 1.15$ is reached.

PACS numbers: 13.75.Cs, 12.39.Pn, 21.30.+y

I. INTRODUCTION

In the previous paper [1], henceforth refered to as paper I, the techniques for the momentum space treatment of the extended soft-core model, hereafter referred to as the ESC-model, are described. This implies first the development of a representation of the ESC-model suitable for the projection onto the Pauli-spinor rotational invariant operators, and secondly the partial wave analysis. This partial wave analysis is organized along similar lines as used for the soft-core OBE-models [2]. In [1] the nucleon-nucleon partial wave contributions have been worked out in detail. These are the analogs of the configuration space two-meson-exchange (TME) potentials given in e.g. [3]. Here, the TME-potentials are defined to contain the planar- and crossed-box two-meson-exchange potentials.

In this second paper on soft-core two-meson-exchange potentials in momentum space, occasiously refered to as paper II in the following, we derive the same representation as in paper I, but now for the contributions to the nucleon-nucleon potentials when either one or both nucleons contains a pair vertex, i.e. the MPE-potentials. We give the partial wave potentials in the similar representation as used in paper I. In Ref. [4] the MPE-contributions to the configuration space nucleon-nucleon potentials, i.e. when either one or both nucleons contains a pair vertex, have been derived. The corresponding "seagull" diagrams are refered to as one-pair and two-pair diagrams. This in order to distinct these from the planar and crossed-box diagrams, which were given Ref. [3].

The two types of two-meson-exchange potentials TME, see I, and MPE presented here are part of our program to extend the Nijmegen soft-core one-boson-exchange potential [5–7] to arrive at a new extended soft-core nucleon-nucleon model, hereafter referred to as the ESC potential [4, 8–10].

In the introduction to Ref. [4] a rather complete description is given of the physical background behind the MPE-potentials, and we refer the interested reader to that reference.

We apply the potentials derived in this work to fit the NN-data. In the TME-potentials we restrict ourselves to the ps-ps exchange. Or, phrased differently, we include only the Goldstone-boson sector. This because it gives the complete long-range contribution, OPEP+TPEP and the inclusion of η etc. is necessary for (i) (approximate) chiral symmetry, and (ii) for completeness in the sense of $SU_f(3)$, which allows an extension to hyperon-nucleon and hyperon-hyperon [10].
In fact, this fit has been performed in the configuration space version. However, the results were checked numerically in momentum space, using the formulas of papers I and II.

This paper is organized as follows. In section II and III, we give the essentials of the procedure followed in deriving the new momentum space representation. In sec-

*Present address: Kyushu International University, Fukuoka 805-8512, Japan

tion IV the projection of the MPE on the Pauli-spinor invariants is worked out for the adiabatic contributions. In section V the same is done for the $1/M$-corrections: the non-adiabatic and the pseudo-vector-vertex terms. In section VI the partial wave analysis is indicated. The procedure for the partial wave projection is completely analogous to that of paper I, and can be transcribed immediately comparing the invariant contributions $\Omega_j(\mathbf{k}^2; t, u)$ for MPE to those for TME in I. In section VII the results from a fit to the NN-data are shown and discussed. Here, phase shifts are given for $T_{Lab} \leq 350$ MeV and the pair-couplings are compared to the values expected from e.g. chiral lagrangians.

In Appendix A the pair-interaction Hamiltonians are listed. In Appendix B the λ-representations for the MPE-denominators are given. In Appendix C we give the integration dictionary for the gaussian integrals that occur in MPE but not in TME. In Appendix D a derivation for the potentials due to the 'derivative scalar pair' interaction, see the $g'_{(\pi\pi)_0}$-coupling in A1a is outlined. This for completeness, since although we do not employ this kind of pair interaction, it occurs occurs often in the current literature. In Appendix E the full $SU_f(3)$ contents of our pair interactions is shown.

II. MOMENTUM SPACE REPRESENTATION MPE-POTENTIALS

Here, we give an outline the essentials of the procedure to derive our new momentum space representation for the MPE-potentilas. These procedures have been described in I, to which we refer for details. Here, we focuss on the peculiar features that occur in the application to the MPE-potentials.

The starting point is the basic convolutive integral

$$\begin{aligned} \widetilde{V}_{M,N}(\mathbf{k}) &= \int\!\!\int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\ \tilde{F}_M(\mathbf{k}_1^2, m_1)\tilde{G}_N(\mathbf{k}_2^2, m_2) \\ &= \int \frac{d^3\Delta}{(2\pi)^3} \tilde{F}_M(\mathbf{\Delta}^2, m_1)\ \tilde{G}_N((\mathbf{k}-\mathbf{\Delta})^2, m_2)\ , \end{aligned} \tag{2.1}$$

where $\tilde{F}_M(\mathbf{k}^2)$ and $\tilde{G}_N(\mathbf{k}^2)$ can be of the form

$$\begin{aligned} M=0:\quad \tilde{F}_0(\mathbf{k}^2) &= \exp\left[-\mathbf{k}^2/\Lambda_1^2\right] \quad , \quad M=2:\quad \tilde{F}_2(\mathbf{k}^2) = \frac{\exp\left[-\mathbf{k}^2/\Lambda_1^2\right]}{\mathbf{k}^2+m_1^2}\ , \\ N=0:\quad \tilde{G}_0(\mathbf{k}^2) &= \exp\left[-\mathbf{k}^2/\Lambda_2^2\right] \quad , \quad N=2:\quad \tilde{G}_2(\mathbf{k}^2) = \frac{\exp\left[-\mathbf{k}^2/\Lambda_2^2\right]}{\mathbf{k}^2+m_2^2}\ , \end{aligned} \tag{2.2}$$

i.e. $M,N=2$ is the modified Yukawa type and $M,N=0$ is the Gaussian type. Below, we give for the different cases the momentum space representation, similar to the one that has been developed in paper I:

(i) $M=N=2$: In paper I using twice the identity

$$\frac{\exp\left[-\mathbf{k}^2/\Lambda^2\right]}{\mathbf{k}^2+m^2} = e^{m^2/\Lambda^2}\int_1^\infty \frac{dt}{\Lambda^2}\exp\left[-\left(\frac{\mathbf{k}^2+m^2}{\Lambda^2}\right)t\right] \tag{2.3}$$

the $\mathbf{\Delta}$-integral has been carried out. After a redefinition of the variables $t \to t/\Lambda_1^2$ and $u \to u/\Lambda_2^2$ the result in I is

$$\begin{aligned} \tilde{V}_{2,2}(\mathbf{k}) &= (4\pi)^{-3/2} e^{m_1^2/\Lambda_1^2} e^{m_2^2/\Lambda_2^2} \int_{t_0}^\infty dt \int_{u_0}^\infty du \frac{\exp[-(m_1^2 t + m_2^2 u)]}{(t+u)^{3/2}}\ \cdot \\ &\times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\ \ (t_0 = 1/\Lambda_1^2, u_0 = 1/\Lambda_2^2)\ . \end{aligned} \tag{2.4}$$

(ii) $M=2, N=0$: Using the identity (2.3) once, and performing similar steps as in paper I, one easily derives that for this case

$$\begin{aligned} \tilde{V}_{2,0}(\mathbf{k}) &= (4\pi)^{-3/2} e^{m_1^2/\Lambda_1^2} e^{m_2^2/\Lambda_2^2} \int_{t_0}^\infty dt \int_{u_0}^\infty du \frac{\exp[-(m_1^2 t + m_2^2 u)]}{(t+u)^{3/2}}\ \cdot \\ &\times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right] \cdot \delta(u-u_0)\ . \end{aligned} \tag{2.5}$$

Here, is defined $\delta(u-u_0) \equiv \lim_{\epsilon\downarrow 0} \delta(u-u_{0,\epsilon})$, where $u_{0,\epsilon} = u_0 - \epsilon$. This definition implies that in (2.5) the u-integration can simply be performed by the substitution $u \to u_0$ in the integrand.
(iii) $M = 0, N = 2$: Similarly to the previous case, one has

$$\begin{aligned} \tilde{V}_{0,2}(\mathbf{k}) &= (4\pi)^{-3/2} e^{m_1^2/\Lambda_1^2} e^{m_2^2/\Lambda_2^2} \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du \frac{\exp[-(m_1^2 t + m_2^2 u)]}{(t+u)^{3/2}} \cdot \\ &\quad \times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right] \cdot \delta(t-t_0) \ . \end{aligned} \tag{2.6}$$

For the integrals $\tilde{V}_{M,N}$ of this section, and similar integrals below in this paper, we introduce the following convenient short-hand notation. We write

$$\tilde{V}_{M,N}(\mathbf{k}) = \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du \ w(t,u) \cdot \left\{ v_{M,N}(t,u) \ \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\right\} \ , \tag{2.7a}$$

with common weight function $w_0(t,u)$ defined as

$$w_0(t,u) \equiv (4\pi)^{-3/2} e^{m_1^2/\Lambda_1^2} e^{m_2^2/\Lambda_2^2} \frac{\exp[-(m_1^2 t + m_2^2 u)]}{(t+u)^{3/2}} \ . \tag{2.7b}$$

The form in which these basic integrals appear in MPE depends on two factors:

(i) The denominators $D(\omega_1, \omega_2)$. In the next section we will give a catalogue of these.

(ii) The operators $\tilde{O}(\mathbf{k}_1, \mathbf{k}_2)$. Also these will be given in the next section.

III. MESON-PAIR EXCHANGE POTENTIALS

In [4] the derivation of the pair-exchange potentials both in momentum and in configuration space is given. In this reference the configuration space potentials are worked out fully. The topic of this paper is to do the same for the momentum space description. In particular, the partial wave analysis is performed leading to a representation which is very suitable for numerical evaluation.

From [4] and equation (3.1) it follows that the momentum space MPE-potential can be represented in general in the form

$$\begin{aligned} \widetilde{V}^{(n)}_{\alpha\beta}(\mathbf{k}) &= C^{(n)}(\alpha\beta) g^{(n)}(\alpha\beta) \int\!\!\int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \\ &\quad \times \delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \ \tilde{F}_0(\mathbf{k}_1^2) \tilde{G}_0(\mathbf{k}_2^2) \\ &\quad \times \sum_p \tilde{O}^{(n)}_{\alpha\beta,p}(\mathbf{k}_1, \mathbf{k}_2) \ D^{(n)}_p(\omega_1, \omega_2), \end{aligned} \tag{3.1}$$

where the index n distinguishes one-pair ($n = 1$) and two-pair ($n = 2$) meson-pair exchange, and $(\alpha\beta)$ refers to the particular meson pair that is being exchanged. The subscript $p = \{ad, na, pv\}$ distinguishes respectively the adiabatic-, the non-adiabatic-, pseudovector vertex-, and off-shell-contributions. Here, the last three are the $1/M$-corrections to the MPE-potentials.

The product of the coupling constants in the cases $n = 1, 2$ is given by

$$\begin{aligned} g^{(1)}(\alpha\beta) &= g_{(\alpha\beta)} g_{NN\alpha} g_{NN\beta}, \\ g^{(2)}(\alpha\beta) &= g^2_{(\alpha\beta)}, \end{aligned} \tag{3.2}$$

with appropriate powers of m_π, depending on the definition of the Hamiltonians given in [4], section II.

The momentum-dependent operators $O^{(n)}_{\alpha\beta,p}$ are given in Tables IV and VI. For completeness, these Tables also contain the isospin factors $C^{(n)}(\alpha\beta)$ as derived in Appendix B of [4]. The momentum operators for $(\pi\pi)_0$ and $(\pi\pi)_1$ both contain a term antisymmetric in $\mathbf{k}_1 \leftrightarrow \mathbf{k}_2$, which only contributes in the nonadiabatic contribution, see [4], section 4. In the adiabatic potential, as explained in [4], they drop out when we integrate over $\mathbf{k}_1$ and $\mathbf{k}_2$.

The energy denominators $D^{(n)}_p$ are also discussed in detail in [4]), section II, in terms of the time-ordered processes involved in one- and two-pair exchange. These denominators depend on the energies of the exchanged mesons, i.e. ω_1 and ω_2. Another sourse of $\omega_{1,2}$-dependence comes from vertices with derivatives, and the non-adiabatic expansion terms. It appears from [4] that in general one can write

$$D^{(n)}_{\{p\}}(\omega_1, \omega_2) = \sum_{p_1,p_2,p_3} c^{(n)}_{p_1,p_2,p_3} \ D_{\{p_1,p_2,p_3\}} \ , \tag{3.3}$$

where in terms of the integer powers p_i $(i = 1, 2, 3)$ the

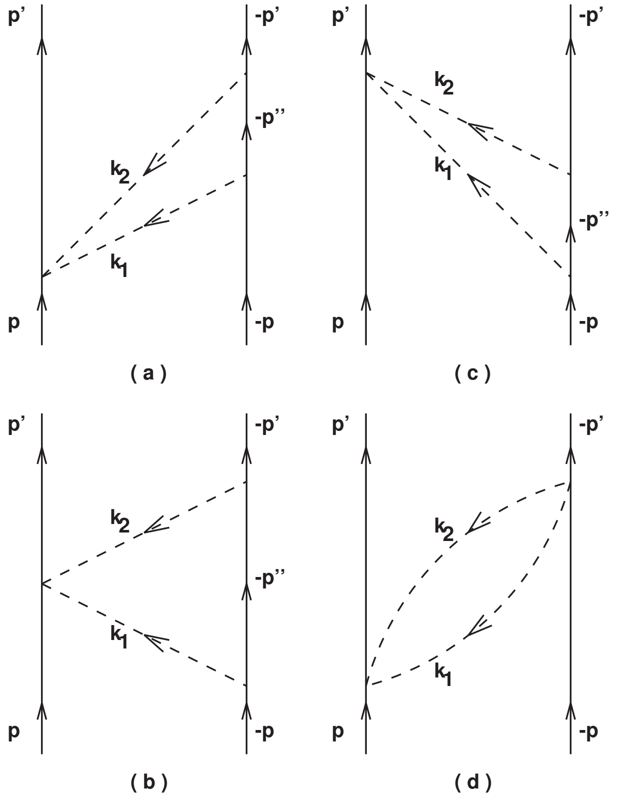

FIG. 1: Time-ordered (a)-(c) one-pair and (d) two-pair diagrams. The dashed line with momentum $\mathbf{k}_1$ refers to the pion and the dashed line with momentum $\mathbf{k}_2$ refers to one of the other (vector, scalar, or pseudoscalar) mesons. To these we have to add the "mirror" diagrams, where for the one-pair diagrams the pair vertex occurs on the other nucleon line.

denominators can be written

$$D_{\{p_1,p_2,p_3\}} = \frac{1}{\omega_1^{p_1}} \frac{1}{\omega_2^{p_2}} \frac{1}{(\omega_1+\omega_2)^{p_3}} \ . \qquad (3.4)$$

The energy denominators $D_p^{(n)}$ are listed in Tables V and VI.

The evaluation of the momentum integrations can now readily be performed using the methods given in [4, 11]. There it was shown that the full separation of the $\mathbf{k}_1$ and $\mathbf{k}_2$ dependence can be achieved in all cases using the λ-integral representation, first introduced in [11]. In Appendix B the occurring λ-integrals are listed. From the listing in B one readily sees that for the derivation of the representation similar to that one in Eqs. (2.4)-(2.6) we need to start out from the generalization of (2.1):

$$\widetilde{V}_{M,N}(\mathbf{k},\lambda) = \frac{2}{\pi}\int_0^\infty d\lambda f_{M,N}(\lambda) \int \frac{d^3\Delta}{(2\pi)^3} \cdot \\ \times \tilde{F}_M(\mathbf{\Delta}^2, \sqrt{m_1^2+\lambda^2})\ \tilde{G}_N((\mathbf{k}-\mathbf{\Delta})^2, \sqrt{m_2^2+\lambda^2}) . \qquad (3.5)$$

In paper I it has been shown that all the occurring λ-integrals can be performed analytically. The result for all cases can be written as

$$\begin{aligned} \widetilde{V}_{p_1,p_2,p_3}(\mathbf{k}) &= \int \frac{d^3\Delta}{(2\pi)^3} \tilde{F}_0(\mathbf{\Delta}^2, m_1)\ \tilde{G}_0((\mathbf{k}-\mathbf{\Delta})^2, m_2)\ D_{\{p_1,p_2,p_3\}}(\omega_1,\omega_2) \\ &= \int_{t_0}^\infty dt \int_{u_0}^\infty du\ w_0(t,u) \cdot \left\{ d_{\{p_1,p_2,p_3\}}(t,u)\ \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\right\} \ . \end{aligned} \qquad (3.6)$$

All functions $d_{\{p_1,p_2,p_3\}}(t,u)$ that occur in this work are given in Table VII. As noted in section II we will use only representations with $M = N = 2$, so that no $\delta(t-t_0)$ or $\delta(u-u_0)$ occurs.

IV. PROJECTION MPE ON SPINOR INVARIANTS I ADIABATIC CONTRIBUTIONS

The MPE-contributions from the adiabatic terms, the non-adiabatic- and pseudovector vertex corrections are the central-, spin-spin-, tensor-, and spin-orbit momentum space analogs of those given in Reference [4] in configuration space. From (3.1), Tables IV-VI, and Table V it is readily verified that the projection onto the potentials V_j , similarly to the paper I, can be written as

$$\widetilde{V}^{(n)}_{\rm pair}(\alpha\beta) \;=\; \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du\ w_0(t,u)\ \left\{\exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\ \Omega^{(n)}_j(\mathbf{k}^2;t,u)\right\}(\alpha\beta)\ . \tag{4.1}$$

The functions $\Omega^{(n)}_j$ are worked out in the subsections below. Like in I, we also introduce for convenience the expansion in $\mathbf{k}^2$:

$$\Omega^{(ad,na,pv)}_j(\mathbf{k}^2;t,u) = C^{(n)}(\alpha\beta) g^{(n)}(\alpha\beta)\cdot\sum_{k=0}^{K}\ \Upsilon^{(ad,na,pv)}_{j,k}(t,u)\ \left(\mathbf{k}^2\right)^k\ . \tag{4.2}$$

Below in this section we give the results for the adiabatic contributions. The coefficients $\Upsilon^{ad}_{j,k}$ are tabulated in Tables VIII-XII.

A. $J^{PC}=0^{++}$: Adiabatic $(\pi\pi)_0$-Exchange potentials

The 1-pair and 2-pair contributions are

$$\begin{aligned}
\Omega^{(1)}_1(\mathbf{k}^2;t,u) \;&=\; 6\left(\frac{g_{(\pi\pi)_0}}{m_\pi}\right)\left(\frac{f^2_{NN\pi}}{m^2_\pi}\right)\cdot d_{\{2,2,0\}}(t,u)\left\{+\frac{3}{2}-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ , &(4.3a)\\
\Omega^{(2)}_1(\mathbf{k}^2;t,u) \;&=\; -3\left(\frac{g_{(\pi\pi)_0}}{m^2_\pi}\right)^2\cdot d_{\{1,1,1\}}(t,u) &(4.3b)
\end{aligned}$$

B. $J^{PC}=1^{--}$: Adiabatic $(\pi\pi)_1$-Exchange potentials

(i) 1-pair exchange:

$$\begin{aligned}
\Omega^{(1)}_1(\mathbf{k}^2;t,u) \;&=\; -4(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m^2_\pi}\right)\left(\frac{f^2_{NN\pi}}{m^2_\pi}\right)\cdot d_{1,1,1}(t,u)\left\{-\frac{3}{2}+\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ , &(4.4a)\\
\Omega^{(1)}_2(\mathbf{k}^2;t,u) \;&=\; -2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m^2_\pi}\right)\left(\frac{f^2_{NN\pi}}{m^2_\pi}\right)\frac{(1+\kappa_1)}{M}\cdot d_{2,2,0}(t,u)\cdot +\frac{1}{3}\mathbf{k}^2\cdot\frac{1}{t+u}\ , &(4.4b)\\
\Omega^{(1)}_3(\mathbf{k}^2;t,u) \;&=\; -2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m^2_\pi}\right)\left(\frac{f^2_{NN\pi}}{m^2_\pi}\right)\frac{(1+\kappa_1)}{M}\cdot d_{2,2,0}(t,u)\cdot -\frac{1}{2}\cdot\frac{1}{t+u}\ , &(4.4c)\\
\Omega^{(1)}_4(\mathbf{k}^2;t,u) \;&=\; -2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m^2_\pi}\right)\left(\frac{f^2_{NN\pi}}{m^2_\pi}\right)\frac{1}{M}\cdot d_{2,2,0}(t,u)\cdot\frac{1}{t+u}\ , &(4.4d)
\end{aligned}$$

(ii) 2-pair exchange:

$$\Omega^{(2)}_1(\mathbf{k}^2;t,u) \;=\; -\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m^2_\pi}\right)^2\cdot\left[d_{1,0,0}+d_{0,1,0}-4d_{0,0,1}\right](t,u)\ . \tag{4.4e}$$

C. $J^{PC}=1^{++}$: Adiabatic $(\pi\rho)_1$-Exchange potentials

(i) 1-pair exchange:

$$\begin{aligned}
\Omega^{(1)}_2(\mathbf{k}^2;t,u) \;&=\; \frac{2}{M}\ (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\rho)_1}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\rho}}{m_\pi}\right)\cdot d_{2,2,0}(t,u)\cdot\\
&\times\left\{\left[\frac{1}{2}+\frac{1}{3}\left(\frac{u^2}{t+u}\right)\mathbf{k}^2\right]+\frac{1}{2}(1+\kappa_\rho)\left[2-\frac{4}{3}\frac{tu}{t+u}\mathbf{k}^2\right]\right\}\cdot\frac{1}{t+u}\ , &(4.5a)\\
\Omega^{(1)}_3(\mathbf{k}^2;t,u) \;&=\; \frac{2}{M}\ (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\rho)_1}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\rho}}{m_\pi}\right)\cdot d_{2,2,0}(t,u)\cdot\\
&\times\left\{\frac{u^2}{t+u}+\frac{1}{2}(1+\kappa_\rho)\frac{2tu}{t+u}\right\}\cdot\frac{1}{t+u}\ , &(4.5b)
\end{aligned}$$

(ii) 2-pair exchange:

$$\Omega_2^{(2)}(\mathbf{k}^2;t,u) \;=\; -(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\rho)_1}}{m_\pi^2}\right)^2 \cdot d_{1,1,1}(t,u) \tag{4.5c}$$

D. $J^{PC}=1^{++}$: Adiabatic $(\pi\sigma)$-Exchange potentials

(i) 1-pair exchange:

$$\begin{aligned}\Omega_2^{(1)}(\mathbf{k}^2;t,u) &= +(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NN\sigma}}{m_\pi}\right)\cdot d_{2,2,0}(t,u)\cdot \\ &\quad\times\left[-2+\frac{2}{3}\left(\frac{tu-u^2}{t+u}\right)\mathbf{k}^2\right]\cdot\frac{1}{t+u}\ , \qquad (4.6a)\\ \Omega_3^{(1)}(\mathbf{k}^2;t,u) &= +2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NN\sigma}}{m_\pi}\right)\cdot d_{2,2,0}(t,u)\cdot \\ &\quad\times\left(\frac{tu-u^2}{t+u}\right)\cdot\frac{1}{t+u}\ , \qquad (4.6b)\end{aligned}$$

(ii) 2-pair exchange:

$$\begin{aligned}\Omega_2^{(2)}(\mathbf{k}^2;t,u) &= -\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\right)^2\cdot d_{1,1,1}(t,u)\cdot \\ &\quad\times\left\{\frac{2}{t+u}+\frac{1}{3}\left(\frac{t-u}{t+u}\right)^2\mathbf{k}^2\right\}\ , \qquad (4.6c)\\ \Omega_3^{(2)}(\mathbf{k}^2;t,u) &= -\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\right)^2\cdot d_{1,1,1}(t,u)\left(\frac{t-u}{t+u}\right)^2\ , \qquad (4.6d)\end{aligned}$$

E. $J^{PC}=1^{+-}$: Adiabatic $(\pi\omega)$-Exchange potentials

(i) 1-pair exchange:

$$\begin{aligned}\Omega_2^{(1)}(\mathbf{k}^2;t,u) &= (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\omega}}{m_\pi}\right)\cdot d_{2,2,0}(t,u)\cdot \\ &\quad\times\left[\frac{2}{3}\left(\frac{tu+u^2}{t+u}\right)\mathbf{k}^2\right]\cdot\frac{1}{t+u}\ , \qquad (4.7a)\\ \Omega_3^{(1)}(\mathbf{k}^2;t,u) &= +2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\omega}}{m_\pi}\right)\cdot d_{2,2,0}(t,u)\cdot \\ &\quad\times\left(\frac{tu+u^2}{t+u}\right)\cdot\frac{1}{t+u}\ , \qquad (4.7b)\end{aligned}$$

(ii) 2-pair exchange:

$$\begin{aligned}\Omega_2^{(2)}(\mathbf{k}^2;t,u) &= -\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)_1}}{m_\pi^2}\right)^2\cdot \\ &\quad\times\left\{d_{1,0,0}+d_{0,1,0}-\frac{1}{3}\mathbf{k}^2 d_{1,1,1}\right\}(t,u)\ , \qquad (4.7c)\\ \Omega_3^{(2)}(\mathbf{k}^2;t,u) &= +\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)_1}}{m_\pi^2}\right)^2\cdot d_{1,1,1}(t,u)\ . \qquad (4.7d)\end{aligned}$$

F. $J^{PC}=1^{++}$: Adiabatic (πP)-Exchange potentials

The treatment of the Pomeron has been explained in [3]. This implies the use of $\widetilde{G}_0/M_N^2$ in section II. Furthermore, w.r.t. σ-exchange there is a $(-)$-sign for P-exchange. Therefore, comparing to (4.6a-4.6d) we obtain the following potentials:
(i) 1-pair exchange:

$$\begin{aligned}\Omega_2^{(1)}(\mathbf{k}^2;t,u) &= -(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NNP}}{m_\pi}\right)\cdot\frac{1}{M_N^2}\cdot d_{2,0,0}(t,u)\cdot \\ &\quad\times\left[-2+\frac{2}{3}\left(\frac{tu-u^2}{t+u}\right)\mathbf{k}^2\right]\cdot\frac{1}{t+u}\cdot\delta(u-u_0)\ , \qquad (4.8a)\\ \Omega_3^{(1)}(\mathbf{k}^2;t,u) &= -2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NNP}}{m_\pi}\right)\cdot\frac{1}{M_N^2}\cdot d_{2,0,0}(t,u)\cdot \\ &\quad\times\left(\frac{tu-u^2}{t+u}\right)\cdot\frac{1}{t+u}\cdot\delta(u-u_0)\ . \qquad (4.8b)\end{aligned}$$

(ii) 2-pair exchange:

$$\begin{aligned}\Omega_2^{(2)}(\mathbf{k}^2;t,u) &= +\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)_1}}{m_\pi}\right)^2\cdot\frac{1}{M_N^2}\cdot d_{1,1,1}(t,u)\cdot \\ &\quad\times\left\{\frac{2}{t+u}+\frac{1}{3}\left(\frac{t-u}{t+u}\right)^2\mathbf{k}^2\right\}\cdot\delta(u-u_0)\ , \qquad (4.8c)\\ \Omega_3^{(2)}(\mathbf{k}^2;t,u) &= +\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)_1}}{m_\pi^2}\right)^2\cdot\frac{1}{M_N^2}\cdot d_{1,1,1}(t,u)\left(\frac{t-u}{t+u}\right)^2\cdot\delta(u-u_0)\ . \qquad (4.8d)\end{aligned}$$

Notice that in (4.8a)-(4.8d) $u_0=1/4m_P^2$.

G. $J^{PC}=0^{++}$: Adiabatic 'derivative' $(\pi\pi)_0$-Exchange potentials

The derivative pair-potentials in coordinate space have been derived in [12] in detail. A summary of this is given in appendix D. A short derivation of the p-space potentials is also can be found there.
(i) 1-pair exchange:

$$\begin{aligned}\Omega_1^{(1)}(\mathbf{k}^2;t,u) &= -12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\cdot d_{2,2,0}(t,u)\cdot \\ &\quad\times\left[\frac{15}{4}+\frac{1}{2}\frac{t^2-8tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\cdot\frac{1}{(t+u)^2}\ , \qquad (4.9a)\end{aligned}$$

(ii) 2-pair exchange:

$$\begin{aligned}\Omega_1^{(2)}(\mathbf{k}^2;t,u) &= -6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)^2\cdot\left\{\left[\frac{15}{4}+\frac{t^2-3tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{d_{1,1,1}(t,u)}{(t+u)^2}\right. \\ &\quad+\frac{1}{2}\left[\frac{3}{2}(m_1^2+m_2^2)+\frac{m_1^2t+m_2^2u}{t+u}\mathbf{k}^2+m_1^2m_2^2(t+u)\right]\frac{d_{1,1,1}(t,u)}{t+u} \\ &\quad\left.+\left[\frac{3}{2}-\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{0,0,1}(t,u)}{t+u}\right\}\ . \qquad (4.9b)\end{aligned}$$

H. $J^{PC}=0^{++}$: Adiabatic $(\sigma\sigma)$-Exchange potentials

(i) 1-pair exchange:

$$\Omega_1^{(1)}(\mathbf{k}^2;t,u) = 2\left(\frac{g_{(\sigma\sigma)}}{m_\pi}\right)\cdot g_{NN\sigma}^2\cdot d_{2,2,0}(t,u)\ , \qquad (4.10a)$$

(ii) 2-pair exchange:

$$\Omega_1^{(2)}(\mathbf{k}^2;t,u) \;=\; -\left(\frac{g_{(\sigma\sigma)}}{m_\pi}\right)^2 d_{1,1,1}(t,u)\ , \tag{4.10b}$$

V. PROJECTION MPE ON SPINOR INVARIANTS II 1/M CORRECTIONS

The non-adiabatic- and pseudovector vertex-corrections have been given in [4], section IV. Similar to Eq. (4.1) we write these contributions in the form

$$\widetilde{V}_{\rm pair}^{(na,pv)}(\alpha\beta) \;=\; \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du\ w_0(t,u)\ \left\{ d^{(n)}_{\{p_1,p_2,p_3\}}(t,u)\ \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\ \Omega_j^{(n)}(\mathbf{k}^2;t,u)\right\}\ . \tag{5.1}$$

A. Non-adiabatic Corrections

From Eqs. (4.5)-4.8) of [4] one readily obtains the momentum space equivalents using the replacements:

$$\int\!\!\int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\ e^{i(\mathbf{k}_1+\mathbf{k}_2)} \rightarrow \int\!\!\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\ \delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$$

Then, by comparison one can easily read off the diverse quantities $\tilde{O}^{(na)}_{\alpha\beta,p}$ and $D_p^{(na)}(\omega_1,\omega_2)$ that occur in Eq. (3.1) for the non-adiabatic potentials. The projections onto the $\Omega_j^{(na)}$ in Eq. (5.1) yield
(i) $(\pi\pi)_0$:

$$\begin{aligned}\Omega_1^{(na)}(\mathbf{k}^2;t,u) \;&=\; -\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \frac{3}{M}\cdot d_{\{na\}}(t,u)\left\{\frac{15}{4}+\right.\\ &\left.+\frac{1}{2}\left(\frac{t^2-8ut+u^2}{t+u}\right)\mathbf{k}^2+\left(\frac{t^2u^2}{(t+u)^2}\right)\mathbf{k}^4\right\}\cdot\frac{1}{(t+u)^2}\ ,\end{aligned} \tag{5.2a}$$

$$\Omega_4^{(na)}(\mathbf{k}^2;t,u) \;=\; -\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \frac{3}{M}\cdot d_{\{na\}}(t,u)\cdot\frac{1}{t+u}\ . \tag{5.2b}$$

(ii) $(\pi\pi)_1$:

$$\begin{aligned}\Omega_1^{(na)}(\mathbf{k}^2;t,u) \;&=\; -2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \frac{1}{M}\cdot d_{\{2,2,0\}}(t,u)\left\{\frac{15}{4}+\right.\\ &\left.+\frac{1}{2}\left(\frac{t^2-8ut+u^2}{t+u}\right)\mathbf{k}^2+\left(\frac{t^2u^2}{(t+u)^2}\right)\mathbf{k}^4\right\}\cdot\frac{1}{(t+u)^2}\ ,\end{aligned} \tag{5.3a}$$

$$\Omega_4^{(na)}(\mathbf{k}^2;t,u) \;=\; -2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \frac{1}{M}\cdot d_{\{2,2,0\}}(t,u)\cdot\frac{1}{t+u}\ . \tag{5.3b}$$

(iii) $(\sigma\sigma)$:

$$\Omega_1^{(na)}(\mathbf{k}^2;t,u) \;=\; \frac{g_{(\sigma\sigma)}}{m_\pi}\frac{g^2_{NN\sigma}}{M}\cdot d_{\{na\}}(t,u)\left\{-\frac{3}{2}+\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ . \tag{5.4}$$

(iv) $(\pi\sigma)$:

$$\begin{aligned}\Omega_2^{(na)}(\mathbf{k}^2;t,u) \;&=\; +(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}\cdot d_{\{na\}}(t,u)\cdot\\ &\times\left\{\frac{5}{2}+\frac{1}{6}\frac{t^2-13tu+6u^2}{t+u}\mathbf{k}^2+\frac{1}{3}\frac{tu^2(t-u)}{(t+u)^2}\mathbf{k}^4\right\}\cdot\frac{1}{(t+u)^2}\ ,\end{aligned} \tag{5.5a}$$

$$\begin{aligned}\Omega_3^{(na)}(\mathbf{k}^2;t,u) \;&=\; +(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}\cdot d_{\{na\}}(t,u)\cdot\\ &\times\left\{\frac{1}{2}\frac{t^2-7tu+6u^2}{t+u}+\frac{tu^2(t-u)}{(t+u)^2}\mathbf{k}^2\right\}\cdot\frac{1}{(t+u)^2}\ .\end{aligned} \tag{5.5b}$$

(v) $(\pi\pi)_0$('derivative'):

$$\Omega_1^{(na)}(\mathbf{k}^2;t,u) = -12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \frac{1}{2M}\cdot\Bigg\{$$

$$+\left[\frac{15}{4}+\frac{1}{2}\left(\frac{t^2-8tu+u^2}{t+u}\right)\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{d_{1,1,1}(t,u)}{(t+u)^2}\ ,$$

$$-\left[\frac{105}{8}+\frac{15}{4}\left(\frac{t^2-5tu+u^2}{t+u}\right)\mathbf{k}^2-\frac{3}{2}tu\left(\frac{t^2-5tu+u^2}{(t+u)^2}\right)\mathbf{k}^4-\frac{t^3u^3}{(t+u)^3}\mathbf{k}^6\right]\frac{d_{na}(t,u)}{(t+u)^3}\Bigg\}\ , \tag{5.6a}$$

$$\Omega_4^{(na)}(\mathbf{k}^2;t,u) = -12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \frac{1}{2M}\cdot\left\{\frac{d_{1,1,1}(t,u)}{t+u}+\left[-5+2\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{na}(t,u)}{(t+u)^2}\right\}\ . \tag{5.6b}$$

Here, $d_{\{na\}}(t,u)$ is defined in (B3).

B. Pseudovector-vertex Corrections

From Eqs. (4.9)-4.11) of [4] likewise as in the case of the non-adiabatic corrections one obtains for the pseudovector-vertex corrections:
(i) $(\pi\pi)_0$:

$$\Omega_1^{(pv)}(\mathbf{k}^2;t,u) = +\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{3}{M}\cdot d_{\{1,1,1\}}(t,u)\left\{3+\left(\frac{t^2+u^2}{t+u}\right)\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ , \tag{5.7a}$$

$$\Omega_4^{(pv)}(\mathbf{k}^2;t,u) = +2\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{3}{M}\cdot d_{\{1,1,1\}}(t,u)\ . \tag{5.7b}$$

(ii) $(\pi\pi)_1$:

$$\Omega_1^{(pv)}(\mathbf{k}^2;t,u) = +(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}\cdot\left[\left(\frac{3}{2}+\frac{u^2}{t+u}\mathbf{k}^2\right)d_{\{2,0,0\}}(t,u)\right.$$
$$\left.+\left(\frac{3}{2}+\frac{t^2}{t+u}\mathbf{k}^2\right)d_{\{0,2,0\}}(t,u)\right]\cdot\frac{1}{t+u} \tag{5.8a}$$

$$\Omega_4^{(pv)}(\mathbf{k}^2;t,u) = +2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}\cdot\left[\frac{u}{t+u}d_{\{2,0,0\}}+\frac{t}{t+u}d_{\{0,2,0\}}\right]\ . \tag{5.8b}$$

(iii) $(\pi\sigma)$:

$$\Omega_2^{(pv)}(\mathbf{k}^2;t,u) = -(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}\cdot d_{\{1,1,1\}}(t,u)\cdot\left\{1+\frac{1}{3}\frac{t^2-tu}{t+u}\mathbf{k}^2\right\}\frac{1}{t+u}\ . \tag{5.9a}$$

$$\Omega_3^{(pv)}(\mathbf{k}^2;t,u) = -(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}\cdot d_{\{1,1,1\}}(t,u)\cdot\left(\frac{t^2-tu}{t+u}\right)\cdot\frac{1}{t+u}\ . \tag{5.9b}$$

(iv) $(\pi\pi)_0$('derivative'):

$$\Omega_1^{(pv)}(\mathbf{k}^2;t,u) = -6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\cdot d_{1,1,1}(t,u)\cdot\Bigg\{\left(m_1^2-m_2^2\right)^2$$
$$-\left[3(m_1^2+m_2^2)+\frac{m_1^2(3t^2-u^2)+m_2^2(3u^2-t^2)}{t+u}\mathbf{k}^2\right]\frac{1}{t+u}$$
$$-\left[\left(\frac{t^2+2tu+u^2}{t+u}\right)\mathbf{k}^2+2tu\left(\frac{t^2+2tu+u^2}{(t+u)^2}\right)\mathbf{k}^4\right]\frac{1}{(t+u)^2}\Bigg\}\ , \tag{5.10a}$$

$$\Omega_4^{(pv)}(\mathbf{k}^2;t,u) = -24\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\cdot d_{1,1,1}(t,u)\cdot$$
$$\times\left\{\left(m_1^2+m_2^2\right)+\left[\frac{3}{2}+\left(\frac{t^2+tu+u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}\right\}\ . \tag{5.10b}$$

The coefficients $\Upsilon^{na,pv}_{j,k}$ defined in (4.2) are tabulated in Tables VIII-XII.

For (πP)-exchange, the $1/M_N$ non-adiabatic and pseudo-vector vertex corrections can be read off from those for $(\pi\sigma)$ and making the same adjustments as given already for the adiabatic contributions. In Table XIII the $\Omega^{(pv,ad)}_i$ for (πP)-pair exchange are given explicitly.

VI. PARTIAL WAVE ANALYSIS

Like the TME-potentials in I, the general form of the MPE-potentials in momentum space is

$$\widetilde{V}^{(n)}_j(\mathbf{k}) \;=\; \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du \; \left\{ w^{(n)}_{\{p_1,p_2,p_3\}}(t,u)\; \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\; \Omega^{(n)}_j(\mathbf{k}^2;t,u)\right\} \;, \tag{6.1}$$

where

$$w^{(n)}_{\{p_1,p_2,p_3\}}(t,u) = w_0(t,u)\; d^{(n)}_{\{p_1,p_2,p_3\}}(t,u) \;.$$

Therefore, the partial wave analysis runs along the same lines as described in sections VI and VII of paper I for the TME-potentials. We refer the reader to this paper for the details.

VII. ESC-MODEL, RESULTS

The momentum space formulas for the potentials of this paper and paper I have been checked numerically. This is done by solving the Lippmann-Schwinger equation and comparing the phase shifts with those obtained by solving the Schrödinger equation using the x-space equivalent of the potentials.

After the completion of the p-space formalism we performed a χ^2-fit with the ESC-model to the 1993 Nijmegen representation of the χ^2-hypersurface of the NN scattering data below $T_{lab} = 350$ MeV [15].

This fitting was executed in x-space using the equivalent x-space potentials. The reason for this is the much faster evaluation of the ESC-model in x-space. We obtained a $\chi^2/Ndata = 1.15$. The phase shifts are shown in Fig.s 2-5. In Table III the results are shown for the ten energy bins, where we compare the results from the updated partial-wave analysis with the ESC potentials.

In Table I we show the OBE-coupling constants and the gaussian cut-off's Λ. The used $\alpha =: F/(F+D)$-ratio's for the OBE-couplings are: pseudo-scalar mesons $\alpha_{pv} = 0.355$, vectormesons $\alpha^e_V = 1.0, \alpha^m_V = 0.275$, and scalar-mesons $\alpha_S = 0.914$, which is computed using the physical $S^* =: f_0(993)$ coupling etc.. In Table II we show the MPE-coupling constants. The used $\alpha =: F/(F+D)$-ratio's for the MPE-couplings are: $(\pi\eta)$ etc. and $(\pi\omega)$ pairs $\alpha(\{8_s\}) = 1.0$, $(\pi\pi)_1$ etc. pairs $\alpha^e_V(\{8\}_a) = 0.4, \alpha^m_V(\{8\}_a) = 0.335$, $(\pi\rho)_1$ etc. pairs $\alpha_A(\{8\}_a) = 0.335$.

We emphasize that we use the single-energy (s.e.) phases and χ^2-surface [17] only as a means to fit the NN-data. As stressed in [15] the Nijmegen s.e. phases have not much significance. The significant phases are the multi-energy (m.e.) ones, see the dashed lines in the figures. One notices that the central value of the s.e. phases do not correspond to the m.e. phases in general, illustrating that there has been a certain amount of noice fitting in the s.e. PW-analysis, see e.g. ϵ_1 and $^1P_1$ at $T_{lab} = 100$ MeV. The m.e. PW-analysis reaches $\chi^2/N_{data} = 0.99$, using 39 phenomenological parameters plus normalization parameters. The related phenomenological PW-potentials NijmI,II and Reid93 [18], with respectively 41, 47, and 50 parameters, all with $\chi^2/Ndata = 1.03$. This should be compared to the ESC-model, which has $\chi^2/N_{data} = 1.15$ using 20 parameters. These are 9 meson-nucleon-nucleon couplings, 8 meson-pair-nucleon-nucleon couplings, and 3 gaussian cut-off parameters. From the figures it is obvious that the ESC-model deviates from the m.e. PW-analysis at the highest energy in particular. If we evalute the χ^2 for the first 9

TABLE I: Meson parameters of the fitted ESC-model. Phases are shown in Figs. 2 to 5. Coupling constants are at $\mathbf{k}^2 = 0$. An asterisk denotes that the coupling constant is not searched, but constrained via $SU(3)$ are simply put to some value used in previous work.

meson	mass (MeV)	$g/\sqrt{4\pi}$	$f/\sqrt{4\pi}$	Λ (MeV)
π	138.04		0.2663	950.69
η	547.45		0.1461*	,,
η'	957.75		0.1789*	,,
ρ	768.10	0.2700	3.6378	688.20
ϕ	1019.41	–1.4717*	0.0149*	,,
ω	781.95	2.6862	0.3255	,,
a_0	982.70	0.9851		734.25
f_0	974.10	–0.7998		,,
ε	760.00	3.7554		,,
A_2	309.10	–0.4317		
Pomeron	309.10	2.5514		

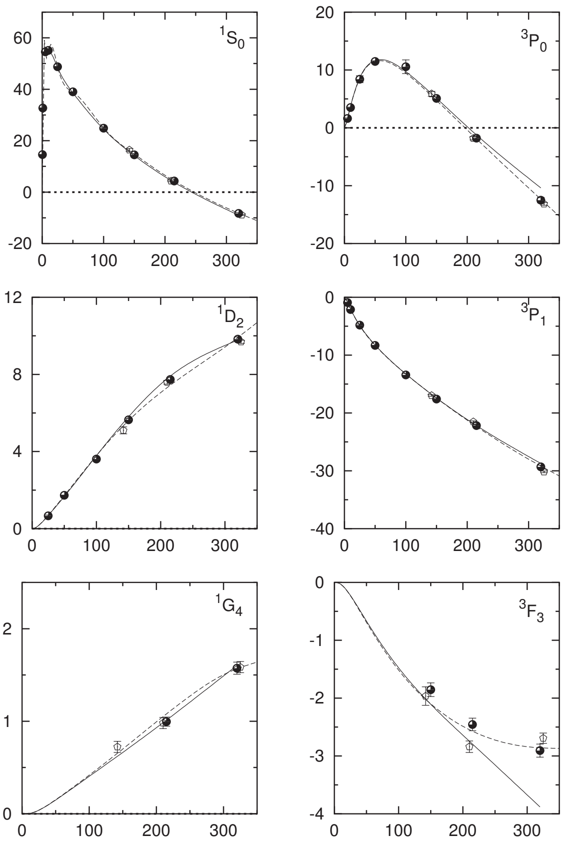

FIG. 2: Solid line: proton-proton $I = 1$ phase shifts for the ESC-model. The dashed line: the m.e. phases of the Nijmegen93 PW-analysis [15]. The black dots: the s.e. phases of the Nijmegen93 PW-analysis. The diamonds: Bugg s.e. [16].

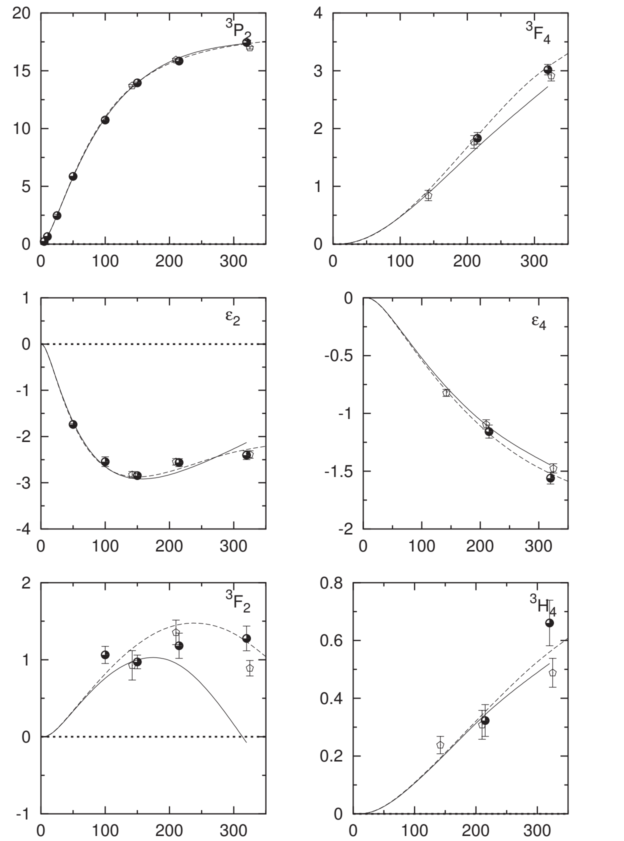

FIG. 3: Solid line: proton-proton $I = 1$ phase shifts for the ESC-model. The dashed line: the m.e. phases of the Nijmegen93 PW-analysis [15]. The black dots: the s.e. phases of the Nijmegen93 PW-analysis. The diamonds: Bugg s.e. [16].

TABLE II: Pair-meson coupling constants employed in the MPE-potentials. Coupling constants are at $\mathbf{k}^2 = 0$.

J^{PC}	$SU(3)$-irrep	$(\alpha\beta)$	$g/4\pi$	$f/4\pi$
0^{++}	$\{1\}$	$(\pi\pi)_0$	0.1567	
0^{++}	,,	$(\sigma\sigma)$	—	
0^{++}	$\{8\}_s$	$(\pi\eta)$	–0.2946	
0^{++}		$(\pi\eta')$	—	
1^{--}	$\{8\}_a$	$(\pi\pi)_1$	0.1093	–0.2050
1^{++}	,,	$(\pi\rho)_1$	0.6950	
1^{++}	,,	$(\pi\sigma)$	0.0140	
1^{++}	,,	(πP)	–0.1604	
1^{+-}	$\{8\}_s$	$(\pi\omega)$	–0.1081	

energies only, we obtain $\chi^2/N_{data} = 1.10$.

We mentioned that we do not include negative energy state contributions. It is assumed that a strong pair suppression is operative at low energies in view of the composite nature of the nucleons. This leaves us for the pseudo-scalar mesons with two essential equivalent interactions: the direct and the derivative one. In expanding the $NN\pi$- etc. vertex in $1/M_N$ these two interactions differ in the $1/M_N^2$-terms, see [3] equations (3.4) and (3.5). Here, we prefer to cancel these $1/M_N^2$ terms by taking

$$\mathcal{H}_{ps} = \frac{1}{2}\left[g_{NN\pi}\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\cdot\boldsymbol{\pi} + (f_{NN\pi}/m_\pi)\gamma_\mu\gamma_5\boldsymbol{\tau}\psi\cdot\partial^\mu\boldsymbol{\pi}\right] , \tag{7.1}$$

where $g_{NN\pi} = (2M_N/m_\pi)f_{NN\pi}$.

As for the OBE-couplings, one notices that $G_E = g_{\rho NN}$ is small, but $G_M = g_\rho + f_\rho$ is okay. One possible explanation would be that part of the ρ-exchange is replaced by the 2-pair $(\pi\pi)_1$-exchange, which has identical quantum numbers. This still leaves room for the interpretation of the 1-pair $(\pi\pi)_1$-exchange as a form factor correction. Another interesting possibility is that leaving out the tensor mesons $a_2(1320)$, $f_2(1270)$, $f_2(1520)$ affects the vector meson couplings. This can be seen as follows. At high energies and low to moderate momentum transfer there is a strong cancellation between the vector and tensor exchange: $(\rho - a_2)$- and $(\omega - f_2)$-cancellation [19]. This is called Exchange-Degeneracy (EXD). Indeed, by changing $g_\rho/\sqrt{4\pi} = 0.3$ to $g_\rho = 0.75/\sqrt{4\pi}$ one can cancel

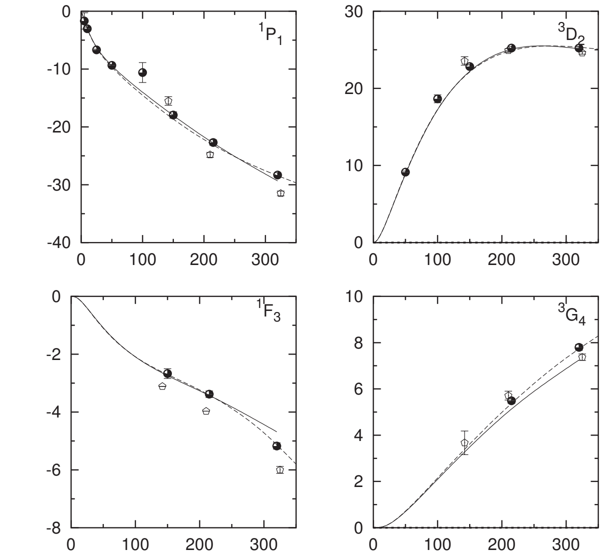

FIG. 4: Solid line: neutron-proton $I = 0$ phase shifts for the ESC-model. The dashed line: the m.e. phases of the Nijmegen93 PW-analysis [15]. The black dots: the s.e. phases of the Nijmegen93 PW-analysis. The diamonds: Bugg s.e. [16].

TABLE III: χ^2 and χ^2 per datum at the ten energy bins for the Nijmegen93 Partial-Wave-Analysis. N_{data} lists the number of data within each energy bin. The bottom line gives the results for the total $0-350$ MeV interval. The χ^2-access for the ESC model is denoted by $\Delta\chi^2$ and $\Delta\hat{\chi}^2$, respectively.

$T_{\rm lab}$	♯ data	χ^2_0	$\Delta\chi^2$	$\hat{\chi}^2_0$	$\Delta\hat{\chi}^2$
0.383	144	137.5549	21.3	0.960	0.148
1	68	38.0187	55.7	0.560	0.819
5	103	82.2257	13.0	0.800	0.127
10	209	257.9946	78.1	1.234	0.269
25	352	272.1971	44.3	0.773	0.126
50	572	547.6727	137.4	0.957	0.240
100	399	382.4493	27.6	0.959	0.069
150	676	673.0548	82.9	0.996	0.123
215	756	754.5248	108.0	0.998	0.143
320	954	945.3772	305.0	0.991	0.320
Total	4233	4091.122	864.2	0.948	0.201

the change in the ρ-exchange potential by the inclusion of a_2-exchange rather completely. The inclusion of mesons with a mass ≥ 1 GeV/c^2, like the axial and tensor mesons we leave as a future project.

Unlike in [3, 4], we did not fix pair couplings using a theoretical model, based on heavy-meson saturation and

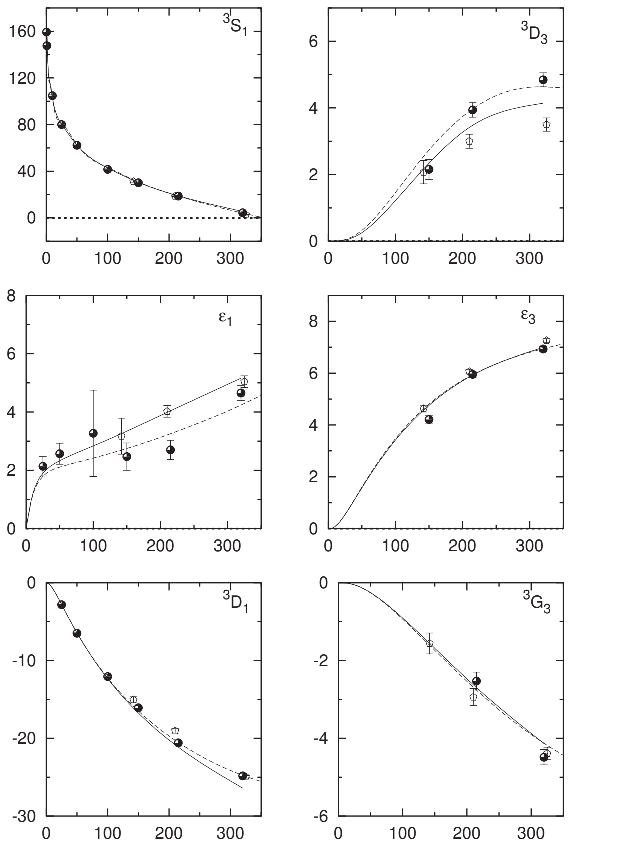

FIG. 5: Solid line: neutron-proton $I = 0$ phase shifts for the ESC-model. The dashed line: the m.e. phases of the Nijmegen93 PW-analysis [15]. The black dots: the s.e. phases of the Nijmegen93 PW-analysis. The diamonds: Bugg s.e. [16].

chiral-symmetry. So, in addition to the 14 parameters used in [3, 4] we now have 6 pair-coupling fit parameters. In Table II the fitted pair-couplings are given. Note that the $(\pi\pi)_0$-pair coupling gets contributions from the $\{1\}$ and the $\{8_s\}$ pairs as well, giving in total $g_{(\pi\pi)} = 0.10$, which has the same sign as in [4]. The $f_{(\pi\pi)_1}$-pair coupling has opposite sign as compared to [4]. In a model with a more complex and realistic meson-dynamics [9] this coupling is predicted as found in the present ESC-fit. The $(\pi\rho)_1$-coupling agrees nicely with A_1-saturation, see [4]. We conclude that the pair-couplings are in general not well understood, and deserve more study.

The ESC-model described here is fully consistent with $SU(3)$-symmetry. In Appendix E we display the full $SU(3)$ contents of the pair interaction Hamiltonians. For example $g_{(\pi\rho)_1} = g_{A_8VP}$, and besides $(\pi\rho)$-pairs one sees also that $(KK^*(I=1)$- and $KK^*(I=0)$-pairs contribute to the NN potentials. All $F/(F+D)$-ratio's are taken fixed with heavy-meson saturation in mind. The approximation we have made in this paper is to neglect the baryon mass differences, i.e. we put $m_\Lambda = m_\Sigma = m_N$. This because we have not yet worked out the formulas for the inclusion of these mass differences, which is straightforward in principle.

VIII. CONCLUSIONS AND OUTLOOK

The presented ESC-model is very succesfull and flexible in describing the NN-data. It can be developed and extended in various ways. First, we plan to extend the OBE-potentials in momentum space by including the full OBE-propagator, i.e.

$$\frac{1}{\omega^2} \rightarrow \frac{1}{\omega(\omega+a)}\ ,\ a = \frac{1}{M}\left[p_f^2 + p_i^2 - 2p_0^2\right]\ . \tag{8.1}$$

This includes retardation at the level of the OBE-potentials. Secondly, one may extend the TME-potentials including besides ps-ps also the ps-vector, ps-scalar, etc. potentials. Thirdly, the inclusion of the axial- and tensor-mesons, which we discussed in connection with EXD.

The momentum space formulation of the ESC-model also suggests a covariant formulation. Consider an Effective Field Theory and suppose that it allows the Wick-rotation. Then, assuming in Euclidean space a Gaussian cut-off, one can use a representation completely akin to 2.3 etc. For example, this opens the way to analyse the expansion in loops in the presence of a strong cut-off. Also, one could evaluate the ESC-model using the Bethe-Salpetyer equation.

The presented ESC-model can be applied in various ways: (i) The study of Few-body systems in momentum space, (ii) The study of Meson-Exchange-Current (MEC) corrections, (iii) The derivation of 3-body forces consistent with the 2-body forces. (iv) G-matrix etc. description of Nuclear Matter.

APPENDIX A: PAIR INTERACTION HAMILTONIANS

The pair hamiltonians are

$$J^{PC} = 0^{++}\ :\ \mathcal{H}_S = (\bar{\psi}'\psi')\left\{g_{\pi\pi)_0}(\boldsymbol{\pi}\cdot\boldsymbol{\pi}) + g_{\sigma\sigma}\sigma^2\right\}/m_\pi + g'_{(\pi\pi)_0}(\bar{\psi}'\psi)(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})/m_\pi^3\ , \tag{A1a}$$

$$J^{PC} = 1^{--}\ :\ \mathcal{H}_V = \left[g_{(\pi\pi)_1}\bar{\psi}'\gamma_\mu\boldsymbol{\tau}\psi' - \frac{f_{(\pi\pi)_1}}{2M}\bar{\psi}'\sigma_{\mu\nu}\boldsymbol{\tau}\psi'\partial^\nu\right](\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})/m_\pi^2\ , \tag{A1b}$$

$$J^{PC} = 1^{++}\ :\ \mathcal{H}_A = g_{(\pi\rho)_1}\bar{\psi}'\gamma_\mu\gamma_5\boldsymbol{\tau}\psi'(\boldsymbol{\pi}\times\boldsymbol{\rho}^\mu)/m_\pi + g_{(\pi\sigma)}\bar{\psi}'\gamma_\mu\gamma_5\boldsymbol{\tau}\psi'\left(\sigma\partial^\mu\boldsymbol{\pi} - \boldsymbol{\pi}\partial^\mu\sigma\right)/m_\pi^2\ , \tag{A1c}$$

$$J^{PC} = 1^{+-}\ :\ \mathcal{H}_B = g_{(\pi\rho)_0}\bar{\psi}'\sigma_{\mu\nu}\gamma_5\psi'\partial^\nu(\boldsymbol{\pi}\cdot\boldsymbol{\rho})/m_\pi^2 + g_{\pi\omega}\bar{\psi}'\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\psi'\partial^\nu(\boldsymbol{\pi}\cdot\omega^\mu)/m_\pi^2\ . \tag{A1d}$$

APPENDIX B: λ-REPRESENTATIONS

The following λ-representations [11] are exploited:

$$D_{1,0,0}(\omega_1,\omega_2) = \frac{1}{\omega_1} = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\omega_1^2+\lambda^2}\ , \tag{B1a}$$

$$D_{0,1,0}(\omega_1,\omega_2) = \frac{1}{\omega_1} = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\omega_2^2+\lambda^2}\ , \tag{B1b}$$

$$D_{0,0,1}(\omega_1,\omega_2) = \frac{1}{\omega_1+\omega_2} = \frac{2}{\pi}\int_0^\infty \frac{\lambda^2 d\lambda}{(\omega_1^2+\lambda^2)(\omega_2^2+\lambda^2)}\ , \tag{B1c}$$

$$D_{1,1,1}(\omega_1,\omega_2) = \frac{1}{\omega_1\omega_2(\omega_1+\omega_2)} = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{(\omega_1^2+\lambda^2)(\omega_2^2+\lambda^2)}\ , \tag{B1d}$$

A special combination occurs in non-adiabatic terms. Here, see Table V, occurs

$$\begin{aligned} D_{na}^{(1)}(\omega_1,\omega_2) &= \frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right] = \\ &= \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\lambda^2}\left[\frac{1}{\omega_1^2\omega_2^2}-\frac{1}{(\omega_1^2+\lambda^2)(\omega_2^2+\lambda^2)}\right] \ . \end{aligned} \tag{B2}$$

Notice that the denominator $D_{na}^{(1)} = 2D_{//}$, see [3]. The corresponding $d_{na}(t,u)$ is, see paper I, section IV.A,

$$d_{na}(t,u) = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\lambda^2}\left[1-e^{-(t+u)\lambda^2}\right] = \frac{2}{\sqrt{\pi}}\sqrt{t+u}\ . \tag{B3}$$

APPENDIX C: INTEGRATION DICTIONARY

In this appendix we give a dictionary for the evaluation of the momentum integrals that occur in the matrix elements of the TME-potentials. The results of the $d^3\Delta$-integration are given apart from a factor $(4\pi a)^{-3/2}$ $(a = t+u)$, common to all integrals. Using the results given in Appendix B of paper I, one obtains:

(i) For the operators $\widetilde{O}^{(1)}_{\alpha\beta,p}$, and the operators $\widetilde{O}^{(2)}_{\alpha\beta,p}$:

$$a.\ (\mathbf{k}_1\cdot\mathbf{k}_2) = \mathbf{\Delta}\cdot\mathbf{k}-\mathbf{\Delta}^2 \Rightarrow \frac{1}{2}\left\{-3+2\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ , \tag{C1a}$$

$$\begin{aligned} &b.\ [\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\times\mathbf{k}_2]\,[\boldsymbol{\sigma}_2\cdot\mathbf{k}_1\times\mathbf{k}_2] = [\boldsymbol{\sigma}_1\cdot\mathbf{\Delta}\times\mathbf{k}]\,[\boldsymbol{\sigma}_2\cdot\mathbf{\Delta}\times\mathbf{k}] \\ &\Rightarrow \frac{1}{2}\left\{\frac{2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ \mathbf{k}^2\ -\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k})\ (\boldsymbol{\sigma}_2\cdot\mathbf{k})-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ \mathbf{k}^2\right]\right\}\cdot\frac{1}{t+u}\ , \end{aligned} \tag{C1b}$$

$$\begin{aligned} &c.\ [(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}_1\times\mathbf{k}_2]\ \mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2) = [(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{\Delta}\times\mathbf{k}]\ \mathbf{q}\cdot(\mathbf{2\Delta}-\mathbf{k}) \\ &\Rightarrow [(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}]\cdot\frac{1}{t+u}\ , \end{aligned} \tag{C1c}$$

$$\begin{aligned} &d.\ (\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\ \boldsymbol{\sigma}_2\cdot\mathbf{k}_2)+(\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\ \boldsymbol{\sigma}_2\cdot\mathbf{k}_1) \\ &\Rightarrow -\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-\frac{2tu}{t+u}(\boldsymbol{\sigma}_1\cdot\mathbf{k})\ (\boldsymbol{\sigma}_2\cdot\mathbf{k})\right\}\cdot\frac{1}{t+u}\ . \end{aligned} \tag{C1d}$$

$$\begin{aligned} &e.\ (\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_1) = (\boldsymbol{\sigma}_1\cdot\mathbf{\Delta})(\boldsymbol{\sigma}_2\cdot\mathbf{\Delta}) \\ &\Rightarrow \frac{1}{2}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{2u^2}{t+u}(\boldsymbol{\sigma}_1\cdot\mathbf{k})\ (\boldsymbol{\sigma}_2\cdot\mathbf{k})\right\}\cdot\frac{1}{t+u}\ , \end{aligned} \tag{C1e}$$

$$\begin{aligned} &f.\ \boldsymbol{\sigma}_1\cdot(\mathbf{k}_1-\mathbf{k}_2)\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_2) = \boldsymbol{\sigma}_1\cdot(2\mathbf{\Delta}-\mathbf{k})\ \boldsymbol{\sigma}_2\cdot(2\mathbf{\Delta}-\mathbf{k}) \\ &\Rightarrow \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\left\{\frac{2}{t+u}+\frac{1}{3}\left(\frac{t-u}{t+u}\right)^2\mathbf{k}^2\right\}+\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k}-\frac{1}{3}\mathbf{k}^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\cdot\left(\frac{t-u}{t+u}\right)^2\ , \end{aligned} \tag{C1f}$$

(ii) For the $1/M$-correction operators $\widetilde{O}^{(na)}_{\alpha\beta}$ etc., not included in the list (C1a)-(C1e):

$$a.\ (\mathbf{k}_1\cdot\mathbf{k}_2)^2 = \left(\boldsymbol{\Delta}\cdot\mathbf{k}-\boldsymbol{\Delta}^2\right)^2$$
$$\Rightarrow \frac{1}{4}\left\{15+2\left(\frac{t^2-8ut+u^2}{t+u}\right)\mathbf{k}^2+4\left(\frac{t^2u^2}{(t+u)^2}\right)\mathbf{k}^4\right\}\cdot\frac{1}{(t+u)^2}\ , \tag{C2a}$$

$$b.\ (\boldsymbol{\sigma}_1\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2) = \boldsymbol{\sigma}_1\cdot(\mathbf{k}-\boldsymbol{\Delta})\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}-\boldsymbol{\Delta})$$
$$\Rightarrow \frac{1}{2}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{2t^2}{t+u}(\boldsymbol{\sigma}_1\cdot\mathbf{k})\ (\boldsymbol{\sigma}_2\cdot\mathbf{k})\right\}\cdot\frac{1}{t+u}\ , \tag{C2b}$$

$$c.\ \mathbf{k}_1^2 = \boldsymbol{\Delta}^2 \Rightarrow \left\{\frac{3}{2}+\frac{u^2}{t+u}\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ , \tag{C2c}$$

$$d.\ \mathbf{k}_2^2 = (\mathbf{k}-\boldsymbol{\Delta})^2 \Rightarrow \left\{\frac{3}{2}+\frac{t^2}{t+u}\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ , \tag{C2d}$$

$$e.\ \mathbf{k}_1^2\ \mathbf{k}_2^2 = \mathbf{k}^2\boldsymbol{\Delta}^2-2\mathbf{k}\cdot\boldsymbol{\Delta}\ \boldsymbol{\Delta}^2+\boldsymbol{\Delta}^4$$
$$\Rightarrow \left[\frac{15}{4}+\frac{1}{2}\frac{(3t^2-4tu+3u^2)}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right\}\cdot\frac{1}{(t+u)^2}\ , \tag{C2e}$$

$$f.\ (\mathbf{k}_1\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_1) = \boldsymbol{\Delta}\cdot(\mathbf{k}-\boldsymbol{\Delta})[\boldsymbol{\sigma}_1\cdot\boldsymbol{\Delta}\ \boldsymbol{\sigma}_2\cdot\boldsymbol{\Delta}]$$
$$\Rightarrow \left\{-\frac{5}{4}+\frac{1}{2}\frac{tu}{t+u}\mathbf{k}^2\right\}\cdot\frac{1}{(t+u)^2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$
$$+(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k})\left\{\frac{1}{2}\frac{(2t-5u)u}{t+u}+\frac{tu^3}{(t+u)^2}\mathbf{k}^2\right\}\cdot\frac{1}{(t+u)^2}\ , \tag{C2f}$$

$$g.\ (\mathbf{k}_1\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2) = \boldsymbol{\Delta}\cdot(\mathbf{k}-\boldsymbol{\Delta})[\boldsymbol{\sigma}_1\cdot\boldsymbol{\Delta}\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}-\boldsymbol{\Delta})]$$
$$\Rightarrow \left\{\frac{5}{4}-\frac{1}{2}\frac{tu}{t+u}\mathbf{k}^2\right\}\cdot\frac{1}{(t+u)^2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$
$$+(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k})\left\{\frac{1}{2}-\frac{7}{2}\frac{tu}{(t+u)^2}+\frac{t^2u^2}{(t+u)^3}\mathbf{k}^2\right\}\cdot\frac{1}{t+u}\ . \tag{C2g}$$

APPENDIX D: DERIVATIVE SCALAR-PAIR POTENTIALS

As pointed out by Ko and Rudaz [13] besides the most simple lagrangian for σ-decay $\mathcal{L}^{(0)}_{\sigma\pi\pi} = g_{\sigma\pi\pi}\sigma\boldsymbol{\pi}\cdot\boldsymbol{\pi}$ also the lagrangian σ-decay $\mathcal{L}^{(1)}_{\sigma\pi\pi} = g'_{\sigma\pi\pi}\sigma\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}$ appears in the linear σ-model. The latter is useful in keeping the scalar meson width's within reasonable bounds as the scalar mass increases. Also, derivative couplings to baryons were considered in the context of an $SU_f(3)$ generalization in [9]. In the $(NN2\pi)$ effective field-theory lagrangian [14] the NN-interaction lagrangian, i.e. the NLO-terms, for the pion-pairs reads

$$\mathcal{L}^{(1)} = -\bar{\psi}\left[8c_1D^{-1}m_\pi^2\frac{\boldsymbol{\pi}}{F_\pi^2}-4c_3\mathbf{D}_\mu\cdot\mathbf{D}^\mu+2c_4\sigma_{\mu\nu}\boldsymbol{\tau}\cdot\mathbf{D}^\mu\times\mathbf{D}^\nu\right]\psi\ , \tag{D1}$$

where $D = 1+\boldsymbol{\pi}^2/f_\pi^2$ and $\mathbf{D}_\mu = D^{-1}\partial_\mu\boldsymbol{\pi}/F_\pi$, with $F_\pi = 2f_\pi = 185$ MeV. The correspondence with the pair terms treated in this paper is that $c_1 \sim g_{(\pi\pi)_0}$, and $c_3 \sim g'_{(\pi\pi)_0}$. The c_4-term has been considered in [9], but not in this paper. The 'derivative' hamiltonian to lowest order in the $\boldsymbol{\pi}$ reads

$$\mathcal{H}_{S'} = g'_{(\pi\pi)_0}(\bar{\psi}'\psi)(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})/m_\pi^3\ . \tag{D2}$$

1. Adiabatic potentials

For the 1-pair graph's in equation (3.1)

$$\tilde{O}^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2) \Rightarrow \tilde{O}^{(S')}_{\alpha\beta,p}\tilde{O}^{(2PS)}_{\alpha\beta} \quad , \quad \tilde{O}^{(2PS)}_{\alpha\beta} = -\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 (\mathbf{k}_1\cdot\mathbf{k}_2 - i\boldsymbol{\sigma}\cdot\mathbf{k}_1\times\mathbf{k}_2)$$
$$\tilde{O}^{(S')}_{\alpha\beta,p} = 2\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\pm\omega_1\omega_2 + \mathbf{k}_1\cdot\mathbf{k}_2\right) \ . \tag{D3}$$

Here, for $p=a,c$ the $(-)$-sign and for $p=b$ the $(+)$-sign applies. Obviously, $\alpha=\beta=\pi$ in (D3). All other quantities in (3.1) are the same as for pion-pair without derivatives. Here, and in the rest of this appendix, we absorb the $g^{(n)}(\alpha,\beta)$-factor in (3.1) into the definition of the O-operators.
Evaluation of the p-sum and including the mirror graph's, one gets collecting all terms and selecting the contributions symmetric in $1\leftrightarrow 2$ the matrix element

$$\sum_p \tilde{O}^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D^{(1)}_p(\omega_1,\omega_2) = -2\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \cdot$$
$$\times(\mathbf{k}_1\cdot\mathbf{k}_2)\left\{\mathbf{k}_1\cdot\mathbf{k}_2 - i\boldsymbol{\sigma}\cdot\mathbf{k}_1\times\mathbf{k}_2\right\}\frac{1}{\omega_1^2\omega_2^2} \ . \tag{D4}$$

For the 2-pair graph's in equation (3.1) one has

$$\tilde{O}^{(2)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D^{(2)}(\omega_1,\omega_2) = -\frac{1}{2\omega_1\omega_2(\omega_1+\omega_2)}\left(-\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2\right)^2 \ . \tag{D5}$$

Using the expressions in this appendix we obtain in p-space the adiabatic 'derivative' $(\pi\pi)_0$-exchange potentials the 1-pair exchange and 2-pair graphs give

$$\Omega^{(1),ad}_1(\mathbf{k}^2;t,u) = -12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 \cdot d_{2,2,0}(t,u)\cdot$$
$$\times\left[\frac{15}{4}+\frac{1}{2}\frac{t^2-8tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\cdot\frac{1}{(t+u)^2} \ , \tag{D6a}$$

$$\Omega^{(2),ad}_1(\mathbf{k}^2;t,u) = -6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)^2\cdot\left\{\left[\frac{15}{4}+\frac{t^2-3tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{d_{1,1,1}(t,u)}{(t+u)^2}\right.$$
$$+\frac{1}{2}\left[\frac{3}{2}(m_1^2+m_2^2)+\frac{m_1^2t+m_2^2u}{t+u}\mathbf{k}^2+m_1^2m_2^2(t+u)\right]\frac{d_{1,1,1}(t,u)}{t+u}$$
$$\left.+\left[\frac{3}{2}-\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{0,0,1}(t,u)}{t+u}\right\} \ . \tag{D6b}$$

2. $1/M$ corrections

The nonadiabatic from the $1/M$-expansion of the energy denominators and the pseudo-vector vertex $1/M$-corrections are described in Ref. [11] and used also in Ref. [4], section IV. Below, we give the results for the evaluation of these $1/m$-corrections for the 1-pair graph's with the 'derivative' pair-interaction.

a. Non-adiabatic contributions: For the 1-pair graph's in equation (3.1) the non-adiabatic operator is

$$\tilde{O}^{(1),na}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2) \Rightarrow -2\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\cdot\frac{1}{2M}\cdot\left[(\mathbf{k}_1\cdot\mathbf{k}_2)^2+\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}_1\times\mathbf{k}_2\ \mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)\right]\cdot$$
$$\times\widetilde{\Gamma}^{(S')}_{\pi\pi,p} \ , \tag{D7}$$

where $\widetilde{\Gamma}^{(S')}_{\pi\pi,p} = \pm\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2$ and the $\pm$-sign has been explained above. The denominators $D^{(na)}_p(\omega_1,\omega_2)$ have been given in [4]. Again, we select the terms symmetric in $1\leftrightarrow 2$ since the asymmetric terms will not contribute, which is easily seen in x-space. The sum over the graph's $p=a,b,c$ yields

$$\sum_p \widetilde{\Gamma}^{(S')}_{\pi\pi,p}\ D^{na}_p(\omega_1,\omega_2) \;=\; \frac{1}{\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}+\frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right]\ (\mathbf{k}_1\cdot\mathbf{k}_2)\,. \tag{D8}$$

Using the expressions in this appendix we obtain in p-space the non-adiabatic 'derivative' $(\pi\pi)_0$-exchange potentials

$$\begin{aligned}
\Omega^{(na)}_1(\mathbf{k}^2;t,u) &= -12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\cdot\Bigg\{\\
&+\left[\frac{15}{4}+\frac{1}{2}\left(\frac{t^2-8tu+u^2}{t+u}\right)\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{d_{1,1,1}(t,u)}{(t+u)^2}\ ,\\
&-\left[\frac{105}{8}+\frac{15}{4}\left(\frac{t^2-5tu+u^2}{t+u}\right)\mathbf{k}^2-\frac{3}{2}tu\left(\frac{t^2-5tu+u^2}{(t+u)^2}\right)\mathbf{k}^4-\frac{t^3u^3}{(t+u)^3}\mathbf{k}^6\right]\frac{d_{na}(t,u)}{(t+u)^3}\Bigg\}\ ,
\end{aligned} \tag{D9a}$$

$$\Omega^{(na)}_4(\mathbf{k}^2;t,u) = -12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\cdot\left\{\frac{d_{1,1,1}(t,u)}{t+u}+\left[-5+2\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{na}(t,u)}{(t+u)^2}\right\}\ . \tag{D9b}$$

Here, $d_{\{na\}}(t,u)$ is defined in (B3).

b. Pseudo-vector contributions: The pseudovector vertex gives $1/M$-terms as can be seen from

$$\bar{u}(\mathbf{p}')\Gamma^{(1)}_P u(\mathbf{p}) = -i\frac{f_{NN\pi}}{m_\pi}\left[\boldsymbol{\sigma}\cdot(\mathbf{p}'-\mathbf{p})\pm\frac{\omega}{2M}\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})\right]\ , \tag{D10}$$

where upper (lower) sign applies for creation (absorption) of the pion at the vertex. For graph (a) the operator for the nucleon line on the right is readily seen to be

$$-\left(\frac{f_P}{m_\pi}\right)^2\frac{1}{2M}\left[\left(\omega_1\mathbf{k}_2^2-\omega_2\mathbf{k}_1^2\right)-2\mathbf{q}\cdot(\omega_1\mathbf{k}_2+\omega_2\mathbf{k}_1)+2i\boldsymbol{\sigma}_2\cdot\mathbf{q}\times(\omega_1\mathbf{k}_2-\omega_2\mathbf{k}_1)\right] \tag{D11}$$

The same expression for gragh (b) is obviously obtained from (D11) by making the the substitution $\omega_1\to-\omega_1$, and for graph (c) the substitution $\omega_{1,2}\to-\omega_{1,2}$. The mirror graphs are included by making the replacement $\boldsymbol{\sigma}_2\to(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)/2$. Combining all this with the adiabatic denominators $D^1_i(\omega_1,\omega_2)$

$$D^1_a(\omega_1,\omega_2) = \frac{1}{2\omega_1\omega_2^2(\omega_1+\omega_2)}\quad,\quad D^1_b(\omega_1,\omega_2) = \frac{1}{2\omega_1^2\omega_2^2}\ , \tag{D12}$$

and $D^1_c(\omega_1,\omega_2) = D^1_a(\omega_2,\omega_1)$. Summing over the 1-pair graphs gives

$$\begin{aligned}
\sum_p \tilde{O}^{(1),na}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D^1_p(\omega_1,\omega_2) &= -\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\cdot\frac{1}{M}\cdot\frac{1}{\omega_1\omega_2\,(\omega_1+\omega_2)}\cdot\\
\times\Bigg[\left\{\frac{1}{2}(m_1^2+m_2^2)(\omega_1^2+\omega_2^2)-2\omega_1^2\omega_2^2\right\}&-(\mathbf{k}_1^2+\mathbf{k}_2^2)(\mathbf{k}_1\cdot\mathbf{k}_2)\\
-\,i\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}\left(\omega_1^2+\omega_2^2+\mathbf{k}_1\cdot\mathbf{k}_2\right)\Bigg]&\ .
\end{aligned} \tag{D13}$$

Using the expressions in this appendix we obtain in p-space the pseudo-vector vertex $1/M$-corrections to the 'derivative' $(\pi\pi)_0$-exchange potentials

$$\begin{aligned}
\Omega^{(1),pv}_1(\mathbf{k}^2;t,u) &= -6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\cdot d_{1,1,1}(t,u)\cdot\Bigg\{\left(m_1^2-m_2^2\right)^2\\
&-\left[3(m_1^2+m_2^2)+\frac{m_1^2(3t^2-u^2)+m_2^2(3u^2-t^2)}{t+u}\mathbf{k}^2\right]\frac{1}{t+u}\\
&-\left[\left(\frac{t^2+2tu+u^2}{t+u}\right)\mathbf{k}^2+2tu\left(\frac{t^2+2tu+u^2}{(t+u)^2}\right)\mathbf{k}^4\right]\frac{1}{(t+u)^2}\Bigg\}\ ,
\end{aligned} \tag{D14a}$$

$$\begin{aligned}
\Omega^{(1),pv}_4(\mathbf{k}^2;t,u) &= -24\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\cdot d_{1,1,1}(t,u)\cdot\\
&\times\left\{\left(m_1^2+m_2^2\right)+\left[\frac{3}{2}+\left(\frac{t^2+tu+u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}\right\}\ .
\end{aligned} \tag{D14b}$$

APPENDIX E: PAIR COUPLINGS AND $SU_f(3)$-SYMMETRY

Below, $\sigma, \mathbf{a}_0, \mathbf{A}_1, \ldots$ are short-hands for respectively the nucleon densities $\bar{\psi}\psi$, $\bar{\psi}\boldsymbol{\tau}\psi$, $\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi, \ldots$.

The $SU_f(3)$ octet and singlet mesons, denoted by the subscript 8 respectively 1, are in terms of the physical ones defined as follows:

(i) Pseudo-scalar-mesons:

$$\begin{aligned} \eta_1 &= \cos\theta_{pv}\eta' - \sin\theta_{pv}\eta \\ \eta_8 &= \sin\theta_{pv}\eta' + \cos\theta_{pv}\eta \end{aligned}$$

Here, η' and η are the physical pseudo-scalar mesons $\eta(957)$ respectively $\eta(548)$.

(ii) Vector-mesons:

$$\begin{aligned} \phi_1 &= \cos\theta_v\omega - \sin\theta_v\phi \\ \phi_8 &= \sin\theta_v\omega + \cos\theta_v\phi \end{aligned}$$

Here, ϕ and ω are the physical vector mesons $\phi(1019)$ respectively $\omega(783)$.

Then, one has the following $SU(3)$-invariant pair-interaction Hamiltonians:

1. $SU(3)$-singlet couplings $S^\alpha_\beta = \delta^\alpha_\beta\sigma/\sqrt{3}$:

$$\mathcal{H}_{S_1PP} = \frac{g_{S_1PP}}{\sqrt{3}}\left\{\boldsymbol{\pi}\cdot\boldsymbol{\pi} + 2K^\dagger K + \eta_8\eta_8\right\}\cdot\sigma$$

2. $SU(3)$-octet symmetric couplings I, $S^\alpha_\beta = (S_8)^\alpha_\beta \Rightarrow (1/4)Tr\{S[P,P]_+\}$:

$$\begin{aligned} \mathcal{H}_{S_8PP} = \frac{g_{S_8PP}}{\sqrt{6}}\Bigg\{ & (\mathbf{a}_0\cdot\boldsymbol{\pi})\eta_8 + \frac{\sqrt{3}}{2}\mathbf{a}_0\cdot(K^\dagger\boldsymbol{\tau}K) \\ & + \frac{\sqrt{3}}{2}\left\{(K_0^\dagger\boldsymbol{\tau}K)\cdot\boldsymbol{\pi} + h.c.\right\} \\ & - \frac{1}{2}\left\{(K_0^\dagger K)\eta_8 + h.c.\right\} \\ & + \frac{1}{2}f_0\left(\boldsymbol{\pi}\cdot\boldsymbol{\pi} - K^\dagger K - \eta_8\eta_8\right)\Bigg\} \end{aligned}$$

3. $SU(3)$-octet symmetric couplings II, $S^\alpha_\beta = (B_8)^\alpha_\beta \Rightarrow (1/4)Tr\{B^\mu[V_\mu,P]_+\}$:

$$\begin{aligned} \mathcal{H}_{B_8VP} = \frac{g_{B_8VP}}{\sqrt{6}}\Bigg\{ & \frac{1}{2}\left[\left(\mathbf{B}_1^\mu\cdot\boldsymbol{\rho}_\mu\right)\eta_8 + \left(\mathbf{B}_1^\mu\cdot\boldsymbol{\pi}_\mu\right)\phi_8\right] \\ & + \frac{\sqrt{3}}{4}\left[\mathbf{B}_1\cdot(K^{*\dagger}\boldsymbol{\tau}K) + h.c.\right] \\ & + \frac{\sqrt{3}}{4}\left[(K_1^\dagger\boldsymbol{\tau}K^*)\cdot\boldsymbol{\pi} + (K_1^\dagger\boldsymbol{\tau}K)\cdot\boldsymbol{\rho} + h.c.\right] \\ & - \frac{1}{4}\left[(K_1^\dagger\cdot K^*)\eta_8 + (K_1^\dagger\cdot K)\phi_8 + h.c.\right] \\ & + \frac{1}{2}H^0\left[\boldsymbol{\rho}\cdot\boldsymbol{\pi} - \frac{1}{2}\left(K^{*\dagger}\cdot K + K^\dagger\cdot K^*\right) - \phi_8\eta_8\right]\Bigg\} \end{aligned}$$

4. $SU(3)$-octet a-symmetric couplings I, $A^\alpha_\beta = (V_8)^\alpha_\beta \Rightarrow (-i/\sqrt{2})Tr\{V^\mu[P,\partial_\mu P]_-\}$:

$$\begin{aligned} \mathcal{H}_{V_8PP} = g_{A_8PP}\Bigg\{ & \frac{1}{2}\boldsymbol{\rho}_\mu\cdot\boldsymbol{\pi}\times\overleftrightarrow{\partial^\mu}\boldsymbol{\pi} + \frac{i}{2}\boldsymbol{\rho}_\mu\cdot(K^\dagger\boldsymbol{\tau}\overleftrightarrow{\partial^\mu}K) \\ & + \frac{i}{2}\left(K_\mu^{*\dagger}\boldsymbol{\tau}(K\overleftrightarrow{\partial^\mu}\boldsymbol{\pi}) - h.c.\right) + i\frac{\sqrt{3}}{2}\Big(K_\mu^{*\dagger}\cdot \\ & (K\cdot\overleftrightarrow{\partial^\mu}\eta_8) - h.c.\Big) + \frac{i}{2}\sqrt{3}\phi_\mu(K^\dagger\overleftrightarrow{\partial^\mu}K)\Bigg\} \end{aligned}$$

5. $SU(3)$-octet a-symmetric couplings II, $A^\alpha_\beta = (A_8)^\alpha_\beta \Rightarrow (-i/\sqrt{2})Tr\{A^\mu[P,V_\mu]_-\}$:

$$\begin{aligned} \mathcal{H}_{A_8VP} = g_{A_8VP}\Bigg\{ & \mathbf{A}_1\cdot\boldsymbol{\pi}\times\boldsymbol{\rho} \\ & + \frac{i}{2}\mathbf{A}_1\cdot\left[(K^\dagger\boldsymbol{\tau}K^*) - (K^{*\dagger}\boldsymbol{\tau}K)\right] \\ & - \frac{i}{2}\left(\left[(K^\dagger\boldsymbol{\tau}K_A)\cdot\boldsymbol{\rho} + (K_A^\dagger\boldsymbol{\tau}K^*)\cdot\boldsymbol{\pi}\right] - h.c.\right) \\ & - i\frac{\sqrt{3}}{2}\left(\left[(K^\dagger\cdot K_A)\phi_8 + (K_A^\dagger\cdot K^*)\eta_8\right] - h.c.\right) \\ & + \frac{i}{2}\sqrt{3}f_1\left[K^\dagger\cdot K^* - K^{*\dagger}\cdot K\right]\Bigg\} \end{aligned}$$

The relation with the pair-couplings of Appendix A is $g_{S_1PP}/\sqrt{3} = g_{(\pi\pi)_0}/m_\pi$, $g_{A_8VP} = g_{(\pi\rho)_1}/m_\pi$ etc.

[1] Th.A. Rijken, H. Polinder, and J. Nagata, *ESC NN-Potentials in Momentum Space. I. PS-PS Exchange Potentials* , preprint 2000, refered to as paper I.

[2] T.A. Rijken, R.A.M. Klomp, and J.J. de Swart, 'Soft-Core OBE-Potentials in Momentum Space', preprint June 1991, Institute for Theoretical Physics, Nijmegen, The Netherlands.

[3] Th.A. Rijken and V.G.J. Stoks, Phys. Rev. **54**, 2851 (1996).

[4] Th.A. Rijken and V.G.J. Stoks, Phys. Rev. **54**, 2869 (1996).

[5] M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. D**17**, 768 (1978).

[6] P.M.M. Maessen, T.A. Rijken, and J.J. de Swart, Phys. Rev. D**40**, 2226 (1989).

[7] T.A. Rijken, Ann. Phys. (NY) **164**, 1 and 23 (1985).

[8] Th.A. Rijken, *Proceedings of the XIVth European Conference on Few-Body Problems in Physics*, Amsterdam 1993, eds. B. Bakker and R. von Dantzig, Few-Body Systems, Suppl. **7**, 1 (1994).

[9] V.G.J. Stoks and Th.A. Rijken, Nucl. Phys. **A613**, (1997) 311.

[10] Th.A. Rijken, *Proceedings of the 1st Asian-Pacific Conference on Few-Body Problems in Physics*, Tokyo 1999, to be published.

[11] Th.A. Rijken, Ann. Phys. (N.Y.) **208**, 253 (1991).

[12] Th.A. Rijken, *Derivative Scalar-Pair Exchange Nucleon-nucleon Potentia*, report University ogf Nijmegen, 2001 (unpublished).
[13] P. Ko and S. Rudaz, Phys. Rev. **D 50** (1994) 6877.
[14] C. Ordóñez and U. van Kolck, Phys. Lett. B **291** (1992) 459; C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett.
[15] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. C**48** (1993) 792.
[16] D.V. Bugg and R.A. Bryan, Nucl. Phys. **A540** (1992) 449.
[17] R.A.M. Klomp, private communication (unpublisjed).
[18] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, and J.J. de Swart, Phys. Rev. C**49** (1994) 2950.
[19] R.C. Arnold, Phys. Rev. Lett. **14** (1965) 657; C. Schmid, Lettere al Nuov. Cim. **1** (1969) 165; H.J. Lipkin, Nucl. Phys. **B9** (1969) 349.

TABLE IV: The one-pair isospin factors $C^{(1)}(\alpha\beta)$ and momentum operators $\tilde{O}^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$. The index p labels the type of denominators. Note that $\kappa_1 = (f/g)_{(\pi\pi)_1}$.

$(\alpha\beta)$	$C^{(1)}(\alpha\beta)$	$O^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$
$(\pi\pi)_0$	6	$-\mathbf{k}_1\cdot\mathbf{k}_2 + \frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}_1\times\mathbf{k}_2)$
$(\sigma\sigma)$	2	1
$(\pi\eta)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$-2\mathbf{k}_1\cdot\mathbf{k}_2$
$(\pi\eta')$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$-2\mathbf{k}_1\cdot\mathbf{k}_2$
$(\pi\pi)_1$	$2i\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$i\left[\mathbf{k}_1\cdot\mathbf{k}_2 - \frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}_1\times\mathbf{k}_2)\right]$
		$+\dfrac{i}{M}\left[(1+\kappa_1)\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1\times\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1\times\mathbf{k}_2)\right.$
		$\left.+\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}_1\times\mathbf{k}_2)\,\mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)\right]$
$(\pi\rho)_1$	$-2i\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\dfrac{i}{M}\left[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1 + \frac{1}{2}(1+\kappa_\rho)\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2\right.\right.$
		$\left.\left.+\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1 - 2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\mathbf{k}_1\cdot\mathbf{k}_2\right)\right]$
$(\pi\sigma)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2 + \boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1 - 2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$
(πP)	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2 + \boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1 - 2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$
$(\pi\rho)_0$	3	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2 + \boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1 + 2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$
$(\pi\omega)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2 + \boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1 + 2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$

TABLE V: The one-pair denominators $D^{(1)}_p(\omega_1,\omega_2)$

$(\alpha\beta)$	$D^{(1)}_{ad}(\omega_1,\omega_2)$	$D^{(1)}_{na}(\omega_1,\omega_2)$	$D^{(1)}_{pv}(\omega_1,\omega_2)$
$(\pi\pi)_0$	$\dfrac{1}{\omega_1^2\omega_2^2}$	$\dfrac{1}{\omega_1^2\omega_2^2}\left[\dfrac{1}{\omega_1}+\dfrac{1}{\omega_2}-\dfrac{1}{\omega_1+\omega_2}\right]$	$\dfrac{1}{\omega_1\omega_2(\omega_1+\omega_2)}$
$(\pi\pi)_1$	$\dfrac{2}{\omega_1\omega_2(\omega_1+\omega_2)}\ ,\ \dfrac{1}{\omega_1^2\omega_2^2}$	$\dfrac{2}{\omega_1^2\omega_2^2}$	$\dfrac{1}{\omega_1^2}\ ,\ \dfrac{1}{\omega_2^2}$
$(\pi\sigma)$	$\dfrac{1}{\omega_1^2\omega_2^2}$	$\dfrac{1}{\omega_1^2\omega_2^2}\left[\dfrac{1}{\omega_1}+\dfrac{1}{\omega_2}-\dfrac{1}{\omega_1+\omega_2}\right]$	$\dfrac{1}{\omega_1\omega_2(\omega_1+\omega_2)}$

TABLE VI: The two-pair isospin factors $C^{(2)}(\alpha\beta)$ and momentum operators $\tilde{O}^{(2)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$, and denominators $D^{(2)}_p(\omega_1,\omega_2)$.

$(\alpha\beta)$	$C^{(2)}(\alpha\beta)$	$\tilde{O}^{(2)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$	$D^{(2)}_p(\omega_1,\omega_2)$
$(\pi\pi)_0$	6	1	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\sigma\sigma)$	2	1	,,
$(\pi\eta)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	1	,,
$(\pi\eta')$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	1	,,
$(\pi\pi)_1$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	1	$-\frac{1}{2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{4}{\omega_1+\omega_2}\right]$
$(\pi\rho)_1$	$2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\sigma)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1-\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_2)$	,,
(πP)	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1-\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_2)$	,,
$(\pi\rho)_0$	3	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$-\frac{1}{2}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)$
		$-\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1+\mathbf{k}_2)$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\omega)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$-\frac{1}{2}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)$
		$-\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1+\mathbf{k}_2)$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$

TABLE VII: The $d_p(t,u)$-functions cooresponding to the denominators $D_p(\omega_1,\omega_2)$, $p=0,1,2,3,4,5$.

$D_{\{p\}}(\omega_1,\omega_2)$			$d_{\{p\}}(t,u))$		
$D_{2,2,0}$	=	$\frac{1}{\omega_1^2\omega_2^2}$	$d_{2,2,0}$	=	1
$D_{2,0,0}$	=	$\frac{1}{\omega_1^2}$	$d_{2,0,0}$	=	$\delta(u-u_0)$
$D_{0,2,0}$	=	$\frac{1}{\omega_2^2}$	$d_{0,2,0}$	=	$\delta(t-t_0)$
$D_{1,0,0}$	=	$\frac{1}{\omega_1}$	$d_{1,0,0}$	=	$\frac{1}{\sqrt{\pi}}t^{-1/2}\ \delta(u-u_0)$
$D_{0,1,0}$	=	$\frac{1}{\omega_2}$	$d_{0,1,0}$	=	$\frac{1}{\sqrt{\pi}}u^{-1/2}\ \delta(t-t_0)$
$D_{0,0,1}$	=	$\frac{1}{\omega_1+\omega_2}$	$d_{0,0,1}$	=	$\frac{1}{2\sqrt{\pi}}(t+u)^{-3/2}$
$D_{1,1,1}$	=	$\frac{1}{\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$	$d_{1,1,1}$	=	$\frac{1}{\sqrt{\pi}}(t+u)^{-1/2}$

TABLE VIII: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\pi)_0$ contributions.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$
	1-pair exchange		
$\Omega_1^{(1),ad}$	$+\frac{3}{2}\frac{1}{t+u}$	$-\frac{tu}{(t+u)^2}$	—
$\Omega_1^{(1),na}$	$-\frac{1}{\sqrt{\pi}}\frac{15}{4}\frac{\sqrt{t+u}}{(t+u)^2}\cdot\frac{1}{M}$	$-\frac{1}{2\sqrt{\pi}}\left(\frac{t^2-8tu+u^2}{t+u}\right)\frac{\sqrt{t+u}}{(t+u)^2}\cdot\frac{1}{M}$	$-\frac{1}{\sqrt{\pi}}\left(\frac{t^2u^2}{(t+u)^2}\right)\frac{\sqrt{t+u}}{(t+u)^2}\cdot\frac{1}{M}$
$\Omega_4^{(1),na}$	$-\frac{1}{\sqrt{\pi}}\frac{\sqrt{t+u}}{t+u}\cdot\frac{1}{M}$	—	—
$\Omega_1^{(1),pv}$	$\frac{3}{2\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\cdot\frac{1}{M}$	$\frac{1}{2\sqrt{\pi}}\left(\frac{t^2+u^2}{t+u}\right)\frac{1}{(t+u)^{3/2}}\cdot\frac{1}{M}$	—
$\Omega_4^{(1),pv}$	$+\frac{3}{\sqrt{\pi}}\frac{1}{(t+u)^{1/2}}\cdot\frac{1}{M}$	—	—
	2-pair exchange		
$\Omega_1^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{1}{(t+u)^{1/2}}$	—	—

TABLE IX: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\pi)_1$ contributions.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$
	1-pair exchange		
$\Omega_1^{(1),ad}$	$+\frac{3}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{2}{\sqrt{\pi}}\frac{tu}{(t+u)^{5/2}}$	—
$\Omega_2^{(1),ad}$	—	$-\frac{1}{3\sqrt{\pi}}\frac{(1+\kappa_1)}{M}\frac{1}{t+u}$	—
$\Omega_3^{(1),ad}$	$+\frac{1}{2\sqrt{\pi}}\frac{(1+\kappa_1)}{M}\frac{1}{t+u}$	—	—
$\Omega_4^{(1),ad}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{M}\frac{1}{t+u}$	—	—
$\Omega_1^{(1),na}$	$-\frac{15}{4}\frac{1}{(t+u)^2}\cdot\frac{1}{M}$	$-\frac{1}{2}\left(\frac{t^2-8tu+u^2}{(t+u)^3}\right)\cdot\frac{1}{M}$	$-\frac{t^2u^2}{(t+u)^4}\cdot\frac{1}{M}$
$\Omega_4^{(1),na}$	$-\frac{1}{t+u}\cdot\frac{1}{M}$	—	—
$\Omega_1^{(1),pv}$	$+\frac{3}{4}\frac{\delta(t-t_0)+\delta(u-u_0)}{t+u}\cdot\frac{1}{M}$	$+\frac{1}{2}\frac{t^2\delta(t-t_0)+u^2\delta(u-u_0)}{(t+u)^2}\cdot\frac{1}{M}$	—
$\Omega_1^{(4),pv}$	$+\frac{t\;\delta(t-t_0)+u\;\delta(u-u_0)}{t+u}\cdot\frac{1}{M}$	—	—
	2-pair exchange		
$\Omega_1^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\left[\frac{\delta(u-u_0)}{\sqrt{t}}+\frac{\delta(t-t_0)}{\sqrt{u}}-\frac{2}{(t+u)^{3/2}}\right]$		—

TABLE X: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\rho)_1$ contributions.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$
		1-pair exchange	
$\Omega_2^{(1),ad}$	$+\frac{1}{M}\left(\frac{3}{2}+\kappa_\rho\right)\frac{1}{t+u}$	$\frac{1}{3M}\left(\frac{u^2}{(t+u)^2}-2(1+\kappa_\rho)\frac{tu}{(t+u)^2}\right)$	—
$\Omega_3^{(1),ad}$	$+\frac{1}{M}\frac{u^2+tu\,(1+\kappa_\rho)}{(t+u)^2}$	—	—
		2-pair exchange	
$\Omega_2^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{1}{(t+u)^{1/2}}$	—	—-

TABLE XI: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\sigma)_1$ contributions.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$
		1-pair exchange	
$\Omega_2^{(1),ad}$	$-\dfrac{2}{t+u}$	$+\dfrac{2}{3}\dfrac{tu-u^2}{(t+u)^2}$	—
$\Omega_3^{(1),ad}$	$+2\dfrac{tu-u^2}{(t+u)^2}$	—	—
$\Omega_2^{(1),na}$	$+\dfrac{5}{\sqrt{\pi}}\dfrac{1}{(t+u)^{3/2}}\cdot\dfrac{1}{M}$	$+\dfrac{1}{3\sqrt{\pi}}\dfrac{t^2-13tu+6u^2}{(t+u)^{5/2}}\cdot\dfrac{1}{M}$	$+\dfrac{2}{3\sqrt{\pi}}\dfrac{tu^2(t-u)}{(t+u)^{7/2}}\cdot\dfrac{1}{M}$
$\Omega_3^{(1),na}$	$+\dfrac{1}{\sqrt{\pi}}\dfrac{t^2-7tu+6u^2}{(t+u)^{5/2}}\cdot\dfrac{1}{M}$	$+\dfrac{2}{\sqrt{\pi}}\dfrac{tu^2(t-u)}{(t+u)^{7/2}}\cdot\dfrac{1}{M}$	—
$\Omega_2^{(1),pv}$	$-\dfrac{1}{\sqrt{\pi}}\dfrac{1}{(t+u)^{3/2}}\cdot\dfrac{1}{M}$	$-\dfrac{1}{3\sqrt{\pi}}\dfrac{t^2-tu}{(t+u)^{5/2}}\cdot\dfrac{1}{M}$	—
$\Omega_3^{(1),pv}$	$-\dfrac{1}{\sqrt{\pi}}\dfrac{t^2-tu}{(t+u)^{5/2}}\cdot\dfrac{1}{M}$	—	—
		2-pair exchange	
$\Omega_2^{(2),ad}$	$-\dfrac{1}{\sqrt{\pi}}\dfrac{1}{(t+u)^{3/2}}$	$-\dfrac{1}{6\sqrt{\pi}}\dfrac{(t-u)^2}{(t+u)^{5/2}}$	—
—	—-		
$\Omega_3^{(2),ad}$	$-\dfrac{1}{2\sqrt{\pi}}\dfrac{(t-u)^2}{(t+u)^{5/2}}$	—	—-

TABLE XII: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\omega)_1$ contributions.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$
	1-pair exchange		
$\Omega_2^{(1),ad}$	—	$+\frac{2}{3}\frac{u}{t+u}$	—
$\Omega_3^{(1),ad}$	$+2\frac{u}{t+u}$	—	—
	2-pair exchange		
$\Omega_2^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\left[\frac{\delta(t-t_0)}{\sqrt{u}}+\frac{\delta(u-u_0)}{\sqrt{t}}\right]$	$+\frac{1}{6\sqrt{\pi}}\frac{1}{\sqrt{t+u}}$	—
$\Omega_3^{(2),ad}$	$+\frac{1}{2\sqrt{\pi}}\frac{1}{\sqrt{t+u}}$	—	—

TABLE XIII: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi P)_1$ contributions. These coefficients have to be multiplied by a factor $-\delta(u-u_0)/M_N^2$.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$
	1-pair exchange		
$\Omega_2^{(1),ad}$	$-\frac{2}{t+u}$	$+\frac{2}{3}\frac{tu-u^2}{(t+u)^2}$	—
$\Omega_3^{(1),ad}$	$+2\frac{tu-u^2}{(t+u)^2}$	—	—
$\Omega_2^{(1),na}$	$+\frac{5}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\cdot\frac{1}{M}$	$+\frac{1}{3\sqrt{\pi}}\frac{t^2-13tu+6u^2}{(t+u)^{5/2}}\cdot\frac{1}{M}$	$+\frac{2}{3\sqrt{\pi}}\frac{tu^2(t-u)}{(t+u)^{7/2}}\cdot\frac{1}{M}$
$\Omega_3^{(1),na}$	$+\frac{1}{\sqrt{\pi}}\frac{t^2-7tu+6u^2}{(t+u)^{5/2}}\cdot\frac{1}{M}$	$+\frac{2}{\sqrt{\pi}}\frac{tu^2(t-u)}{(t+u)^{7/2}}\cdot\frac{1}{M}$	—
$\Omega_2^{(1),pv}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\cdot\frac{1}{M}$	$-\frac{1}{3\sqrt{\pi}}\frac{t^2-tu}{(t+u)^{5/2}}\cdot\frac{1}{M}$	—
$\Omega_3^{(1),pv}$	$-\frac{1}{\sqrt{\pi}}\frac{t^2-tu}{(t+u)^{5/2}}\cdot\frac{1}{M}$	—	—
	2-pair exchange		
$\Omega_2^{(2),ad}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{1}{6\sqrt{\pi}}\frac{(t-u)^2}{(t+u)^{5/2}}$	—
—	—		
$\Omega_3^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{(t-u)^2}{(t+u)^{5/2}}$	—	—

TABLE XIV: Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\pi)_0('derivative')$ contributions.

	$\Upsilon_0(//)(t,u)$	$\Upsilon_1(//)(t,u)$	$\Upsilon_2(//)(t,u)$	$\Upsilon_3(//)(t,u)$
$\Omega_1^{(1),ad}$	$-\frac{15}{4}\frac{1}{t+u}$	$-\frac{1}{2}\frac{t^2-8tu+u^2}{(t+u)^3}$	$-\frac{t^2u^2}{(t+u)^4}$	—
$\Omega_1^{(1),na}$	$-\frac{45}{2\sqrt{\pi}}\frac{1}{(t+u)^{5/2}}\cdot\frac{1}{M}$	$+\frac{1}{2\sqrt{\pi}}\frac{14t^2-67tu+14u^2}{(t+u)^{7/2}}\cdot\frac{1}{M}$	$-\frac{tu}{\sqrt{\pi}}\frac{3t^2-14tu+3u^2}{(t+u)^{9/2}}\cdot\frac{1}{M}$	$-\frac{1}{\sqrt{\pi}}\frac{t^3u^3}{(t+u)^{1/2}}\cdot\frac{1}{M}$
$\Omega_4^{(1),na}$	$+\frac{9}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\cdot\frac{1}{M}$	$-\frac{4}{\sqrt{\pi}}\frac{tu}{(t+u)^{5/2}}\cdot\frac{1}{M}$	—	—
$\Omega_1^{(1),pv}$	$-\frac{1}{2\sqrt{\pi}}\frac{(m_1^2-m_2^2)^2}{(t+u)^{1/2}}\cdot\frac{1}{M}$	$\frac{1}{2\sqrt{\pi}}\frac{m_1^2(3t^2-u^2)-m_2^2(t^2-3u^2)}{(t+u)^{5/2}}\cdot\frac{1}{M}$	$\frac{1}{\sqrt{\pi}}\frac{tu(t^2+2tu+u^2)}{(t+u)^{9/2}}\cdot\frac{1}{M}$	—
$\Omega_4^{(1),pv}$	$+\frac{3}{2\sqrt{\pi}}\frac{m_1^2+m_2^2}{(t+u)^{3/2}}\cdot\frac{1}{M}$ $-\frac{2}{\sqrt{\pi}}\frac{(m_1^2+m_2^2)}{(t+u)^{1/2}}\cdot\frac{1}{M}$ $-\frac{3}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\cdot\frac{1}{M}$	$+\frac{1}{2\sqrt{\pi}}\frac{t^2+2tu+u^2}{(t+u)^{7/2}}\cdot\frac{1}{M}$ $-\frac{2}{\sqrt{\pi}}\frac{(t^2+tu+u^2)}{(t+u)^{5/2}}\cdot\frac{1}{M}$	—	—
$\Omega_1^{(2),ad}$	$+\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{1}{2\sqrt{\pi}}\frac{tu}{(t+u)^{5/2}}$	—	—